Macroscopic test of quantum mechanics versus stochastic electrodynamics

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We identify a test of quantum mechanics versus macroscopic local realism in the form of stochastic electrodynamics. The test uses the steady-state triple quadrature correlations of a parametric oscillator below threshold. [S1050-2947(97)07901-8]

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Local hidden variable theories are known to be inconsistent with quantum mechanics at the microscopic level, from the Bell inequalities [1]. Experimental tests at this level have decided in favor of quantum mechanics [2], although there are still some experimental problems with low detection efficiency. The situation is different as particle numbers increase. There are no *macroscopic* tests of quantum measurement theory versus hidden variable theories. While it is possible to obtain Bell-type inequalities [3], these are difficult to implement experimentally. However, it is in this region that quantum mechanical measurement theory is most open to question, as Bell [1] has most cogently pointed out.

It is the purpose of this paper to demonstrate that an efficient test of quantum mechanics is possible, in a regime involving quantum correlations with large particle numbers. The test is a simple extension of a recent parametric oscillator Einstein-Podolsky-Rosen (EPR) [4] experiment, which first demonstrated the EPR [5] paradox of quantum mechanics in its original form. That is, the experiment was the first to employ observables having the Heisenberg algebra of $[\hat{x}_i, \hat{p}_j] = \hbar\delta_{ij}$, as used in the original EPR paper. Reid has recently shown [6] that this experiment is also intrinsically multiparticle in nature, since the quadrature operator measurements involve multiple particle detection.

In testing quantum mechanics, it is useful to have an alternative as a comparison to the quantum mechanical predictions. We choose to compare quantum mechanical predictions with those of stochastic electrodynamics [7]—a classical theory with added vacuum fluctuations. This is known already to reproduce many features of quantum mechanics. In fact, theories of this type, in the guise of approximate Wigner [8] representations with a positive Wigner function, have been used in quantum optics [9] to obtain convenient approximations to quantum theory at large photon number. In this regime, the theory is sometimes called the semiclassical method [10]. Of course, we do not regard this as a practical alternative at *small* photon number, as it cannot violate the Bell inequality. However, stochastic electrodynamics is a possible alternative to quantum theory at large particle number.

Either theory produces identical predictions for many experiments involving second-order correlations, including both squeezing (quadrature noise reduction) and the original EPR proposal. An EPR experiment in its original form shows that *either* quantum mechanics is incomplete *or* we must abandon local realism. It gives no information on which alternative is preferred. In this paper, we show that, by taking additional measurements on the parametric oscillator pump output beam, it is possible to differentiate the two principal alternatives in an operational measurement at large photon number.

Suppose that the parametric oscillator experiments previously used to demonstrate squeezing or EPR [4,11] correlations are extended to include pump output phase quadrature measurements. These can then be correlated with the products of the signal and idler quadratures. This information is always present implicitly, but was ignored in previous experiments. The crucial triple correlation of this type is the triple correlation of pump phase quadrature, together with orthogonal (uncorrelated) quadratures of the signal and idler beams. For this measurement, the two theories predict quite different results below threshold. The essential difference is that the semiclassical theory predicts that vacuum fluctuations behave as real fields, causing measurable correlations in the absence of a driving field. This is not found in quantum theory, which predicts smaller correlations—proportional to the input intensity well below threshold.

There are different results also predicted for other quantities, like squeezing, but only to higher orders in the coupling. In the case of triple correlations, we find that there is a difference predicted even to lowest nonvanishing order in the calculation. Since this difference is not dependent on the strength of the driving field, it survives in the macroscopic regime, where the semiclassical result might be expected to be correct. While this test cannot presently rule out all other hidden variable theories, the experiment would provide an additional test of quantum mechanics versus a typical hidden variable theory, in a different multiphoton regime.

The theory presented here deals with an idealized three-mode parametric oscillator, triply resonant in a cavity. The standard interaction Hamiltonian is [4,11]

$$H = i\hbar\left(\mathcal{E}\hat{a}_3 + g\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3 + \sum_j \hat{\Gamma}_j\hat{a}_j\right) + \text{H.c.}$$

The terms $\hat{\Gamma}_j$ represent damping reservoirs or output mirrors, g is the nonlinear coupling due to a $\chi^{(2)}$ nonlinear material, while $\mathcal{E}$ is the external driving for the pump field

$\hat{a}_3$. This field is resonant at frequency ω_3 both with the cavity and with the sum-frequency $(\omega_1+\omega_2)$ of the signal and idler modes $\hat{a}_1$ and $\hat{a}_2$.

The simplest treatment within quantum mechanics is to use the coherent-state expansion, leading to stochastic equations in the positive-P representation [11]. This is valid when boundary terms are negligible, which we have checked computationally to be valid for large pump threshold photon numbers (i.e., $g^2 \ll \gamma_1\gamma_2$).

This method results in stochastic equations for the twin amplitudes α_j, α_j^+, representing an off-diagonal coherent-state projector in the system density matrix:

$$\dot{\alpha}_1 = (-\gamma_1\alpha_1 + g\alpha_2^+\alpha_3) + (g\alpha_3)^{1/2}\xi_1(t),$$

$$\dot{\alpha}_2 = (-\gamma_2\alpha_2 + g\alpha_1^+\alpha_3) + (g\alpha_3)^{1/2}\xi_2(t),$$

$$\dot{\alpha}_3 = (\mathcal{E} - \gamma_3\alpha_3 - g\alpha_1\alpha_2).$$

There is a similar set of equations for α_j^+, with the replacement of ξ_i by ξ_i^+, where

$$\langle \xi_i(t)\xi_j(t')\rangle = \langle \xi_i^+(t)\xi_j^+(t')\rangle = \delta_{i,3-j}\delta(t-t').$$

The noise-sources are complex Gaussian stochastic processes, and all their other correlations vanish. While these equations are numerically soluble, more insight is obtained on expanding them analytically as a power-series in the coupling g, which is applicable for driving fields below the critical region (i.e., $E < E_T \equiv \sqrt{\gamma_1\gamma_2\gamma_3/g}$):

$$\alpha_i(t) = \sum_{n=0}^{\infty} \alpha_i^{(n)}(t) g^{n-1}.$$

It is of most interest to calculate the steady-state correlations of quadratures below threshold, which we define as usual to be

$$x_i = \alpha_i + \alpha_i^+, \quad y_i = (\alpha_i - \alpha_i^+)/i.$$

To zeroth order, the usual classical result is obtained of $x_1^{(0)} = y_1^{(0)} = x_2^{(0)} = y_2^{(0)} = y_3^{(0)} = 0$; $x_3^{(0)} = 2\bar{\mathcal{E}}$. We regard $\bar{\mathcal{E}} = \mathcal{E}g/\gamma_3$ as of order unity in the expansion [12].

To obtain results to the next order, it is simplest to consider the combinations (for $j \leq 2$)

$$\overline{x_j} = \alpha_j + \alpha_{3-j}^+, \quad \overline{y_j} = (\alpha_j - \alpha_{3-j}^+)/i.$$

In the symmetric case of $\gamma_1 = \gamma_2 = \gamma$, these diagonalize the stochastic equations, allowing their immediate solution:

$$\overline{x_j^{(1)}}(\omega) = \sqrt{\bar{\mathcal{E}}}[\xi_j(\omega) + \xi_{3-j}^+(\omega)]/(-i\omega + \gamma_-),$$

$$\overline{y_j^{(1)}}(\omega) = \sqrt{\bar{\mathcal{E}}}[\xi_j(\omega) - \xi_{3-j}^+(\omega)]/(\omega + i\gamma_+).$$

Here $x(\omega) = \int \exp(i\omega t)x(t)dt/\sqrt{2\pi}$, and we have defined new damping rates $\gamma_\pm \equiv (\gamma \pm \bar{\mathcal{E}})$. The steady-state pump quadrature solutions are $x_3^{(1)} = y_3^{(1)} = 0$, to first-order in the expansion. It is simple to verify from the above solutions that external quadrature measurements of $x_1(\omega)$ are strongly correlated with $x_2(-\omega)$, and similarly with $y_1(\omega)$ and $y_2(-\omega)$. This is the reason for the EPR paradox observed in this type of experiment.

If results of higher order again are calculated, a new result is obtained. Of most interest is the triple correlation between three distinct quadratures in the external fields $X_j(\omega), Y_j(\omega)$, where $X_j(\omega) = \sqrt{2\gamma_j}x_j(\omega)$. These are

$$\langle X_1(\omega_1)Y_2(\omega_2)Y_3(\omega_3)\rangle_Q$$

$$= \frac{\gamma}{2}\sqrt{2\gamma_3}\langle[\overline{x_1}(\omega_1)\overline{y_2}(\omega_2) + \overline{x_2}(\omega_1)\overline{y(}\omega_1)$$

$$+ \overline{y_1}(\omega_1)\overline{x_2}(\omega_2) + \overline{y_2}(\omega_1)\overline{x_1}(\omega_1)]y_3(\omega_3)\rangle.$$

The twelve other terms of form $\langle \overline{x_1}(\omega_1)\overline{x_2}(\omega_2)y_3(\omega_3)\rangle$ or $\langle \overline{x_1}(\omega_1)\overline{y_1}(\omega_2)y_3(\omega_3)\rangle$ (etc.), all vanish owing to the symmetry properties of the Hamiltonian. On calculating the lowest order nonvanishing triple correlation, we find

$$\langle X_1(\omega_1)Y_2(\omega_2)Y_3(\omega_3)\rangle_Q \simeq g\bar{\mathcal{E}}^2\Delta(\vec{\omega}) + (\omega_1 \leftrightarrow \omega_2),$$

where

$$\Delta(\vec{\omega}) \equiv \frac{2\gamma\sqrt{\gamma_3/\pi}\,\delta(\omega_1+\omega_2+\omega_3)}{(\omega_1^2+\gamma_-^2)(\omega_2^2+\gamma_+^2)(\gamma_3 - i\omega_3)}.$$

In the stochastic electrodynamics or semiclassical theory, the calculation is more complicated. Second-order terms exist in all three quadratures, and there are additional terms arising from reflected vacuum fields—giving rise to 192 different combinations. Most vanish, as before, owing to symmetry properties. With a little algebra, we obtain the following simple result to lowest order in the expansion:

$$\langle X_1(\omega_1)Y_2(\omega_2)Y_3(\omega_3)\rangle_S$$

$$\simeq g\left(\bar{\mathcal{E}}^2 + \frac{1}{4}(\gamma_- + i\omega_1)(\gamma_+ + i\omega_2)\right)\Delta(\vec{\omega}) + (\omega_1 \leftrightarrow \omega_2).$$

This is like the quantum theory prediction, except for an extra term which is *independent* of the driving field $\bar{\mathcal{E}}$, as $\bar{\mathcal{E}} \to 0$.

This effect also shows up in the total intracavity triple correlation, which can be obtained on integrating the above results over all frequencies. Here, we find that quantum theory predicts that

$$\langle x_1y_2y_3\rangle_Q = M_Q = \frac{g\bar{\mathcal{E}}^2}{2(\gamma^2 - \bar{\mathcal{E}}^2)(\gamma_3 + 2\gamma)},$$

while the SED (semiclassical) theory predicts that

$$\langle x_1y_2y_3\rangle_S = M_Q + \frac{g}{2(\gamma_3 + 2\gamma)}.$$

These results are easily verified computationally, simply by integrating the relevant stochastic equations numerically. We find that the simulation results agree very well with the analytic calculations, except for the obvious critical fluctuation divergence that occurs in the vicinity of threshold at

$\bar{\mathcal{E}} = \gamma$, where this simple perturbation theory no longer holds. In fact, other technical problems occur at threshold, so for this test it is preferable to use a finite fraction of threshold intensity—even though the triple correlations are greatly increased at threshold.

While the triple correlation varies with the coupling g (which is small in current experiments), any operational measurement of X_j effectively turns it into a Schwinger operator $\hat{a}_j\hat{b}_j^\dagger + \hat{a}_j^\dagger\hat{b}_j$, thus amplifying it by the local-oscillator amplitude $\hat{b}_j$. In particular, a phase-shifting interferometric measurement of the pump output phase would increase X_3 by a local oscillator term proportional to the intracavity pump amplitude $\sqrt{n_3}$, which varies with $(1/g)$ at any finite proportion of threshold intensity. This implies that the discrepancy in the final current correlations is always finite, even for small g, and relatively intense driving fields. Despite this, practical limitations, such as thermal refractive-index fluctuations and background counts due to random triple correlations, would strongly indicate that large g (small n_3) values are preferable.

In summary, the two theories presented here give completely different results for triple correlations. It seems possible that these predictions could be tested in ultrasmall waveguide-based parametric oscillators with integrated mirrors, in order to reduce the size of the intracavity photon number at threshold.

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