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Optical mesons

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An ''optical meson''(two-photon quantum soliton) is proven to exist in a parametric waveguide. This could provide an ideal quantum soliton environment, because of more realistic formation lengths and much larger binding energies than $\chi^{(3)}$ quantum solitons. We estimate the binding energy, radius, and interaction length in comparison to the $\chi^{(3)}$ case in optical fibers. [S1050-2947(97)51008-0]

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Quantum solitons can be regarded as the most fundamental objects in quantum-field theory. These quantum structures, bound together solely through the self-interactions of the quantum fields, have been an area of many theoretical investigations since the early work of Lieb [1], McGuire [2], Yang [3], Coleman [4], and Lee [5], chiefly in the area of bosonic quantum solitons. The simplest object of this type has just two bosons bound together. It is found, for example, in the well-known case of a one-dimensional Bose gas with an attractive δ-function interaction. Very simple quantum objects of this type, if available experimentally, would provide an ideal testing ground for some well-known concepts of particle formation in quantum fields. However, to date it has not proved possible to directly observe the existence of elementary quantum bound states of this type, due to the relatively low nonlinearities found in practice.

Instead, the prediction [6] and observation [7] of quantum effects in fiber-optical solitons has resulted from the realization that quantum soliton properties may be tested experimentally in laser physics, using states that are superpositions of different boson numbers. This led to the first demonstrations of quantum effects both in the free time evolution [7] and in the collisions [8] of optical solitons. In fiber-optical solitons, the $\chi^{(3)}$ nonlinearity in optical fibers provides an environment that is equivalent to an attractive δ-function interaction between the photons [9].

Despite this, there are some drawbacks to silica optical fibers. These are the low nonlinearity of $\chi^{(3)}$ effects, and the existence of thermal background effects due to Raman and Brillouin scattering. The first problem means that photon numbers are typically large (of the order of 10×10^{7} or so) when fiber-optical solitons form. Deutsch *et al.* suggested that two-photon quantum solitons (bound states) could be observed in near-resonant atomic vapor or related systems [10]. These proposals, however, may also introduce losses and have not yet resulted in observation of quantum solitons with low photon numbers. In addition, thermal effects can provide a classical background that masks the unique effects due to quantum mechanics. The latter effect was reduced in the first quantum soliton experiments by using short interaction lengths and low temperatures.

To overcome the problem of low nonlinearity, we propose the investigation of $\chi^{(2)}$ parametric quantum solitons (or simultons), which require lower power levels in principle, since the phase shift is proportional to E^2, not E^3. A wide range of *classical* topological [11,12] and nontopological parametric soliton solutions [12–18] have recently been found. This leads to the interesting problem of whether the corresponding *quantum*-field theory has bound states, and whether these can have a large enough binding energy to become experimentally observable.

In this Rapid Communication, we will show that an exactly soluble one-dimensional two-photon quantum bound state does exist, and has a binding energy of at least ten orders of magnitude greater than the corresponding optical-fiber diphoton soliton. Such a solution is the basis for the further study of low-photon-number solutions. The relatively large binding energy, combined with low-temperature experimental techniques, could make it feasible to observe this simple quantum soliton in experiment. The new object consists of a superposition of a second-harmonic photon with a localized pair of subharmonic photons. These are bound together by an exchange force due to the parametric coupling. In effect, the system is analogous to the quark model of the meson. The subharmonic photons act as quarks, and the second-harmonic photon takes the role of the gluon. Thus, these new theoretical entities can be called ''optical mesons.''

We start by remarking that the effective Hamiltonian describing a nonlinear parametric waveguide in a moving frame of reference, using the rotating-wave approximation (RWA), can be written [19] as

$$\hat{H}=\hbar\int\Bigg[\frac{\hbar}{2m_1}\partial\phi_1^\dagger\partial\phi_1+\frac{\hbar}{2m_2}\partial\phi_2^\dagger\partial\phi_2+\rho\phi_2^\dagger\phi_2 +\frac{\chi}{2}(\phi_2^\dagger\phi_1^2+\phi_2\phi_1^{\dagger 2})\Bigg]dx. \tag{1}$$

Raman effects have been neglected for simplicity. We have included two group-velocity matched carrier frequencies at ω_1 and $\omega_2=2\omega_1$. The effective mass $m_i=\hbar/\omega_i''$ is caused by the dispersion ω_i'' in the ith frequency band. In addition, ρ is the phase mismatch, while the nonlinearity χ is given as

$$\chi=\frac{\chi^{(2)}\omega_1}{n^3}\left(\frac{\hbar\omega_2}{2\epsilon_0}\right)^{1/2}\int d^2x(u^{(1)}(x))^2(u^{(2)}(x))^*. \tag{2}$$

Here n is the refractive index and $\chi^{(2)}$ is the Bloembergen [20] expansion coefficient of nonlinear optics, in S.I. units.

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The functions $u^{(i)}(x)$ refer to normalized transverse-mode functions, which are assumed plane polarized in the appropriate direction that maximizes the coupling, and the following commutation laws for the photon fields ϕ_i are satisfied:

$$[\phi_m(x),\phi_n(x')^\dagger]=\delta(x-x')\delta_{mn}. \quad (3)$$

Traditionally, a two-photon eigenstate for the interacting Bose-gas equation is constructed by virtue of cancellations between δ-function terms from the kinetic-energy (linear) part of the Hamiltonian and δ-function terms from the interaction operator acting on the state [2,9,21]. In this case, a more subtle method is called for. The interaction here transforms pairs of subharmonic photons into single second-harmonic photons. That is, it does not conserve particle number. However, it does conserve a generalized particle number, or Manly-Rowe invariant, equal to N_1+2N_2. Thus, it is essential to construct energy eigenstates from the eigenstates of this invariant quantity. In this way, a general candidate for a two-photon eigenstate with energy E can be given as

$$|\psi\rangle=\left(\int P(x_1)\phi_2^\dagger(x_1)dx_1 + \int\int Q(x_1,x_2)\phi_1^\dagger(x_1)\phi_1^\dagger(x_2)dx_1dx_2\right)|0\rangle. \quad (4)$$

Operating on Eq. (4) with the Hamiltonian, Eq. (1), and rearranging, yields the following two equations:

$$\frac{\hbar^2}{2m_1}\left(\frac{\partial^2 Q(x_1,x_2)}{\partial x_1^2}+\frac{\partial^2 Q(x_1,x_2)}{\partial x_2^2}\right)+EQ(x_1,x_2) = \frac{\hbar\chi}{2}P\left(\frac{x_1+x_2}{2}\right)\delta(x_1-x_2), \quad (5a)$$

$$\frac{\hbar^2}{2m_2}\frac{\partial^2 P(x_1)}{\partial x_1^2}+(E-\rho\hbar)P(x_1)=\hbar\chi Q(x_1,x_1). \quad (5b)$$

To solve Eq. (5a), one introduces relative and center-of-mass coordinates according to

$$r=x_1-x_2,$$

$$R=\frac{x_1+x_2}{2}. \quad (6)$$

We next assume that $Q(x_1,x_2)$ is separable, and has an R dependence given entirely by the corresponding single-photon wave function, so that

$$Q(x_1,x_2)=P(R)\psi(r). \quad (7)$$

Substituting the above expression into Eq. (5a) gives the following two equations in the new coordinate system:

$$-\frac{\hbar^2}{4m_1}\frac{d^2P(R)}{dR^2}=E_cP(R), \quad (8a)$$

$$\frac{\hbar^2}{m_1}\frac{d^2\psi(r)}{dr^2}+E_r\psi(r)=\frac{\hbar\chi}{2}\delta(r). \quad (8b)$$

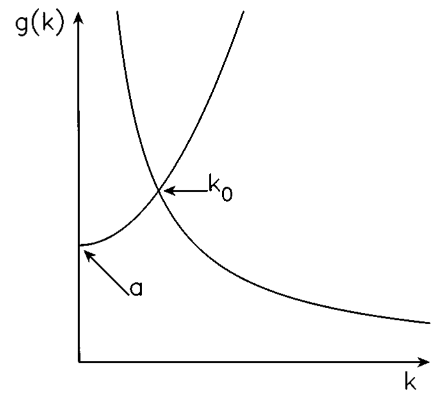

FIG. 1. Graphical solution of the cubic equation for the bound state, showing $g(k)$ vs k.

Here the energy is divided into center-of-mass and relative components in the usual way, so that $E=E_c+E_r$.

Solving the above equations in the case of solutions that are localized gives

$$P(R)=P_0e^{iKR}, \quad (9a)$$

$$\psi(r)=-\frac{m_1\chi}{4\hbar k}e^{-k|r|}. \quad (9b)$$

where $K=\sqrt{4m_1E_c}/\hbar$ and $k=\sqrt{-m_1E_r}/\hbar$. Substituting $P(x)=P_0\exp(iKx)$ into Eq. (5b) gives

$$E=\frac{\hbar^2K^2}{2m_2}+\rho\hbar-\frac{m_1\chi^2}{4k}=\frac{\hbar^2K^2}{4m_1}-\frac{\hbar^2k^2}{m_1}. \quad (10)$$

Here, k must be real and positive, for a localized bound state. It is easier to understand this system if we write the above equation in the form $E_r=-\hbar^2k^2/m_1$, where

$$g(k)=k^2+a=\frac{2b}{k}, \quad (11)$$

while $a=-K^2(m_2-2m_1)/(4m_2)+m_1\rho/\hbar$ and $b=[m_1\chi/(\hbar)]^2/8$.

The solutions for k are illustrated in Fig. 1. Since $b>0$, there is always a positive solution regardless of the value of a. When $a<0$, it is possible to have negative solutions; however, these unbounded solutions are scattering states that are the quantum analog of the classical continuum part of the pulse, and are not considered here.

The theory of cubic equations shows that the positive solution of k for a bound state can be written as

$$k_0=[b+\sqrt{(a/3)^3+b^2}]^{1/3}+[b-\sqrt{(a/3)^3+b^2}]^{1/3}. \quad (12)$$

A particularly simple case is when the dispersions have the relationship of $m_2=2m_1$. In this case the equations have a symmetry under boost transformations, so they behave rather like particle systems in which every eigenstate has a continuous set of corresponding boosted eigenstates, whose energy increases by the center-of-mass kinetic energy. This can be seen from the fact that the equation for k is then independent of the equation for K, while the total energy can always be written as a sum of two terms, involving k and K separately.

If there is also perfect phase matching, so that $\rho=0$, then $k_0=[m_1\chi/(2\hbar)]^{2/3}$ and the binding energy is given by $E_b=-E_r=\hbar^{2/3}m_1^{1/3}(\chi/2)^{4/3}$. Thus, it is clear that the binding energy increases with effective mass and nonlinear coupling. Ideally, the binding energy should be greater than the thermal phonon energy k_BT, in order to minimize coupling to phonons.

Currently available waveguides use $LiNbO_3$, in which $\chi^{(2)}=11.9\times10^{-12}$ m/V. This gives a binding energy that is low compared to thermal phonon energies. However, recent experiments indicate that three orders of magnitude greater nonlinearity might be feasible in semiconductor devices (for example, using GaAs asymmetric quantum wells and related systems [22,23]). Another factor that influences observability is the soliton radius $r_0=1/k$, which scales as $[\omega''/\chi]^{2/3}$ in parametric waveguides. The corresponding $\chi^{(3)}$ diphoton would have a radius that scales with $[\omega''/\chi]$, where we note that the coupling constant has different dimensions in this model. The radius, in turn, influences the characteristic interaction length, which scales as $z_0=c/(k_0^2\omega'')$. Clearly, r_0 should be less than the waveguide length, and z_0 less than an absorption length.

In order to compare the various possibilities, the table below gives order-of-magnitude parameters for typical silica fibers, together with estimates for parametric waveguides with nonlinearities corresponding to $LiNbO_3$ and GaAs asymmetric quantum wells (AQWs), respectively [assuming phase matching, a 1-μm fundamental wavelength, ideal conditions, and a mode area of (5 μm)^2]. For simplicity, we take $\omega''=0.2$ m^2/s in all cases, although smaller values are possible.

	Diphoton in silica	Optical meson in $LiNbO_3$	Optical meson in GaAs AQWs
χ	6×10^{-6} m/s	6×10^{4} $\sqrt{\text{m}}$/s	6×10^{7} $\sqrt{\text{m}}$/s
r_0	30 km	0.25 mm	2.5 μm
z_0	10^{15} km	60 m	6 mm
E_b	10^{-22} eV	10^{-9} eV	10^{-5} eV

The above table clearly illustrates the advantages of observing quantum ‘‘optical mesons.’’ The silica fiber two-photon soliton is simply unobservable, since the interaction length is about 14 orders of magnitude greater than the absorption length of 10 km in silica. The optical meson is nearly achievable even with present $LiNbO_3$ techniques. It appears possible, with improvements in semiconductor technology, that quantum solitions with very small numbers of photons could be experimentally observed in a strongly coupled parametric waveguide environment. A simple experiment would be, for example, to observe the autocorrelation function of the subharmonic photons. This opens the way to a new paradigm for testing quantum-field theory. In these experiments, photonic ‘‘particles’’ would be created and interact with each other, using lasers rather than particle accelerators.

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