Entanglement creation using quantum interrogation

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We present some applications of high-efficiency quantum interrogation ("interaction-free measurement") for the creation of entangled states of separate atoms and of separate photons. The quantum interrogation of a quantum object in a superposition of object-in and object-out leaves the object and probe in an entangled state. The probe can then be further entangled with other objects in subsequent quantum interrogations. By then projecting out those cases in which the probe is left in a particular final state, the quantum objects can themselves be left in various entangled states. In this way, we show how to generate two-, three-, and higher-qubit entanglement between atoms and between photons. The effect of finite efficiency for the quantum interrogation is delineated for the various schemes.

I. INTRODUCTION

Quantum-information processing is currently receiving considerable attention [1,2], with significant effort focused on finding applications. Known applications include quantum computation [3,4], quantum communication [5], quantum cryptography [6–9], quantum teleportation [10–12], quantum dense coding [13], and high-precision measurements [14,15]. At the heart of many of these applications is entanglement, which is generally thought to be one of the key resources required in quantum-information processing. The characterization of entangled states and entanglement is a challenging problem and a considerable theoretical effort has been invested in characterizing entanglement in a variety of physical situations [15–24]. Likewise, there has been considerable experimental effort in developing techniques for creating highly entangled resources (e.g., entangled photons [25] or ions [26]), including the ability to produce arbitrary entangled states [27,28].

In this paper, we propose several schemes using quantum interrogation (QI) to generate entanglement between the states of separate particles (see *Note added*), expanding on a suggestion in Ref. [29]. The technique of quantum interrogation (also known as "interaction-free measurement") has its roots in "negative results" measurements originally discussed by Renninger [30], and later by Dicke [31], who analyzed the change in an atom's wave function by the *nonscattering* of a photon from it. In 1993, Elitzur and Vaidman (EV) proposed a particularly dramatic version where a photon was used to ascertain the presence of a light-sensitive bomb without the bomb exploding, hence seemingly without interacting with it [32]. The EV scheme works with an efficiency of at best 50%, i.e., at most 50% of the measurements are "interaction free." High-efficiency schemes making use of the quantum Zeno effect [33] were proposed by Kwiat

et al. [34] and achieved an efficiency of 74%. An alternative scheme using high finesse resonators was introduced by [35] and achieved a comparable efficiency. The above efficiency values take into account other losses that we will not consider, so to avoid confusion we will characterize out figures of merit against the number of cycles in a QI.

Consider an idealized high-efficiency quantum interrogation scheme, of the type presented in [34], in the limit of perfect efficiency. We shall take the absorbing object to be a quantum device that can be in one of two states: $|0\rangle_a$ representing object-out, i.e., a completely transparent object; and $|1\rangle_a$ representing object-in, i.e., a completely absorbing object. We shall probe the state of the object using a photon which can be in one of the two states $|0\rangle_p$ or $|1\rangle_p$, which can be represented schematically as two ports to the device as in Fig. 1(a). The two states of the photon could be, for example, different polarization states as in Fig. 1(b) (figure taken from Kwiat *et al.* [34]) or different spatial modes.

Quantum interrogation functions in the following way: with the object-out, a probing photon initially in state $|0\rangle_p$ or $|1\rangle_p$ remains unchanged and exits the device in the same state [as in Fig. 1(b) with the addition of a 90° polarization rotation at the end]. With the absorbing object in state $|1\rangle_a$ (object-in), then a photon initially in state $|0\rangle_p$ will evolve to state $|1\rangle_p$ without changing the state of the object (an "interaction-free measurement"). If we probe the object with a photon initially in state $|1\rangle_p$, the photon will certainly get absorbed by the object—this event was dramatized as a bomb exploding in the EV scheme. Note that the convention we have chosen here, where the photon changes state when the absorbing object is in, is opposite from the usual presentation of quantum interrogation. We have used this convention so that the logic structure of the operation is more apparent.

With this representation, the behavior of the quantum interrogation is tantalizingly close to the operation of a controlled-NOT (CNOT) gate. That is, we have the mapping Q:

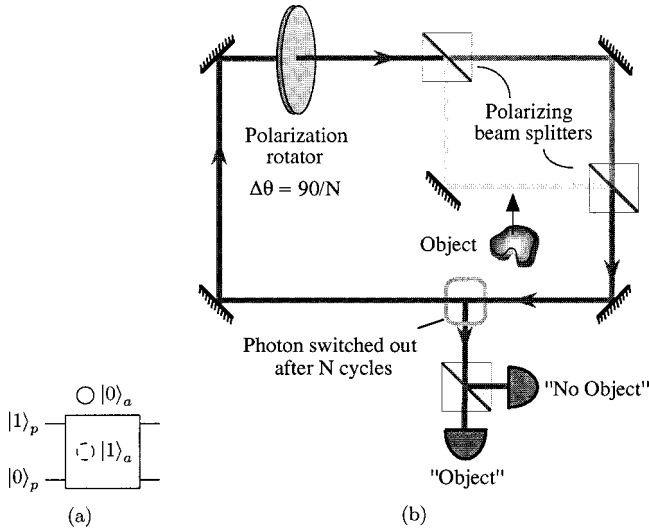*Electronic address: alexei@physics.uq.edu.au

FIG. 1. (a) An idealized quantum interrogation and the labeling of logical qubits. $|0\rangle_p$ and $|1\rangle_p$ are the logical states of the probe particle and $|0\rangle_a$ and $|1\rangle_a$ are the logical states of the object. (b) An optical implementation of high-efficiency quantum interrogation. The probe particle is a photon for which horizontal and vertical polarization represent the target qubit state and the presence or absence of an absorbing object represents the control qubit state (after Kwiat *et al.* [34]).

$$Q: \quad \begin{aligned} |00\rangle &\rightarrow |00\rangle \\ |01\rangle &\rightarrow |01\rangle \\ |10\rangle &\rightarrow |11\rangle \\ |11\rangle &\rightarrow |\text{boom!}\rangle, \end{aligned} \qquad (1)$$

where the first mode represents the state of the object and the second the state of the photon. We could equally have flipped the interpretation of the two ports so that with the object-in we would have $|11\rangle \rightarrow |10\rangle$ and $|10\rangle \rightarrow |\text{boom!}\rangle$. We shall represent this alternative map as Q_r. It should be noted that since only a single combination of the terms in the map (1) fails, if we can detect the failure event (detecting the bomb exploding) then we could in principle recreate the appropriate state. We shall, however, assume that this is not possible for the purposes of this paper.

Despite not having access to the full logic table for a CNOT gate, the device proves remarkably useful as can be seen from some of the quantum circuits that can be constructed using it depicted in Fig. 2. There are three principal obstructions to performing these ideal circuits.

(i) The effect of finite efficiency in the quantum interrogation scheme.

(ii) The potential inability to switch the roles of the control and target. For instance, it is much easier to have an interferometer using photons (the target) and a suitable atom as the quantum object (the control) than to have an atom interferometer repeatedly probing the state of a single photon.

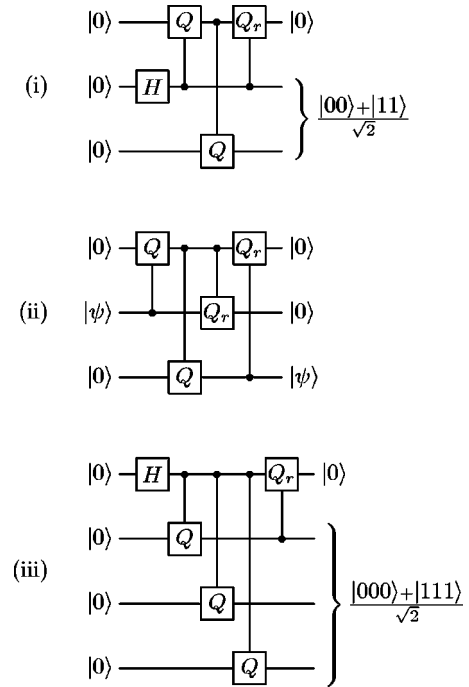(iii) The effect of a semitransparent object; see, for instance, Refs. [36,37].

FIG. 2. Several quantum circuits constructed using a perfect efficiency quantum interrogation measurement, Q and Q_r, which are explained in the text. H is a Hadamard gate. Circuit (i) creates Bell states. Circuit (ii) is a quantum bus, which swaps a qubit from one channel to another. Circuit (iii) creates a GHZ state.

In this paper, we shall examine the first two issues and leave the third for a subsequent work. In the schemes that follow, we shall restrict ourselves to using the state of some atom as the control qubit and the state of a photon as the target qubit. In Sec. II, we present a simple model of a quantum interrogation measurement of a specific quantum object. In Sec. III, we propose three conditional schemes to generate Bell-, W-, and GHZ-type entanglement in the state of two and three atoms using photons as mediators. In Sec. IV, we propose using an atom to generate Bell- and GHZ-type entanglement between separate photons.

II. THE MODEL

We can represent the quantum interrogation apparatus as a series of N Mach-Zender interferometers laid end on end as in Fig. 3, where it is understood that the absorbing object labeled $\hat{A}$ in the figure is the same object each time. This is
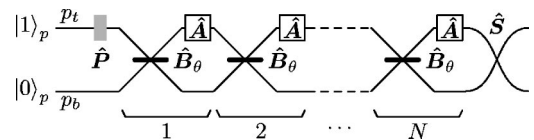

FIG. 3. Schematic representation of a high-efficiency quantum interrogation. A single photon probes the state of an atom $\hat{A}$ through repeated passes through a Mach-Zender interferometer. The flow of time is to the right. Note that there is an initial 180° phase shift $\hat{P}$ applied to the top arm and a final interchange of the modes in order to achieve a more convenient logic structure.
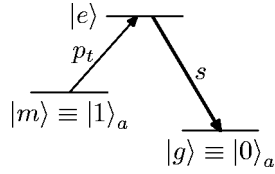
FIG. 4. Model of the interaction with the atom and the labeling of the logical basis. The levels $|m\rangle$, $|e\rangle$, and $|g\rangle$ are metastable, excited, and ground states, respectively. A photon in the top mode of the quantum interrogation (p_t) can induce a coherent evolution between states $|m\rangle$ and $|e\rangle$. State $|e\rangle$ experiences rapid decay to the ground state, releasing a scattered photon s.

equivalent to the experimental arrangement in Fig. 1(b). We shall label the light modes above and below the beam splitters as modes p_t and p_b, respectively. Thus a photon in the top mode ($|1\rangle_{p_t}|0\rangle_{p_b}$) will be used to code a logical $|1\rangle_p$ for the photon qubit, and a photon in the bottom mode ($|0\rangle_{p_t}|1\rangle_{p_b}$) will code a logical $|0\rangle_p$ for the photon qubit.

We shall take as our model of the absorbing object a three-level atom, similar to that introduced in [38], depicted in Fig. 4. The atom can start in a metastable state $|m\rangle$, from which it will absorb a photon from mode p_t with unit efficiency. After absorbing a photon, the atom immediately decays from the excited state $|e\rangle$ to the ground state $|g\rangle$, which is far off-resonance from the metastable state. We are able to neglect the reabsorption of the emitted photon so this forms an essentially irreversible process. We can then label the metastable state as our logical $|1\rangle_a$ (object-in) for the atom qubit. The atom in its ground state is transparent to the p_t photons, and so we can label the ground state as our logical $|0\rangle_a$ (object-out) for the atom qubit. Note that filtering off the higher-frequency scattered photons removes the problems of forward scattering [37].

In what follows, the atom is always considered to be the control qubit, and the photon the target qubit, and we shall always write them in that order. We shall use the subscripts p and a to denote photon and atom only if necessary.

The effects of the atom and beam splitters on the modes (in the logical basis) are then

$$\hat{A}: \begin{cases} |1\rangle_a|0\rangle_p \rightarrow |1\rangle_a|0\rangle_p, \\ |1\rangle_a|1\rangle_p \rightarrow |0\rangle_a|s\rangle_p, \end{cases} \tag{2}$$

$$\hat{B}_\theta: \begin{cases} |0\rangle_p \rightarrow \cos\theta|0\rangle_p + \sin\theta|1\rangle_p, \\ |1\rangle_p \rightarrow \cos\theta|1\rangle_p - \sin\theta|0\rangle_p, \end{cases} \tag{3}$$

where the reflectivity $R = \cos^2\theta$, and $|s\rangle_p$ represents a scattered photon. Note that a photon being absorbed and scattered by the atom removes the system from the logical basis (there will be no photon in either p_t or p_b) and in writing the state $|s\rangle$ we are using a convenient shorthand to denote this event.

After N cycles within the quantum interrogation, with the atom and photon initially in state $|\phi_0\rangle$, we will evolve to the state

$$|\phi_N\rangle = \hat{S}\hat{A}_N\hat{B}\hat{A}_{N-1}\cdots\hat{B}\hat{P}|\phi_0\rangle, \tag{4}$$

where $\hat{P}$ is a 180° phase shift and $\hat{S}$ represents the final swap of the modes—these operations are done to achieve a more traditional logic structure.

With the atom in state $|0\rangle_a$ (object-out) after N cycles we have

$$|0\rangle_p \rightarrow \cos(N\theta)|1\rangle_p + \sin(N\theta)|0\rangle_p, \tag{5}$$

$$|1\rangle_p \rightarrow -\cos(N\theta)|0\rangle_p + \sin(N\theta)|1\rangle_p. \tag{6}$$

We choose $\theta = \pi/2N$ so that $|0\rangle_p \rightarrow |0\rangle_p$ and similarly $|1\rangle_p \rightarrow |1\rangle_p$.

Now consider the atom initially in the state $|1\rangle_a$ (object-in). After N cycles, Eq. (4) yields

$$|1\rangle_a|0\rangle_p \rightarrow \cos^N\theta|11\rangle + \sin\theta\sum_{j=0}^{N-1}\cos^j\theta|0s_j\rangle, \tag{7}$$

$$|1\rangle_a|1\rangle_p \rightarrow \sin\theta\cos^{N-1}\theta|11\rangle - \cos\theta|0s'\rangle$$
$$+ \sin^2\theta\sum_{j=0}^{N-2}\cos^j\theta|0s_j\rangle, \tag{8}$$

where we have dropped the subscripts for the kets on the right.

Although in the cases in which the photon is scattered the control qubit is changed, in the limit of high-efficiency QI ($N\rightarrow\infty$), all the terms with a $\sin\theta$ disappear and Eqs. (5)–(8) show the psuedo-CNOT logic given in Eq. (1).

III. ATOM ENTANGLEMENT PREPARATION

In this section, we present schemes for generating several types of entangled states between atoms of the type described in Sec. II using photons as a mediating particle. These schemes allow the entanglement of separated atoms without ever bringing them into direct interaction with each other. All the schemes are nondeterministic in that they will work only a certain percentage of the time, when a specific result is obtained upon measuring the photon. This is a limitation that is common to many entanglement generation schemes. There is an added advantage in using a conditioned scheme in our case. Detecting the final state of the photon in either $|0\rangle_p$ (photon in mode p_b) or in $|1\rangle_p$ (photon in mode p_t) means we condition out those cases in which the atom absorbs a photon since the photon will be removed from both modes of the interferometer. This guarantees that we generate a pure, entangled state.

To characterize the success of a scheme in generating a particular entangled state, we shall use the fidelity [43] F, which is simply

$$F = |\langle\psi_{\text{desired}}|\psi_{\text{actual}}\rangle|, \tag{9}$$

and the tangle τ, which is the square of the concurrence [17] from which the entanglement of formation can be calculated. For a mixed state ρ of two qubits, the concurrence C is given by

$$C = \max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0), \tag{10}$$
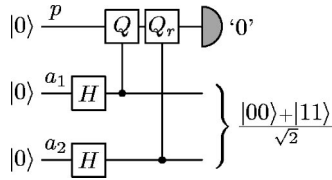
FIG. 5. Nondeterministic generation of the Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$. Two atoms are initially prepared in superposition states by Hadamard transformations. A photon is then used to probe each atom in turn using quantum interrogation. In the subensemble of cases in which the final state of the photon is measured to be $|0\rangle_p$ (mode p_t), the atoms have been left in the required Bell state.

where the λ_i are the square roots of the eigenvalues, in decreasing order, of $\rho\tilde{\rho} = \rho\sigma_y^A \otimes \sigma_y^B \rho^* \sigma_y^A \otimes \sigma_y^B$, and ρ^* denotes the complex conjugation of ρ in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

The tangle is valid for two qubits; for three qubits in a pure state we will use the 3-tangle, τ_3 [39], which gives the purely three-way entanglement of the system,

$$\tau_3 = \tau_{A(BC)} - \tau_{AB} - \tau_{AC}, \tag{11}$$

where τ_{AB} and τ_{AC} are the tangle between the resulting systems when qubits C and B are traced out, respectively, and $\tau_{A(BC)}$ is calculated from $\tau_{A(BC)} = 4 \det \rho_A$, which is valid when the state of ABC is pure. ρ_A is the reduced density matrix of qubit A alone.

The 3-tangle can be understood loosely to embody the amount of entanglement of qubit A with qubits B and C over and above the amount of entanglement of qubit A with B and of A with C.

Consider the scheme depicted in Fig. 5. Two atoms are initially placed into in a superposition state. A photon makes a QI of the first atom, and is then used to make another QI of the second atom, where upon it is measured in the state $|0\rangle_p$ (i.e., exiting in mode p_b of the last QI). In the limit of high-efficiency QI, the two atoms will be left in a maximally entangled Bell state.

If initially we let the atoms be in arbitrary superposition states, i.e., we have

$$|\psi_0\rangle = (\alpha_1 |0\rangle_{a_1} + \beta_1 |1\rangle_{a_1})(\alpha_2 |0\rangle_{a_2} + \beta_2 |1\rangle_{a_2})|0\rangle_p, \tag{12}$$

then after N cycles within each QI, the final state of the system *conditioned* on a successful measurement of the state $|0\rangle_p$ is

$$|\psi_N\rangle = \mathcal{N}\{\alpha_1\alpha_2|00\rangle + \beta_1\beta_2 c^{2N}|11\rangle + sc^{N-1}\alpha_1\beta_2|01\rangle\}, \tag{13}$$

where $c = \cos\theta$, $s = \sin\theta$, and the normalization $\mathcal{N}$ is determined by the requirement that $\langle\psi_N|\psi_N\rangle = 1$ after the state is conditioned on a successful measurement.

In Fig. 6, we plot the probability of successful operation P, the fidelity F, and the tangle τ against the number of cycles in each QI for generating the Bell state $(|00\rangle$
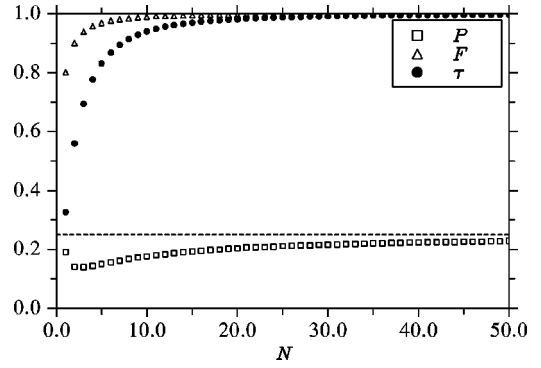

FIG. 6. The conditional generation of a Bell state, following the scheme in Fig. 5. Plotted as a function of the number of cycles N through each QI is (i) the probability of successful operation P, which has a limiting value of $\frac{1}{4}$ (dashed line), (ii) the fidelity against the desired Bell state F, and (iii) the tangle τ of the output state.

$+ |11\rangle)/\sqrt{2}$, with $\alpha_{1,2} = \beta_{1,2} = 1/\sqrt{2}$. We can access the other Bell states $[(|01\rangle \pm |10\rangle)/\sqrt{2}]$ by either swapping the second quantum interrogation from Q to Q_r and conditioning on the detection of $|1\rangle_p$, which amounts to swapping the ports of one of the quantum interrogations, or by using local operations on the final state. We can therefore tune our device to produce a desired type of entanglement.

We can extend the technique to three atoms, and generate an entangled three-qubit state. We will present two schemes to generate two types of three-qubit entanglement, which are inequivalent under local operations and classical communication (LOCC) [40].

First, we will examine the scheme in Fig. 7 for generating the W entangled state, $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$. With three atoms initially in superpositions, the photon probes each atom in turn with a QI before being detected in the state $|1\rangle_p$.

The $|W\rangle$ state has only pairwise entanglement, so we plot the tangle between pairs of qubits in Fig. 8 together with the probability of success and the fidelity. For an ideal $|W\rangle$ state, the tangle between pairs of qubits is $\tau = \frac{4}{9}$.

If each atom starts in an arbitrary superposition of $\alpha_j |0\rangle_{a_j} + \beta_j |1\rangle_{a_j}$, where j indexes the atoms, then after N cycles in each QI following the scheme in Fig. 7, we obtain
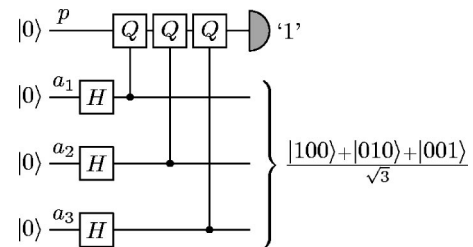

FIG. 7. A nondeterministic preparation of the W state using an auxiliary mode. Atoms initially prepared in superposition states are probed in turn using QI. In the cases in which the final state of the photon is $|1\rangle_p$, the atoms have been left in a W entangled state.
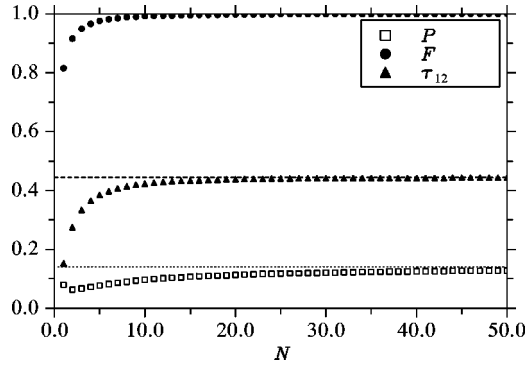
FIG. 8. The conditional generation of a W state following the scheme in Fig. 7. Plotted as a function of the number cycles N through each QI is (i) the probability of successful operation P, with a limiting value of 9/64 shown as a dotted line; (ii) the fidelity against the target W state F; and (iii) the tangle between pairs of qubits τ (all pairs have equal tangle). The theoretical limiting value of 4/9 is shown as a dashed line.

$$\begin{aligned}|\psi_N\rangle = \mathcal{N}\{c^N(&\beta_1\alpha_2\alpha_3|100\rangle + \alpha_1\beta_2\alpha_3|010\rangle \\ &+ \alpha_1\alpha_2\beta_3|001\rangle)sc^{2N-1}(\beta_1\beta_2\alpha_3|110\rangle \\ &+ \beta_1\alpha_2\beta_3|101\rangle + \alpha_1\beta_2\beta_3|011\rangle) \\ &+ s^2c^{3N-2}\beta_1\beta_2\beta_3|111\rangle\}.\end{aligned} \qquad (14)$$

In Fig. 8 are plotted various performance parameters against N for generating the $|W\rangle$ state starting with a symmetric superposition in each atom.

As before, we can access other W states either by changing a Q to a Q_r and conditioning on a $|1\rangle_p$, or by using local operations on the final state. By extending the circuit in Fig. 7 in the obvious way to more modes, we can create higher-order W states such as $(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle)/2$.

Finally, we can use this technique to induce a GHZ state in three separated atoms by using two auxiliary photons as depicted in Fig. 9. Here, with the atoms prepared in superposition states, the first photon probes atoms one and two in

turn, and the second photon probes atoms two and three in turn, before both photons are detected in the joint state $|00\rangle_{p_1 p_2}$.

With the atoms each initially in the arbitrary superposition states $\alpha_j|0\rangle_{a_j} + \beta_j|1\rangle_{a_j}$, where j indexes the atoms, then after N cycles in each QI we get

$$\begin{aligned}|\psi_N\rangle = \mathcal{N}\{&\alpha_1\alpha_2\alpha_3|000\rangle + c^{4N}\beta_1\beta_2\beta_3|111\rangle \\ &+ sc^{N-1}\alpha_1\alpha_2\beta_3|001\rangle + sc^{3N-1}\alpha_1\beta_2\beta_3|011\rangle\}.\end{aligned}$$

In Fig. 10, we characterize the success of generating the state $(|000\rangle + |111\rangle)/\sqrt{2}$ with the three-way tangle τ_3, and the fidelity F, for atoms initially in equal superposition states.

It should be noted that the circuit in Fig. 5 is embedded within the circuit in Fig. 9 and in fact the construction can be extended recursively to generate states of the form $(|0000\rangle + |1111\rangle)/\sqrt{2}$ and higher. Also, as in the previous cases, we can access other GHZ states.

It should be emphasized that in the three schemes presented in this section, the postselection ensures that the final states are pure states, as it selects specifically the cases in which incoherent evolution has not occurred.

FIG. 10. The conditional generation of a GHZ state following the scheme in Fig. 9. Plotted as a function of the number cycles N through each QI is (i) the probability of successful operation P, which has a limiting value of 2/64, (ii) the fidelity against the target Bell state F, and (iii) the 3-tangle τ_3 of the output state.

IV. PHOTON ENTANGLEMENT PREPARATION

In the preceding section, we used a photon to entangle separate atoms. In this section, we will present a scheme to use an atom to entangle independent photons. With an atom
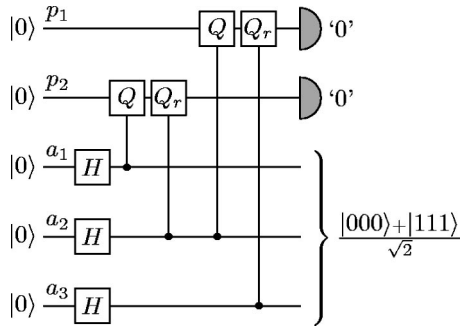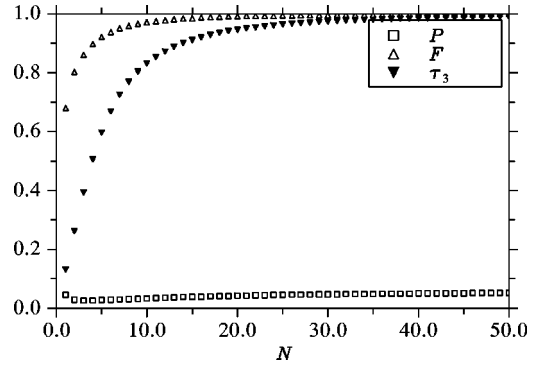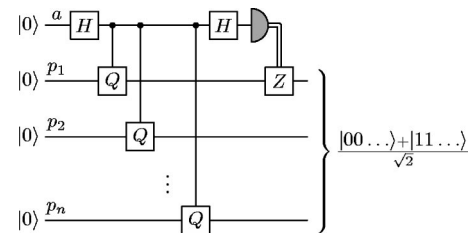

FIG. 9. A nondeterministic preparation of the GHZ state using two auxiliary modes. After three atoms have been prepared in superposition states, a photon probes atoms a_1 and a_2 using QI. Another photon probes atoms a_2 and a_3 also using QI. In the cases in which the final state of both photons is $|00\rangle_{p_1 p_2}$, the atoms are left in the GHZ state shown.

FIG. 11. Using a measurement with a classically conditioned Z gate (Pauli σ_z) to replace one of the quantum interrogations.
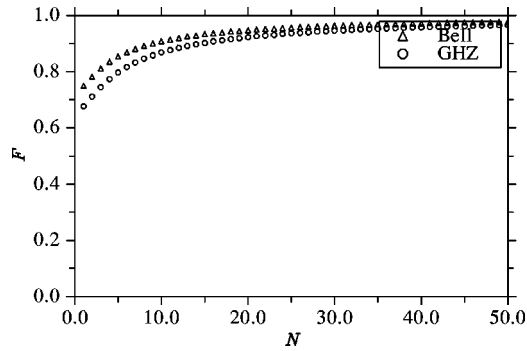
FIG. 12. Fidelity of output state for the circuits in Fig. 11 to generate $|00\rangle+|11\rangle$ and $|000\rangle+|111\rangle$, when compared against those states.

in a superposition state, we probe its state using n photons, in n consecutive QI's as in Fig. 11. Measurement of the final state of the atom can be used to classically condition a gate (a Pauli σ_z transformation) on one of the photons.

An advantage of this scheme is that in the ideal quantum interrogation limit, it works deterministically—it is not conditioned on the detection of a particular result.

For this scheme and for finite N we have a more limited group of measures of how close we are to the ideal scheme. Whereas in the previous, atom entangling, schemes the postselection ensured the final states would be pure, this is not the case for the photon entangling scheme. This means that not only will we end up with mixed states if we trace over the environment, but some of those states will be outside our logical basis (for instance, the case in which there are no photons in either the top or the bottom mode). For this reason, we shall only plot the fidelity against the desired state (in the ideal case in which there are no absorptions). In Fig. 12, we plot the fidelity of the output state compared with the desired state, for circuits to generate a Bell state $((|00\rangle+|11\rangle)/\sqrt2)$ and a GHZ state $((|000\rangle+|111\rangle)/\sqrt2)$. Although the convergence is not as rapid as for the atom entangling schemes, we still approach the desired state in relatively few cycles.

V. CONCLUSIONS

To conclude, in this paper we have described how high-efficiency quantum interrogation can be used to generate entangled particles. The protocols provide a mechanism by which two or more atoms can be entangled via a mediating photon (the photon can be thought of acting as a bus) or alternatively how two or more photons can be entangled via a mediating atom.

Although we have required strong interaction between the atom and the light in our schemes, the quantum interrogation ensures that the photon is not absorbed by the atom in the high-efficiency limit. Indeed, the requirements on the interaction are sufficiently general that it may be possible to realize such an interaction in a system quite different from a single atom such as a quantum dot.

The attractive aspects of the proposal are that the entanglement is created without making use of prior entangled states, the entanglement is tunable (i.e., using the same apparatus allows you to set the degree and type of entanglement, including accessing different classes of higher-order entanglement), and for the atoms the entanglement is achieved *in situ*, without needing to bring the atoms in proximity to each other.

Although the scheme presented here is idealized (perfect optical elements and no losses), a high degree of entanglement is achieved in remarkably few cycles in the quantum interrogation, leading to a hope that in real applications, entanglement by these schemes may be achievable with current technology.

Note added. Alternative approaches, not involving quantum interrogation, have been proposed for entangling separated atoms [41] and separated atomic ensembles [42].

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[43] It is also possible to define the fidelity as $F = |\langle \psi_{\text{desired}} | \psi_{\text{actual}} \rangle|^2$, and both definitions are used in the literature. Here we have chosen to follow the convention in [1].