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Spring-neap tide-induced beach water table fluctuations in a sloping coastal aquifer

D.-S. Jeng,[1] X. Mao,[2] P. Enot,[2] D. A. Barry,[3] and L. Li[4]

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[1] Predictions of water table fluctuations in coastal aquifers are needed for numerous coastal and water resources engineering problems. Most previous investigations have been based on the Boussinesq equation for the case of a vertical beach. In this note an analytical solution based on shallow water expansion for the spring-neap tide-induced water table fluctuations in a coastal aquifer is presented. Unlike most previous investigations, multitidal signals are considered with a sloping coastal aquifer. The new solution is verified by comparing with field observations from Ardeer, Scotland. On the basis of the analytical approximation the influences of higher-order components on water table elevation are examined first. Then, a parametric study has been performed to investigate the effects of the amplitude ratio (λ), frequency ratio (ω), and phases (δ_1 and δ_2) on the tide-induced water table fluctuations in a sloping sandy beach.

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1. Introduction

[2] Tidal dynamics in coastal aquifers plays a role in numerous environmental issues in coastal and estuarine areas, such as saltwater intrusion, contaminant transformation and migration, control of erosion and biological activities [*Cheng and Ouazar*, 2004]. Numerous analytical solutions for modeling of tide-induced groundwater fluctuations are available, which take into account the effect of the vertical beach, sloping beach, aquifer leakage, density differences and varying tidal signal along the estuary [e.g., *Nielsen*, 1990; *Li et al.*, 2000a; *Teo et al.*, 2003]. Most analytical solutions are based on the assumption of monochromatic tides, which may over simplify the tidal wave conditions. In reality, tides are more complicated and often bichromatic, containing oscillations of at least two different frequencies. For example, in Ardeer, Scotland, a semidiurnal solar tide has period $T_1 = 12$ hours and frequency $\omega_1 = 0.5236$ rad/h, while $T_2 = 12.42$ hours and $\omega_2 = 0.5059$ rad/h for a semidiurnal lunar tide (X. Mao et al., Tidal influence on behaviour of a coastal aquifer adjacent to a low-relief estuary, submitted to *Journal of Hydrology*, 2005, hereinafter referred to as Mao et al., submitted manuscript, 2005). As a result, the spring-neap cycle (i.e., the tidal envelope) is formed with a longer period, $T_{sn} = 2\pi/(\omega_1 - \omega_2) = 14.78$ days. The nonlinear propagation of the bichromatic tides in the aquifer results in low-frequency water table fluctuations over the spring-neap period, as has been measured in the field by *Raubenheimer et al.* [1999] and demonstrated mathematically by *Li et al.* [2000b]. These low-frequency water table fluctuations, called spring-neap tidal water table fluctuations hereafter, propagated much further inland than the primary tidal signals (i.e., diurnal and semi-diurnal tides). Such fluctuations have been analyzed recently [*Li et al.*, 2000b; *Su et al.*, 2003], with results demonstrating the effects of interacting tidal components. However, these results were based on only the zeroth-order shallow water expansion, i.e., the Bouniessq equation, which may be insufficient for some tidal conditions [*Teo et al.*, 2003].

[3] The objective of this note is to extend these results by deriving an analytical solution for spring-neap tide-induced water table fluctuations in a sloping sandy beach, based on a higher-order shallow water expansion. The proposed analytical solution will be compared briefly with field observations from Adreer, Scotland (Mao et al., submitted manuscript, 2005), and previous analytical solution based on Boussinesq equation [*Li et al.*, 2000b]. Then a parametric study to investigate the influence of amplitude ratio, frequency ratio and phases is conducted.

2. Theoretical Formulation

2.1. Analytical Solution

[4] In this study, the flow is assumed to be homogeneous and incompressible in a rigid porous medium. The configuration of the groundwater flow in the coastal aquifer is shown in Figure 1. In Figure 1, $h(x, t)$ is the total tide-induced water table height, D is the thickness of the aquifer and β is the beach slope. Seepage face effects are ignored. Since the fluid is incompressible, the free surface flow of groundwater satisfies the conservation of mass, leading to Laplace's equation [e.g., *Bear*, 1972]:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \qquad (1)$$

where ϕ is the potential head.

[1] Department of Civil Engineering, University of Sydney, Sydney, New South Wales, Australia.
[2] Contaminated Land Assessment and Remediation Research Centre, Institute for Infrastructure and Environment, School of Engineering and Electronics, University of Edinburgh, Edinburgh, UK.
[3] School of Architecture, Civil and Environmental Engineering, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.
[4] School of Engineering, University of Queensland, Brisbane, Queensland, Australia.

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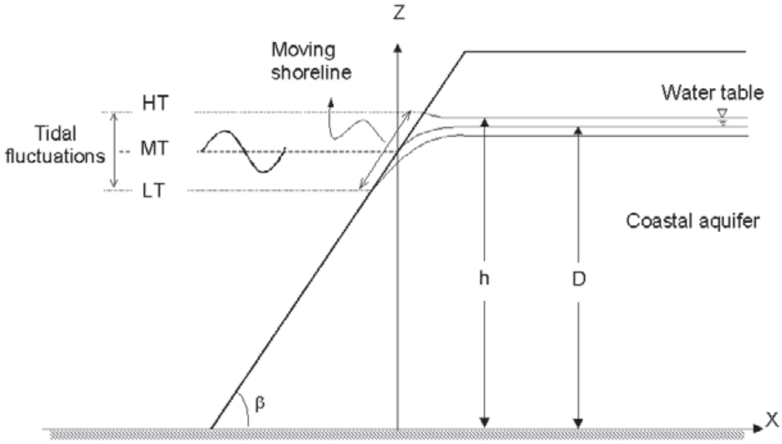

Figure 1. Sketch of tidal dynamics in coastal aquifers.

[5] Equation (1) is to be solved subject to the following boundary conditions,

$$\frac{\partial \phi}{\partial z} = 0, \qquad \text{at } z = 0, \tag{2a}$$

$$\phi = h, \qquad \text{at } z = h, \tag{2b}$$

$$n_e \frac{\partial \phi}{\partial t} = K\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right] - K\frac{\partial \phi}{\partial z}, \qquad \text{at } z = h, \tag{2c}$$

$$h(x_0, t) = D + A_1 \cos(\omega_1 t + \delta_1) + A_2 \cos(\omega_2 t + \delta_2)$$
$$\text{at } x_0 = [A_1 \cos(\omega_1 t + \delta_1) + A_2 \cos(\omega_2 t + \delta_2)] \cot \beta, \tag{2d}$$

$$\frac{\partial \phi}{\partial x} \rightarrow 0, \qquad \text{as } x \rightarrow \infty. \tag{2e}$$

Note that the soil properties are defined by the soil porosity (n_e) and hydraulic conductivity (K).

[6] Following *Teo et al.* [2003], the tide-induced water table fluctuation can be expressed as

$$h(x,t) = D\left[1 + \left(\alpha H_{01} + \alpha^2 H_{02}\right) + \varepsilon\left(\alpha H_{11} + \alpha^2 H_{12}\right) + \varepsilon^2 \alpha H_{21}\right], \tag{3}$$

where $\varepsilon = \sqrt{\frac{n_e \omega_1 D}{2K}}$ is the shallow water parameter and $\alpha = \frac{A_1}{D}$ is the amplitude parameter. Detailed derivation of the analytical solutions and H_{ij} coefficients are available in the auxiliary material.[1]

2.2. Comparisons With Field Data

[7] To test the analytical solutions, field data of water table fluctuations at Ardeer, Scotland, are used as an example. Ardeer is a former industrial site containing a substantial waste deposit. The low-relief estuary adjacent to the site has a mildly sloping sandy beach. Between low and high tide the beach length varies by 180 m. Field monitoring was conducted to characterize the tidal influence on the groundwater dynamics and contaminant migration as well as the saltwater intrusion. Detailed information on the field observations is available elsewhere (Mao et al., submitted manuscript, 2005).

[8] Analysis on tidal signals shows that the fluctuation amplitude decreases exponentially with distance from the estuary, accompanied by a phase lag, as has been shown by the FFT analysis of the observed data (Mao et al., submitted manuscript, 2005). FFT analyses of the estuarine tides have shown the dominant frequencies are M2, S2 and O1. To simplify the analytical solution, we only considered M2 and S2 components. The fitted expression modeling the estuarine tides is

$$h(x,t) = D + 1.1745\cos\left(\omega_1 t - 1.6676 + \frac{\pi}{2}\right) + 0.08\cos \cdot \left(\omega_2 t - 1.3377 + \frac{\pi}{2}\right), \tag{4}$$

where h is measured in meters, $\omega_1 = 2\pi/0.52$ rad/d and $\omega_2 = 4\pi$ rad/d.

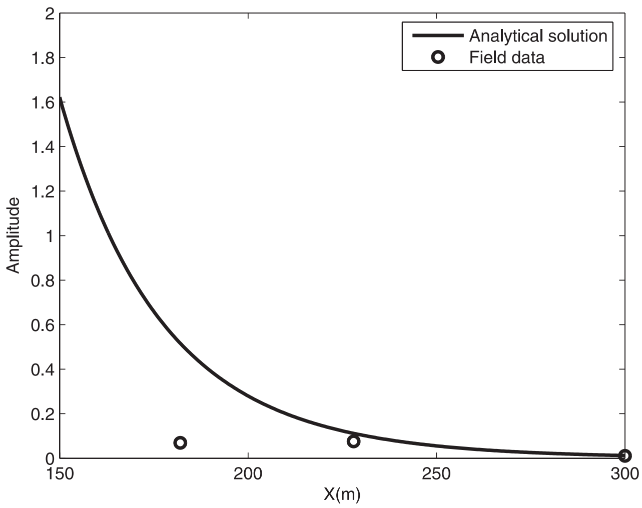

Figure 2. Comparison of analytical solution with field data. See color version of this figure in the HTML.

[1] Auxiliary material is available at ftp://ftp.agu.org/apend/wr/2005WR003945.

Table 1. Input Data for Figures 3–6

Parameter	Value
Soil porosity n_e	0.22
Hydraulic conductivity K, m/d	50
Slope of the beach β, rad	0.02
Thickness of aquifer D, m	5
Amplitude of the first tidal wave A_1, m	2
Frequency of the first tidal wave ω_1	4π
Amplitude parameter ε	0.372
Shallow water parameter α	0.2

[9] The calculated groundwater table fluctuation based on the analytical solution with the above estuarine tide is shown in Figure 2 where the damping of the amplitude is compared against observed data. In the development of the analytical solutions, we assume the coastal aquifer is homogeneous with a uniformly sloped beach. However, the real aquifer is multilayered and inhomogeneous in both vertical and horizontal direction near the intersection of the ocean and the aquifer. It has been reported that inhomogeneity will affect water table fluctuations [*Trefry*, 1999]. Furthermore, the variations of beach slope are expected to affect the water table level, but no solution is available yet. In addition, seepage face effects have been ignored. Finally, we consider two main frequencies of tidal waves (M2 and S2) in equation (4), based on FFT analysis. The inclusions of other tidal components may improve the prediction of the analytical solutions.

3. Results and Discussions

[10] The major difference between the present solution and previous solution [*Li et al.*, 2000b] is the higher-order component. It is of interesting to examine the effects of higher-order component here. The input data for the comparison are listed in Table 1. As seen in Figure 3, the previous solution [*Li et al.*, 2000b] overestimates the water table elevation.

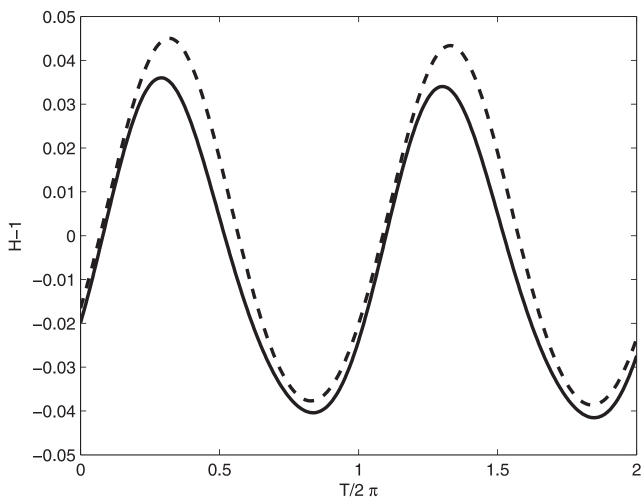

Figure 3. Effects of higher-order components on water table fluctuations in coastal aquifers (solid line is from the present solution, and dashed line is from *Li et al.* [2000b]). X_1 = 2, λ = 0.5, ω = 0.5, and (δ_1, δ_2) = (0, 0). See color version of this figure in the HTML.

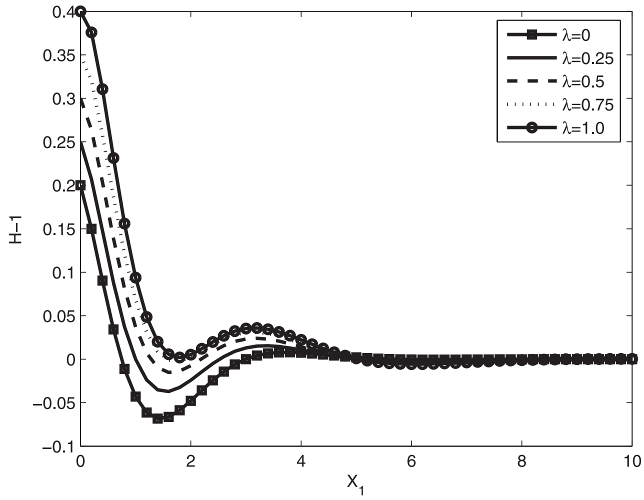

Figure 4. Effects of amplitude ratio (λ) on water table fluctuations in coastal aquifers. Here ω = 0.5, and (δ_1, δ_2) = (0, 0), t = 0. See color version of this figure in the HTML.

[11] As shown in analytical solutions presented in section 2, numerous parameters are involved in the solutions. The objective of this parametric study is to investigate three parameters. These are (1) amplitude ratio (λ = A_2/A_1), (2) frequency ratio (ω = ω_2/ω_1), and (3) phases (δ_1 and δ_2). Although spring-neap tides normally have the frequencies ration (ω) close to unity, the variation of frequency ration (ω) is also considered here for the general applications of other cases rather than limited to spring-neap tides. Recently, *Li et al.* [2000b] discussed part of the above parameters briefly based on Boussinesq equation, it is worthwhile to reexamine the effects of eth above parameters with the new solution presented in section 2. The input data of numerical examples are given in Table 1.

[12] The amplitude ratio (λ) is the ratio of the amplitudes of two tidal components (A_2/A_1). Here we allow λ to vary

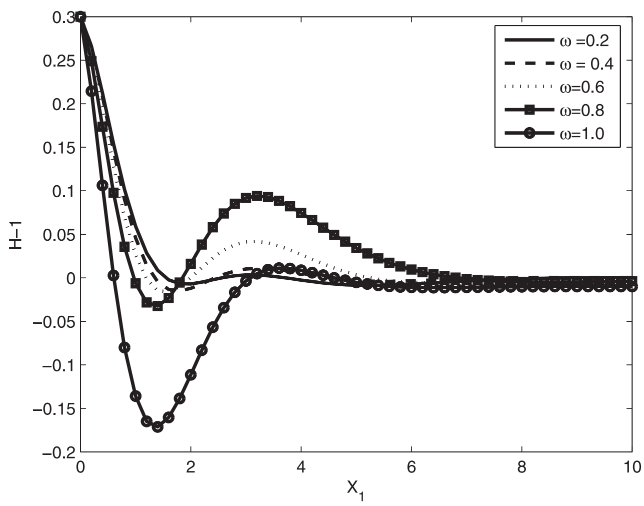

Figure 5. Effects of frequency ratio (ω) on water table fluctuations in coastal aquifers. Here λ = 0.5 and (δ_1, δ_2) = (0, 0), t = 0. See color version of this figure in the HTML.

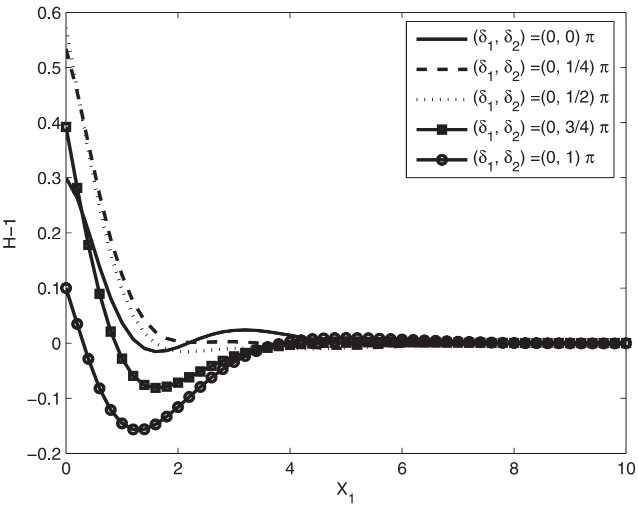

Figure 6. Effects of phases (δ_1 and δ_2) on water table fluctuations in coastal aquifers. Here $\lambda = 0.5$, $\omega = 0.5$, $t = 0$. See color version of this figure in the HTML.

from zero (0) to unity (1). $\lambda = 0$ represents the case without the second tidal component, which is the case reported by *Teo et al.* [2003], while $\lambda = 1$ represents the case of equal weight of two tidal signals.

[13] Figure 4 illustrates the effects of the amplitude ratio (λ) on the tide-induced water table height above the mean thickness of aquifer at $T = 0$, i.e., $H - 1 = (h - D)/D$. As shown in Figure 4, the water table height increases as the amplitude ratio (λ) increases, i.e., it increases as the amplitude of the second tidal signal increases.

[14] Besides the amplitude ratio (λ), the frequency ratio (ω) is another factor, which may affect the tide-induced water table fluctuations. The distribution of water table heights versus the horizontal distances (X) for various values of frequency ratio (ω) is presented in Figure 5. As shown in Figure 5, the water table height decreases as ω increases when $X < 1.8$. When X increases ($X > 1.8$), the influence of ω perform an irregular trend, which may require more advanced theories.

[15] Another major difference between the previous solution [*Teo et al.*, 2003] and the present solution is the phase differences between two tidal signals. Figure 6 illustrates the effects of phase differences of two tidal components on the tide-induced water table heights. To see the influence of phase difference, we fix $\delta_1 = 0$, and vary δ_2 from zero (0) to π. Generally speaking, the phase difference significantly affects the water table height. For example, the water table height decreases as δ_2-$\delta 1$ increases.

4. Conclusions

[16] In this note, an analytical solution up to the second-order shallow water expansion for spring-neap tides-induced water table fluctuations in a sloping coastal aquifer has been derived. The analytical solution was verified by the field observation in Ardeer, Scotland. A parametric study indicates that the amplitude ratio, frequency ratio and phase differences significantly affect the water table elevations in a sloping coastal aquifer.

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D. A. Barry, School of Architecture, Civil and Environmental Engineering, École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland. (andrew.barry@epfl.ch)

P. Enot and X. Mao, Contaminated Land Assessment and Remediation Research Centre, Institute for Infrastructure and Environment, School of Engineering and Electronics, University of Edinburgh, Edinburgh EH9 3JL, UK. (patricia.enot@ed.ac.uk; xmao@ed.ac.uk)

D.-S. Jeng, Department of Civil Engineering, University of Sydney, Sydney, NSW 2006, Australia. (d.jeng@civil.usyd.edu.au)

L. Li, School of Engineering, University of Queensland, Brisbane, Qld 4072, Australia. (l.li@uq.edu.au)