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Causal Loops and the Independence of Causal Facts

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According to Hugh Mellor in *Real Time II* (1998, Ch. 12), assuming the logical independence of causal facts and the 'law of large numbers', causal loops are impossible because if they were possible they would produce inconsistent sets of frequencies. I clarify the argument, and argue that it would be preferable to abandon the relevant independence assumption in the case of causal loops.

1. Mellor's Argument Against the Possibility of Causal Loops. Suppose we have a causal loop between particular events (or facts about particulars) P and Q; i.e., P causes Q and Q causes P. According to Mellor P causes Q only if $ch_P Q = q$ and $ch_{\sim P} Q = q'$ exist; and Q causes P only if the following chances exist: $ch_Q P = p$ and $ch_{\sim Q} P = p'$, where '$ch_Q P$' is read as 'the chance that Q gives P'. Mellor argues that such a scenario produces contradictions, and therefore the possibility of such loops should be rejected (1998, Ch. 12). His argument utilizes two important assumptions.

Mellor's first key assumption is Independence, i.e., that causal facts are logically independent of each other. Since the fact that P causes Q is logically independent of what causes P and what Q causes, if the P-Q loop is possible, then all combinations of individually possible values for p, p′, q, q′ should be possible, apart from the requirement that $p > p'$ and $q > q'$.

Mellor's second key assumption we may call the Frequency Condition. Chances are connected to frequencies by laws of large numbers. The relative frequency of Q-type events in partitions of P-type events $f(Q|P)$ in

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the same circumstances should be very close to q in a large enough sample, in the sense that "as [the number of trials] n increases without limit, f_n's chance of differing from q by less than a given amount, however small, while it will never be 1, will eventually differ from 1 by less than any given amount, however small" (Mellor 1998, 133). So, in large samples,

$$\begin{aligned} f(Q|P) &\approx ch_P Q = q \\ f(Q|{\sim}P) &\approx ch_{\sim P} Q = q' \\ f(P|Q) &\approx ch_Q P = p \\ f(P|{\sim}Q) &\approx ch_{\sim Q} P = p'. \end{aligned} \tag{1}$$

From this it follows (see Appendix 1) that, in large samples, we have the following approximations of the number of Q-type events N(Q), of P-type events N(P), and likewise for the number of times P and Q don't occur:

$$N(Q) \approx q\ N(P) + q'\ N({\sim}P) \tag{4a}$$
$$N(P) \approx p\ N(Q) + p'\ N({\sim}Q) \tag{4b}$$
$$N({\sim}Q) \approx (1 - q)N(P) + (1 - q')N({\sim}P) \tag{4c}$$
$$N({\sim}P) \approx (1 - p)N(Q) + (1 - p')N({\sim}Q). \tag{4d}$$

Mellor considers a sample of 20 million (M) cases of individuals with the chances q, q′, p, p′. The chances q and q′ are logically independent (provided $q > q'$); suppose $q = 0.6$ and $q' = 0.2$. Suppose also that the number of P, N(P) = N(∼P) = 10M; i.e., $f(P) = f({\sim}P) = 1/2$. Then, by (4a) and (4c) N(Q) ≈ 8M and N(∼Q) = 12M. But p and p′ are independent of q and q′. So we can take any values, say $p = 0.5$, $p' = 0.25$. Then, by (4b) and (4d) N(P) = 7M and N(∼P) = 13M. This contradicts the initial frequencies. Since the possibility of such a loop entails the compatibility of any set of values for q, q′, p, p′ given the independence of causal facts and the Frequency Condition (1), the independence of causal facts and the Frequency Condition together entail that such a loop is not possible.

It will be helpful to have an example. (Here P and Q are not direct causes as in Mellor's loop, but the example should be instructive all the same.) Suppose we have a wormhole whose mouth opens sporadically, and which when open allows a photon to pass through and to reappear from the other end a short time earlier. Suppose that we consider cases where a certain kind of atom, which can absorb and radiate photons, is put in the vacinity of the wormhole when it is open. Let t_2 be the small time segment for which the wormhole mouth is open, and t_1 a small time segment some time before t_2 during which a photon could appear from the wormhole.

Take P to be the absorption of a photon by the atom at t_1, and Q to be the emission of a photon by the atom at t_2. Suppose we have 20M cases where a wormhole is open in the presence of such an atom. Assume we have 10M cases where a photon is absorbed at t_1, and 10M where one is not. Suppose the chance that an atom which has absorbed a photon at t_1 will emit a photon at t_2 is $ch_P Q = q = 0.6$, and the chance that it will emit at t_2 when it hasn't absorbed a photon at t_1 is $ch_{\sim P} Q = q' = 0.2$; and that the chance that the atom will absorb a photon at t_1 when it emits one at t_2 is $ch_Q P = p = 0.5$, and finally, the chance that it will absorb at t_1 when it hasn't emitted a photon at t_2 is $ch_{\sim Q} P = p' = 0.25$. Then, by the above reasoning, it will follow that there are 7M cases where a photon is absorbed at t_1 and 13M cases where one is not, contradicting the original initial frequency according to which a photon was absorbed in half the cases.

So, to recap the argument, assuming (1) the independence of causal facts and thence of the chances in a loop and (2) the Frequency Condition, it follows that the possibility of causal loops entails inconsistent frequencies. Therefore causal loops are not possible.

2. Revised Formulation. However, the argument of the last section, as formulated, is invalid—the assumptions as stated do not entail a contradiction. In fact we need a stronger independence condition to derive a contradiction. Recall that the Independence assumption allowed us to choose any set of chances p, p′, q, q′. But this alone will not give a contradiction, because we can show that for any set of chances in a loop, there is a unique value for the 'initial' frequency which does not yield contradictory frequencies (this result was first given in Berkovitz 2001; for my proof see Appendix 2). To gain a contradiction we must also assume that for a given set of four chances, we can set any initial frequency. In fact our derivation of the contradiction in the previous section does assume this.

Firstly, (4a)–(4d) entail (5a) and (5b) (see Appendix 2):

$$N(Q)/N(\sim Q) \approx (p'q + q' - p'q')/(1 - pq - q' + pq') \quad (5a)$$
$$N(P)/N(\sim P) \approx (pq' + p' - p'q')/(1 - pq - p' + p'q). \quad (5b)$$

In other words, the initial and final frequencies in a loop are each a direct function of the set of four chances p, p′, q, q′. For any set of chances there is a unique value for $f(P)$ and a unique value for $f(Q)$. Clearly, to choose a different value of the initial frequency will lead to a contradiction.

Given Mellor's choice of four chances, {q = 0.6, q′ = 0.2, p = 0.5, p′ = 0.25}, (5a) and (5b) entail that $N(Q)/N(\sim Q) = N(P)/N(\sim P) = 1/2$. Equivalently, $f(P) = f(Q) = 1/3$. We can get consistency by taking $N(P, \sim P) = (10M, 20M)$, from which it follows from the four chances and the frequency condition that $N(Q, \sim Q) = (10M, 20M)$, which in turn entails that $N(P, \sim P) = (10M, 20M)$, i.e., no contradiction. In other words, given these chances, Mellor's choice of initial frequencies $f(P) =$

$f(Q) = 1/2$ is not allowed by the laws of large numbers. It is not surprising then that it yields a contradiction.

Alternatively, a choice of chances q and q′ together with initial frequencies $f(P)$ and $f(\sim P)$ fixes the chances p and p′. Given Mellor's choices, these chances are constrained in large sequences as follows:

$$8M(1 - p) + 12M(1 - p') \approx 10M \tag{6}$$

which, together with the requirement that if Q causes P then $p > p'$, entails the restriction that $p > 1/2$, contrary to Mellor's assumption.

So the argument that the independence of the four chances together with the frequency condition gives a contradiction in the case of loops is invalid. However, we can strengthen the independence condition thus:

Input–Independence Condition: For any set of chances in a loop, we are free to choose any initial frequency.

Input–Independence and the Frequency Condition together entail contradictions in loops. This is what Mellor's argument achieves, as shown in the previous section. This argument is valid, but it raises the question, why should we take the Input–Independence Condition to be at all plausible in the case of loops? Why not simply reject it, rather than the possibility of loops?

3. Independence of Initial Frequencies Without Loops. First, some comments about Input–Independence in the linear case. Consider a 'normal' case of causation, where P causes Q, and where $ch_P Q = q = 0.6$ and $ch_{\sim P} Q = q' = 0.2$. Here the Input-Independence assumption is plausible. The Frequency Condition entails that (4a) applies:

$$N(Q) \approx q\, N(P) + q'\, N(\sim P) \tag{4a}$$

so that we see clearly how the output frequency depends logically on the chances and the input frequency.

However, logically, for a given pair of chances either input frequencies or output frequencies (but not both) can be taken as independent. Since $N(P) + N(\sim P) = N(Q) + N(\sim Q)$, (4a) entails that:

$$N(P) \approx \{(1 - q')/(q - q')\}\, N(Q) - \{(q')/(q - q')\}\, N(\sim Q). \tag{6}$$

In other words, the input frequency depends logically on the two chances and the output frequency. For $q = 0.6$ and $q' = 0.2$, if we take the output frequency to be $N(Q, \sim Q) = (8M, 12M)$, then (6) gives us $N(P, \sim P) = (10M, 10M)$.

Normally, of course, we take the inputs as independent and the outputs as dependent (Hausman 1998). But as we have just seen, this is not required logically. The logical requirement is that either inputs or outputs (exclusive 'or') are dependent on the chances. So the Input–Independence assumption is not a logical requirement. Part of it comes from the further assumption that causes rather than effects should be treated as the free variables.

For example, suppose we have our atom without the wormhole, but with the same chances for Q, the emission of a photon, contingent on the earlier absorption P. We can manipulate P to control Q by controlling the circumstances so as to affect the chance of P. So we increase the rate of incident photons. This doesn't affect the chances q, q′; by increasing $f(P)$ we simply increase $f(Q)$. We don't think we can do the same with the output frequencies. We don't think that by changing the output frequency, while holding the chances of Q fixed (if we think this is even possible), that we can control the input frequencies. Whether this is justified, and if so what justifies it, is beyond the scope of this paper (Price 1996). But it is not justified by the logic alone. But, of course, there is no question that we do follow this practice in normal cases. If smoking causes cancer, then we control the cancer by influencing the frequency of smoking, which leaves the chances of cancer for smokers and non-smokers untouched. If cancer causes weight loss, we don't think we could control cancer by influencing the frequency of weight loss.

Our results also ensure transitivity. Suppose, additionally, that Q causes R, again in the absence of the possibility of any loops, and that $ch_Q R = r = 0.5$ and $ch_{\sim Q} R = r' = 0.25$. We assume, as we did in the case of loops, that the circumstances in which Q gives R this chance is essentially the same as the circumstances in which P gives Q that chance.

Since $f(R)$ depends on r, r′, and $f(Q)$, and $f(Q)$ depends on q, q′, and $f(P)$, it follows that $f(R)$ depends on r, r′, q, q′, and $f(P)$. For example, for these values of the chances, if N(P, ~P) = (10M, 10M) then by the Frequency Condition (4a) it follows that N(R, ~R) = (7M, 13M). Alternatively, again, it is also true that $f(P)$ depends on r, r′, q, q′, and $f(R)$. For the same chances, N(R, ~R) = (7M, 13M) entails N(P, ~P) = (10M, 10M).

4. Independence of 'Initial' Frequencies in Loops. The argument against the possibility of loops derives a contradiction from the Frequency Condition together with the Input–Independence Condition (that the initial frequency $f(P)$ can be set independently of the four chances in the loop). So we face a choice: granting the Frequency Condition and the original Independence assumption (i.e., that C causes E is logically independent of what causes C and what E causes), we must reject either the possibility of causal loops or the Frequency Condition. In this section I offer a case for dropping the Input–Independence Condition for loops, rather than rejecting the possibility of loops. I argue that in the case of loops we have ample reason to abandon the Input–Independence Condition.

First, there is a prima facie case which exists purely in virtue of the logic. As shown in the previous section, in the case of normal causation logic allows that having set the chances and granting the Frequency Condition, we are still free to set initial frequency, providing we haven't set the output frequency. However, we have also seen that in the case of loops setting the chances logically determines the initial frequency. This in itself is a prima facie case for rejecting Input–Independence, unless we can come up with a good independent reason for keeping it despite the logic. However, as we shall see, the usual reasons for holding Independence seem inappropriate in the case of loops.

Second, given that it is supposed to establish the *impossibility* of loops, for Mellor's argument to work the Input–Independence needs to be necessarily true (strictly, that necessarily Input or Output-Independence is true). How do we prove or disprove that such a claim is necessarily true? One way to disprove it is to give a possible consistent case which violates it. Well, the consistent-frequency loop case given above is just such a case. It is consistent, and it violates both Input and Output-Independence. To overcome this Mellor needs to offer an independent argument for the necessity of the principle. Again, the burden of proof clearly lies with the defender of Input–Independence.

Mellor's rationale concerns the original Independence principle (that C causes E is logically independent of what causes C and what E causes). But, as we have seen, this is besides the point. We can accept this principle without contradiction, because it does not entail Input–Independence. Are there other arguments available? I will now canvas some possible candidates, but argue that they do not give us reason to apply Input-Independence to loops.

One argument for applying Independence to loops might be that, by analogy, since we normally take causes as independent and effects as dependent, and in loops we claim to be dealing with causes and effects, i.e., P in our loop is an initial cause, so we should allow $f(\mathrm{P})$ in the loop to be a free variable. This is an argument by analogy: as in the linear case, so also for loops; input frequencies are free in the linear case, so they should be in loops; $f(\mathrm{P})$ is input frequency, therefore it should be free. But it's not clear why the reasoning should run this way. After all, P in the loop is both a cause and an effect. Why don't we reason: since effects are normally taken to be dependent variables, in the loop P is an effect and $f(\mathrm{P})$ is an output, so we should treat it as an effect, and not a free variable. This seems to be an equally good argument by analogy: Why should we want to treat $f(\mathrm{P})$ as if it were like a linear input rather than a linear output? Further, the argument by analogy faces the obvious objection that there is a suspiciously relevant-looking point of disanalogy between the linear case and loops: precisely that logically in the linear case inputs can be treated as free whereas in loops they cannot.

Another reason we normally take input frequencies to be independent is that we can manipulate them to control output frequencies, generally leaving the chances unaffected. We control the frequency of cancer by influencing the frequency of smoking, which does not affect the chances of cancer for smokers or non-smokers. But, (i) in loops we don't have the same capacity to manipulate input frequencies, and (ii) even to the extent that we do, we can't use it to control output frequency in the normal way.

One reason we can't manipulate input frequency $f(P)$ in the loop is simply that a cause of P in an actual loop is Q. For example, in the wormhole example, if we actually have a particular loop P-Q-P then the photon absorbed comes via the wormhole from the future emission by the atom. The photon in our actual loop has no history or future. It appears from the wormhole, is absorbed by the atom, is emitted by the atom, and disappears down the wormhole, from where it appears.

In such a case we do not bring about P by controlling the background incidence of photons. So in general, in our example if there are 10M cases of P, then 5M of them are caused by Q and 5M of them arise in the absence of Q. Supposing that we have control over those cases where P occurs in the absence of Q, we still have only limited control over $f(P)$, because although we can increase the relative number of P's in the absence of Q, this gives us no control over the relative incidence of P-with-Q compared to cases where P does not occur. For this reason, in general we cannot control $f(P)$.

Another argument for applying Input–Independence to loops is a definitional argument. Suppose we assert that the meaning of 'cause' is in part 'an event whose frequency is independent of the chances it and its absence gives its effect, and of the frequency of the effect'; and the meaning of 'effect' is in part 'an event whose frequency is dependent on its chances and the frequency of its cause'. Then it will follow that neither P nor Q in our loop deserve the label 'cause', or 'effect'.

But why should we accept such a definition? There may be reasons why we should, roughly of the form that if we accept the definition certain features of causation would be explained. However, as arguments for applying Input–Independence to loops, these reasons beg the question.

To illustrate, I give one example. The definition explains why the causal relation is asymmetric in the logical sense. (Every two-place relation is either symmetric, asymmetric, or non-symmetric. Relation Rab is symmetric if Rab entails Rba, asymmetric if Rab entails not-Rba, and non-symmetric if it entails neither.) Our definition indeed entails that the singular causal relation is asymmetric. But why should we accept that the causal relation is asymmetric? Clearly causation is not symmetric, but is it asymmetric or non-symmetric? The only way I know of to determine such formal properties of a relation is by inspection of instances. But we have a consistent instance of causation where P is the cause and the effect of Q. Therefore, even if in most cases causes are not the effects of their effects, since there is one case which is, it follows that the causal relation is non-symmetric. Thus to suppose that causation is asymmetric in the logical sense simply begs the question against loops, so we have no reason to accept the definition under consideration.

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Appendix 1

From the standard probability calculus we know that the frequency $f(Q)$ of Q type events in given circumstances is given by

$$f(Q) = f(Q|P)f(P) + f(Q|{\sim}P)f({\sim}P) \tag{2a}$$

and

$$f(P) = f(P|Q)f(Q) + f(P|{\sim}Q)f({\sim}Q). \tag{2b}$$

Also, since $f({\sim}Q|P) = 1 - f(Q|P)$, $f({\sim}Q|{\sim}P) = 1 - f(Q|{\sim}P)$, $f{\sim}(P|Q) = 1 - f(P|Q)$, and $f({\sim}P|{\sim}Q) = 1 - f(P|{\sim}Q)$, we also have, from (2a)

$$f({\sim}Q) = [1 - f(Q|P)]f(P) + [1 - f(Q|{\sim}P)]f({\sim}P) \tag{2c}$$

and, from (2b)

$$f({\sim}P) = [1 - f(P|Q)]f(Q) + [1 - f(P|{\sim}Q)]f({\sim}Q). \tag{2d}$$

So, in large samples, we have the approximations that:

$$f(Q) \approx q\,f(P) + q'\,f({\sim}P) \tag{3a}$$
$$f(P) \approx p\,f(Q) + p'\,f({\sim}Q) \tag{3b}$$
$$f({\sim}Q) \approx (1 - q)f(P) + (1 - q')f({\sim}P) \tag{3c}$$
$$f({\sim}P) \approx (1 - p)f(Q) + (1 - p')f({\sim}Q). \tag{3d}$$

To express this in terms of the number of cases of Q, P, ~Q, and ~P; since $f(Q) = N(Q)/\{N(Q) + N({\sim}Q)\}$ and $f(P) = N(P)/\{N(Q) + N({\sim}Q)\}$ (and similarly for ~Q and ~P):

$$N(Q) \approx q\ N(P) + q'\ N(\sim P) \tag{4a}$$
$$N(P) \approx p\ N(Q) + p'\ N(\sim Q) \tag{4b}$$
$$N(\sim Q) \approx (1 - q)N(P) + (1 - q')N(\sim P) \tag{4c}$$
$$N(\sim P) \approx (1 - p)N(Q) + (1 - p')N(\sim Q). \tag{4d}$$

Appendix 2

The 'initial' and 'final' frequencies in a loop are each a function of the four chances p, p', q, q'. To see this, we note firstly that (4a)–(4d) entail (5a) and (5b) as follows. Substituting (4b) and (4d) into (4a),

$$\begin{aligned} N(Q) &= q[pN(Q) + p'N(\sim Q)] + q'[(1 - p)N(Q) + (1 - p')N(\sim Q)] \\ &= (qp + q' - pq')N(Q) + (qp' + q' - q'p')N(\sim Q), \end{aligned}$$

from which we get (5a), and by similar reasoning, (5b):

$$N(Q)/N(\sim Q) \approx (p'q + q' - p'q')/(1 - pq - q' + pq') \tag{5a}$$
$$N(P)/N(\sim P) \approx (pq' + p' - p'q')/(1 - pq - p' + p'q). \tag{5b}$$