Response of a two-level atom to a narrow-bandwidth squeezed-vacuum excitation

A. Messikh,[1] R. Tanaś,[1] and Z. Ficek[2,3]
[1]Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[2]Department of Physics and Centre for Laser Science, The University of Queensland, Brisbane, Australia 4072
[3]Department of Applied Mathematics and Theoretical Physics, The Queen's University of Belfast, Belfast BT7 1NN, Northern Ireland
(Received 12 October 1998; revised manuscript received 3 September 1999; published 15 February 2000)

Using the coupled-system approach we calculate the optical spectra of the fluorescence and transmitted fields of a two-level atom driven by a squeezed vacuum of bandwidths smaller than the natural atomic linewidth. We find that in this regime of squeezing bandwidths the spectra exhibit unique features, such as a hole burning and a three-peak structure, which do not appear for a broadband excitation. We show that the features are unique to the quantum nature of the driving squeezed vacuum field and do not appear when the atom is driven by a classically squeezed field. We find that a quantum squeezed-vacuum field produces squeezing in the emitted fluorescence field which appears only in the squeezing spectrum while there is no squeezing in the total field. We also discuss a nonresonant excitation and find that depending on the squeezing bandwidth there is a peak or a hole in the spectrum at a frequency corresponding to a three-wave-mixing process. The hole appears only for a broadband excitation and results from the strong correlations between squeezed-vacuum photons.

PACS number(s): 42.50.Dv, 42.50.Lc

I. INTRODUCTION

The spontaneous-emission spectrum of a two-level atom damped into an ordinary vacuum is a Lorentzian function of frequency with the width given by the atomic linewidth. Gardiner [1] found that the spectrum can be fundamentally altered by placing the atom into a squeezed vacuum. In this case the decay rate of one of the two quadrature components of the atomic polarization can be slower than the normal decay rate, leading to a subnatural linewidth of the spontaneous-emission spectrum. The linewidth narrowing is one of the nonclassical effects which reveal the quantum nature of the squeezed-vacuum field. With the addition of a coherent driving field the spectrum is a triplet [2] with the linewidths depending on the relative phase between the coherent and the squeezed-vacuum fields [3]. In particular, the central peak of the triplet can either be much narrower or much broader than the atomic linewidth. Apart from the dependence on the phase, the squeezed vacuum can lead to the qualitative changes in the spectrum such as a suppression of the spectral lines [4,5], hole burning, and dispersive profiles [6–8].

A standard procedure to calculate the squeezing effects is to derive an appropriate master equation describing atomic evolution, which together with the quantum regression theorem is then used to calculate the spectrum. The derivation of the master equation is based on the assumption of the Born and Markov approximations, the later requires a broadband spectrum of the reservoir modes. However, present sources of squeezed light, which are subthreshold optical parametric oscillators [9–11], generate finite rather than broadband squeezed fields of bandwidths typically of the order of the atomic linewidth. This experimental fact has led to the investigation of new theoretical methods, different than the Born-Markov master equation method, of calculating the problem of a two-level atom in a squeezed vacuum.

The effect of a finite squeezing bandwidth on the atom-squeezed light interaction was investigated both numerically [12–14] and analytically [15–22]. The numerical approach was based on stochastic methods whereas the analytical approach involved a Markovian master equation in a dressed-atom basis of a driven system. The approaches were applied to analyze the narrowing of the fluorescence and absorption lines and confirmed the effect of the squeezed-vacuum field on the spectral linewidth, with the overall conclusion that a finite squeezing bandwidth degrades the narrowing of the spectral lines. Later, Gardiner [23] and Carmichael [24] have proposed the *coupled-system* approach in which the parametric oscillator producing the squeezed field is included to the system and the master equation describing the parallel evolution of the atom and the cavity field is obtained. This approach has been applied by Gardiner and Parkins [25] and Smyth *et al.* [26] to study the effect of squeezing bandwidth on the inhibition of atomic phase decays, spectral linewidth narrowing, and the anomalous features of the resonance fluorescence spectrum. The results show that the squeezing-induced effects disappear for squeezing bandwidths comparable to the atomic linewidth.

In this paper we show that a squeezed vacuum of bandwidths smaller than the natural linewidth can produce certain effects that are unique to narrow-bandwidth excitations and the quantum nature of squeezed light. Using the Gardiner-Parkins [25] master equation, we examine the fluorescence and transmitted spectra and find that the squeezed vacuum of bandwidths smaller than the atomic linewidth can produce a structure with three peaks, similar to the Mollow triplet, or can burn a hole at the line center. We explain the features as arising from squeezing produced by the atom which, on the other hand, results from quantum nature of the driving squeezed-vacuum field. The hole burning of squeezing origin has been reported previously in absorptive optical bistability [27], the field transmitted through a cavity [28], and resonance fluorescence [29]. However, in the systems considered the squeezing was produced by driving the atom with a weak coherent field. In the case considered here, the squeezing is produced without the presence of the coherent driving field. In stark contrast to the squeezing resulting from coherent excitation, we find that the squeezing produced by a quantum squeezed vacuum appears only in the squeezing spectrum, and there is no squeezing in the total fluorescence field.

II. COUPLED-SYSTEMS APPROACH

An experimentally realistic model of the atom–squeezed-light interaction must take into account the finite bandwidth of a squeezed-light source. The simplest and most convenient means of calculating the effects of finite squeezing bandwidth is provided by the coupled-system approach [25]. In this approach one considers a quantum system consisting of two subsystems. A field $b_{in}(1,t)$ drives the first system, and gives rise to an output $b_{out}(1,t)$ which, after a propagation delay τ, becomes the input field $b_{in}(2,t)$ to the second system. In the coupled-system approach it is assumed that the output from the first system drives the second system without there being any coupling back from the second system to the first, which experimentally can be achieved by appropriate isolation techniques. Such a one-way coupling is described in terms of an appropriately chosen Hamiltonian [23].

In our case the first system is a degenerate parametric oscillator (DPO), the output of which drives a two-level atom. The Hamiltonian of this system is given by [25]

$$H=H_{sys}+H_B+H_{int}, \tag{1}$$

where

$$H_{sys}=\hbar\omega_s a^\dagger a+\frac{i\hbar}{2}(\epsilon a^{\dagger 2}e^{-i2\omega_s t}-\epsilon^* a^2\, e^{i2\omega_s t})+\frac{1}{2}\hbar\omega_A\sigma_z, \tag{2}$$

$$H_B=\hbar\int_{-\infty}^{\infty}d\omega|\omega|b^\dagger(\omega)b(\omega), \tag{3}$$

$$\begin{aligned}H_{int}=&\,i\hbar\int_{-\infty}^{\infty}d\omega\,\kappa_1(\omega)[b^\dagger(\omega)\,a-a^\dagger\,b(\omega)]\\&+i\hbar\int_{-\infty}^{\infty}d\omega\,\kappa_2(\omega)[\sigma_- b^\dagger(\omega)e^{-i\omega\tau}-\sigma_+ b(\omega)e^{i\omega\tau}].\end{aligned} \tag{4}$$

The system Hamiltonian (2) describes the cavity mode at frequency ω_s which is pumped nonlinearly by a classical field with the amplitude ϵ and frequency $2\omega_s$. This consists of the degenerate parametric oscillator (the first system of the two-coupled systems), and the two-level atom with the transition frequency ω_A (the second system). In Eqs. (3) and (4), the operators $b(\omega)$ and $b^\dagger(\omega)$ are the boson annihilation and creation operators for the bath, $\kappa_1(\omega)$ describes the coupling of the cavity mode to the bath, and $\kappa_2(\omega)$ describes the coupling of the atom to the bath.

Using standard procedures [25,30] one can derive from the Hamiltonian (1) the master equation, which in the frame rotating with ω_s has the following form:

$$\begin{aligned}\dot\rho=&\frac{1}{2}[i\delta\sigma_z+(\epsilon a^{\dagger 2}-\epsilon^* a^2),\rho]+\frac{\kappa}{2}\{2\,a\rho\,a^\dagger-\rho\,a^\dagger\,a\\&-a^\dagger\,a\rho\}+\frac{\gamma}{2}[1+(1-\eta)N(\omega_s)]\{2\,\sigma_-\rho\,\sigma_+-\rho\,\sigma_+\,\sigma_-\\&-\sigma_+\,\sigma_-\rho\}+\frac{\gamma}{2}(1-\eta)N(\omega_s)\{2\,\sigma_+\rho\,\sigma_--\rho\,\sigma_-\,\sigma_+\\&-\sigma_-\,\sigma_+\rho\}-\sqrt{\eta\kappa\gamma}\{[\sigma_+,a\,\rho]+[\rho\,a^\dagger,\sigma_-]\},\end{aligned} \tag{5}$$

where $\delta=\omega_s-\omega_A$ is the detuning of the squeezing carrier frequency from the atomic resonance, $\kappa=2\pi\kappa_1(\omega_s)^2$ is the DPO cavity bandwidth and $\gamma=2\pi\kappa_2(\omega_s)^2$ is the natural atomic linewidth. The parameter η $(0<\eta\leq 1)$ describes the matching of the incident squeezed vacuum to the modes surrounding the atom. For perfect matching $\eta=1$, whereas $\eta<1$ for an imperfect matching, which is always the case in experimental situations [9–11]. We assume here an imperfect matching ($\eta<1$) and that the remaining nonsqueezed modes are in a thermal state with the mean photon number $N(\omega_s)$. However, in order to observe the effects of the squeezed vacuum on the atom the parameter η should be as close to unity as possible. This requirement could be difficult to achieve in experiments, although some schemes involving optical cavities have been proposed [19,31] and experimentally tested [32]. On the other hand, if the fluorescent field radiated by the atom to the nonsqueezed modes is to be observed, η cannot be exactly unity because the radiation rate to the nonsqueezed modes, which is $(1-\eta)\gamma$, would be zero and no fluorescence would be observed. In the coupled-systems approach one has a choice of detecting either transmitted light, or the fluorescent light radiated by the atom to the modes of the ordinary vacuum. The transmitted light is a superposition of the squeezed-vacuum field coming from the DPO and the field radiated by the atom to the modes occupied by the squeezed vacuum.

The master equation (5), compared to the standard Gardiner-Parkins form, contains extra terms $(1-\eta)N(\omega_s)$ which represent an external thermal noise to the atom. In other words, we assume that the atom ''sees'' a fraction $(1-\eta)$ of modes that are in a thermal state. We have added the thermal field to the master equations to model a classically squeezed field driving the atom. The classically squeezed field can be modeled by real Gaussian fields [33,34] or by the output from an empty two-sided cavity in which one of the two mirrors is driven by a broadband white-noise field [35]. We propose a different scheme which allows us to use the same master equation to model the interaction of the atom with a quantum or a classically squeezed field. For $N=0$ the atom is driven by the output of the DPO, which is a quantum squeezed-vacuum field with positive fluctuations in one of the quadrature components and negative fluctuations in the other component. When $N\neq 0$ the atom is simultaneously driven by a thermal field and the output from the

DPO. By a suitable choice of the value of N, the effective field driving the atom can have positive fluctuations in one of the field quadrature components and no fluctuations in the other, which corresponds to a classically squeezed field.

In Sec. III we solve the master equation (5) numerically to find the evolution of the atomic and cavity fields, and use the solution as a starting point to calculate the steady-state spectra for both the transmitted and resonance fluorescence fields. We focus on the cases of the squeezing bandwidths smaller than the natural atomic linewidth. In this regime of squeezing bandwidths the broadband squeezing assumption is not valid, and analytical results based on the standard Born-Markov master equation are not applicable.

III. OPTICAL SPECTRA FOR TRANSMITTED AND FLUORESCENT FIELDS

Effective numerical solutions of the master equation (5) are possible when the mean number of photons $\langle a^\dagger a\rangle$ in the cavity is small ($\langle a^\dagger a\rangle<1$) [25,36]. In this case it is sufficient to take about ten lowest photon states as a basis of the photon Hilbert space and the two atomic states that form the atomic Hilbert space. Steady-state solutions of the master equation (5) together with the quantum regression theorem allow us to find optical spectra for the transmitted and fluorescent fields as well as atomic quadrature noise spectra for the cases when squeezing bandwidth is smaller or comparable to the natural atomic linewidth.

The transmitted field can be described by the (collapse) operator [24,37]

$$C=\sqrt{\kappa}\, a+\sqrt{\eta\gamma}\,\sigma_- , \tag{6}$$

which is a superposition of the incident squeezed-vacuum field and the field radiated by the atom into the squeezed-field modes. The rate of the atomic radiation that goes to the squeezed modes is equal to $\eta\gamma$, and the fraction $(1-\eta)\gamma$ of the radiation that goes to the remaining (ordinary vacuum) modes constitutes the resonance fluorescence. The photon flux of the transmitted light is given by

$$\begin{aligned}\langle C^\dagger C\rangle_{ss}&=\kappa\langle a^\dagger a\rangle_{ss}+\eta\gamma\langle\sigma_+\sigma_-\rangle_{ss}\\&\quad+\sqrt{\eta\kappa\gamma}\langle a^\dagger\sigma_-+\sigma_+ a\rangle_{ss},\end{aligned} \tag{7}$$

and the flux of fluorescent photons that goes to the ordinary vacuum modes is $(1-\eta)\gamma\langle\sigma_+\sigma_-\rangle_{ss}$, where $\langle\dots\rangle_{ss}$ denotes the steady-state mean value. The total flux is thus

$$\begin{aligned}\langle C^\dagger C\rangle_{ss}+(1-\eta)\gamma\langle\sigma_+\sigma_-\rangle_{ss}&=\kappa\langle a^\dagger a\rangle_{ss}+\gamma\langle\sigma_+\sigma_-\rangle_{ss}\\&\quad+\sqrt{\eta\kappa\gamma}\langle a^\dagger\sigma_-+\sigma_+ a\rangle_{ss}.\end{aligned} \tag{8}$$

Since the photon flux incident on the atom is $\kappa\langle a^\dagger a\rangle_{ss}$, the last two terms in Eq. (8) must cancel each other to conserve the energy. This means that the steady-state correlations between the cavity field and atomic operators play an important role in the process.

The steady-state spectrum of the transmitted field can be defined as the Fourier transform of the correlation function

$$\mathcal{T}(\omega)=2\,\mathrm{Re}\left\{\int_0^\infty\langle C^+(0),C(\tau)\rangle_{ss}e^{i(\omega-\omega_s)\tau}d\tau\right\}, \tag{9}$$

where Re denotes the real part of the integral, and we use the notation $\langle a,b\rangle\equiv\langle ab\rangle-\langle a\rangle\langle b\rangle$ for the covariance.

The incoherent part of the stationary fluorescence spectrum of a two-level atom is given by the Fourier transform of the two-time atomic correlation function as

$$\mathcal{F}(\omega)=(1-\eta)\gamma\, 2\,\mathrm{Re}\left\{\int_0^\infty\langle\sigma^+(0),\sigma^-(\tau)\rangle_{ss}e^{i(\omega-\omega_s)\tau}d\tau\right\}, \tag{10}$$

where we take into account that only a fraction $(1-\eta)\gamma$ of the radiation goes to the nonsqueezed modes.

We can relate the incoherent part of the fluorescence spectrum to the quadrature noise spectrum (squeezing spectrum) as [28,29]

$$\mathcal{F}(\omega+\omega_s)=S_X(\omega)+S_Y(\omega)+S_A(\omega), \tag{11}$$

where

$$\begin{aligned}S_X(\omega)=(1-\eta)\gamma\,\mathrm{Re}\int_0^\infty&\cos(\omega\tau)[\langle\sigma_+(0),\sigma_-(\tau)\rangle_{ss}\\&+\langle\sigma_+(0),\sigma_+(\tau)\rangle_{ss}]\,d\tau,\end{aligned} \tag{12}$$

$$\begin{aligned}S_Y(\omega)=(1-\eta)\gamma\,\mathrm{Re}\int_0^\infty&\cos(\omega\tau)[\langle\sigma_+(0),\sigma_-(\tau)\rangle_{ss}\\&-\langle\sigma_+(0),\sigma_+(\tau)\rangle_{ss}]\,d\tau,\end{aligned} \tag{13}$$

are, respectively, in-phase and out-of-phase quadrature components of the noise spectrum, and

$$S_A(\omega)=-2(1-\eta)\gamma\int_0^\infty\sin(\omega\tau)\mathrm{Im}\,\langle\sigma_+(0),\sigma_-(\tau)\rangle_{ss}\,d\tau \tag{14}$$

is the asymmetric contribution to the spectrum. If the atomic two-time correlation function is real, $S_A(\omega)=0$, and then the fluorescence spectrum is symmetric. The squeezing spectra for the transmitted field can be defined in a similar way by replacing σ_- and σ_+ operators by C and $C^\dagger$ operators and omitting the factor $(1-\eta)\gamma$.

Integrating the squeezing spectrum components over all frequencies gives the variances of the total fluorescence field

$$F_X=(1-\eta)\gamma[\langle\sigma_+\sigma_-\rangle_{ss}-|\langle\sigma_+\rangle_{ss}|^2-\langle\sigma_+\rangle_{ss}^2], \tag{15}$$

$$F_Y=(1-\eta)\gamma[\langle\sigma_+\sigma_-\rangle_{ss}-|\langle\sigma_+\rangle_{ss}|^2+\langle\sigma_+\rangle_{ss}^2]. \tag{16}$$

Squeezing in the total fluorescence field is defined by the requirement that either F_X or F_Y is negative, which can happen only if the stationary atomic dipole moment $\langle\sigma_+\rangle_{ss}$ is different from zero. For a two-level atom driven by the output of a DPO the atomic dipole moment $\langle\sigma_+\rangle_{ss}=0$ indepen-

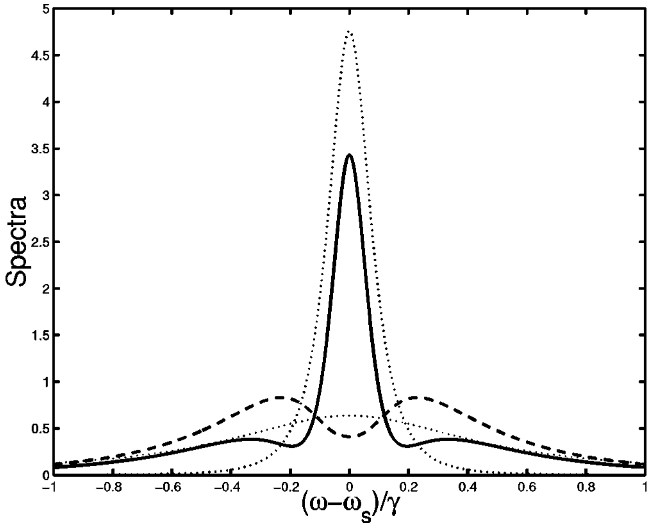

FIG. 1. Optical spectra of the fluorescent (solid line) and transmitted (dashed line) fields for $\gamma=1$, $\kappa=0.3$, $\epsilon=\kappa/6$, and $\eta=0.9$. For reference we have added with dotted lines the Lorentzian with the atomic linewidth $\gamma=1$ (broader) and the spectrum of the DPO (narrower). All the parameters are scaled to the atomic linewidth γ, and the optical spectra are normalized to give unit area under the curve.

dent of the parameters used, indicating that the total field variances F_X and F_Y are always positive. It follows that the total fluorescence field does not exhibit squeezing. Nevertheless, we will show that even in this case there is a strong squeezing possible in the squeezing spectrum.

In Sec. IV, we will plot the transmitted and fluorescence spectra for narrow squeezing bandwidths and explain their unusual features using the squeezing spectra. For a better comparison of the linewidths and shapes, we will normalize the spectra to the unit area.

IV. RESULTS

Gardiner and Parkins [25] have found that the squeezing-induced line narrowing in the fluorescence spectrum appears only for the cavity linewidths κ sufficiently large with respect to the atomic natural linewidth γ. They have shown that the narrowing decreases with decreasing κ and disappears for $\kappa\approx\gamma$. Here, we calculate optical spectra of the fluorescence and transmitted fields for the case when the cavity damping rate κ is smaller than γ. In Fig. 1 we show the spectra for $\delta=0$, $\kappa=0.3$ (κ is measured in units of γ), the pump field $\epsilon=\kappa/6$, $N=0$, and $\eta=0.9$. We see that in this regime of the cavity linewidths the resonance fluorescence spectrum exhibits a three-peak structure and there is a hole at the center of the transmitted light spectrum. For reference, we plot the Lorentzian with the atomic linewidth (broader) and the the spectrum of the DPO output field (narrower). One can see that the fluorescence spectrum has two components: a broad background with the natural linewidth at the wings and a narrow peak with the width narrower than the DPO bandwidth at the center. In Fig. 2 we present both the resonance fluorescence and transmitted field spectra for $\epsilon=\kappa/10$, $\eta=0.9$, and different κ. For sufficiently large κ ($\kappa=4\gamma$), the spectrum is composed of a single Lorentzian peak, whose shape changes as κ decreases. When κ approaches γ a hole starts to appear at the center of the spectral line and then a three-peak structure emerges as κ decreases below γ.

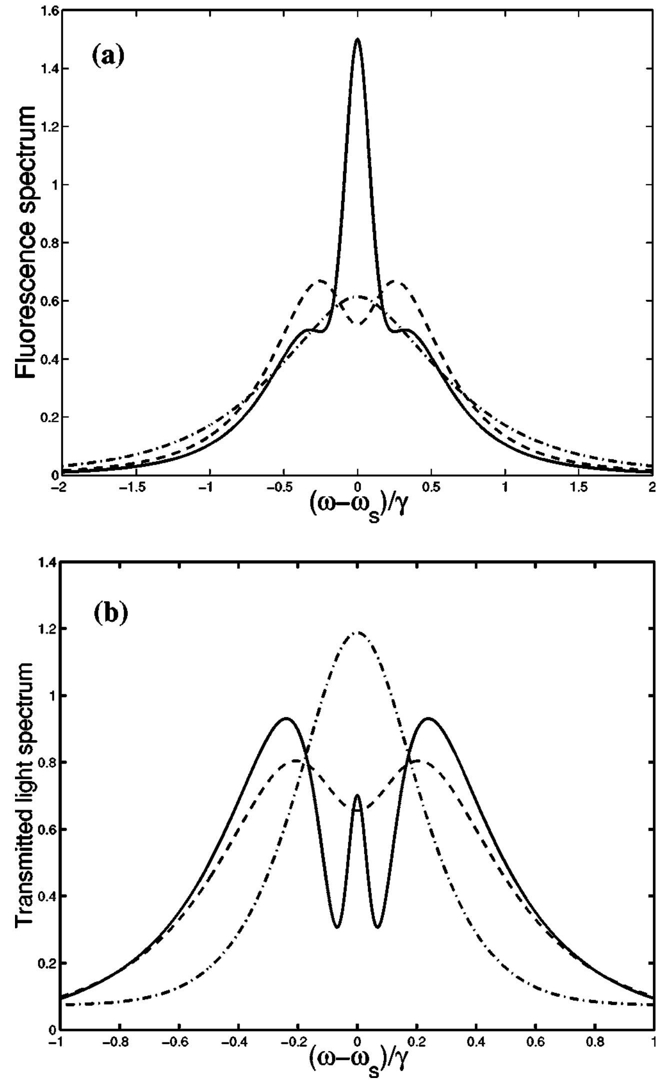

FIG. 2. Normalized (a) resonance fluorescence and (b) transmitted field spectra for various values of the cavity damping κ: $\kappa=0.2$ (solid line), $\kappa=0.5$ (dashed line), and $\kappa=1$ (dashed-dotted line) for $\epsilon=\kappa/10$ and $\eta=0.9$.

According to Eq. (11), the appearance of the unusual features in the fluorescence spectrum can be explained by analyzing the squeezing spectra of the fluorescence field. In Fig. 3 we plot the squeezing spectra for the fluorescent field defined by Eqs. (12) and (13). The $S_Y(\omega)$ quadrature is negative for frequencies near the carrier frequency ω_s i.e., it shows a quantum squeezing [38,39]. The $S_X(\omega)$ quadrature is positive, and adding the two quadrature components gives the fluorescence spectrum shown in Fig. 1. Clearly, the negative values of $S_Y(\omega)$ are responsible for the unusual shape of the resonance fluorescence spectrum. Thus the three-peak structure and hole burning result from quantum squeezing in the fluorescence field. In Fig. 3 we also show the squeezing

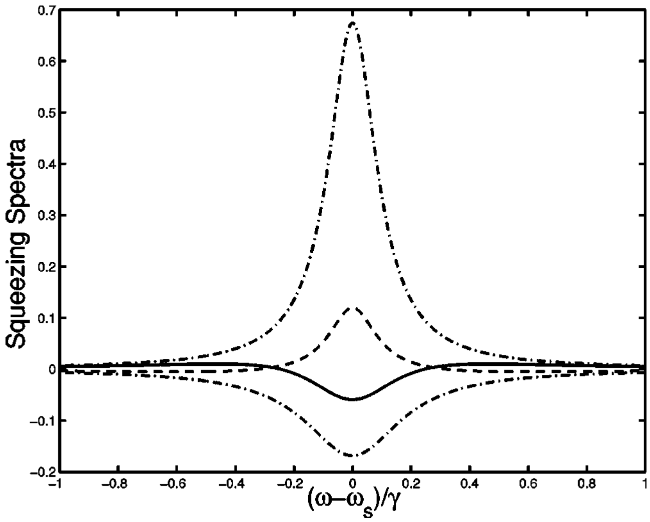

FIG. 3. Squeezing spectra $S_X(\omega)$ (dashed line) and $S_Y(\omega)$ (solid line) of the fluorescent field, and the DPO output field (dashed-dotted lines) for the same parameters as in Fig. 1. The squeezing spectra are plotted according to their definitions without additional normalization.

spectra of the DPO output field, which drives the atom. We see that the quantum squeezing in the fluorescence field results from a quantum squeezing in the DPO output field. If we replace the driving field by an analog of a classical squeezed-vacuum field the emitted fluorescence field does not exhibit quantum squeezing and consequently the unusual features in the fluorescence spectrum disappear. This is shown in Fig. 4, where we plot the fluorescence spectrum [Fig. 4(a)] for quantum and classically squeezed driving fields and the squeezing spectra [Fig. 4(b)] of the fluorescence field for the classically squeezed driving field. It is evident from Fig. 4 that for the classically squeezed driving field the spectra of the fluorescence field are manifestly different from that of the quantum driving field.

In Fig. 5 we plot the squeezing spectra for the transmitted field and compare them with squeezing spectra of the DPO output field. The spectra of the transmitted field show a dip in one quadrature and a peak in the other quadrature at the central frequency ω_s. This feature of the transmitted light noise spectra is caused by the atomic resonance fluorescence to the nonsqueezed modes. For a small κ ($\kappa=0.5$), shown in Fig. 5(a), the noise spectra of the transmitted field are essentially broadened with respect to their counterparts in the DPO field. The appearance of the hole in one quadrature is also evident. Even for sufficiently large value of $\kappa=10$, as seen in Fig. 5(b), there is still an important difference at the center with a very pronounced peak with the width of the atomic linewidth and the dip in the other quadrature.

The features discussed here depend crucially on the value of η, which should be as close to unity as possible to have the coupling between the two subsystems as high as possible. On the other hand, there is only a fraction $(1-\eta)\gamma$ of the radiation that goes to the modes different from the squeezed-vacuum modes, and this rate must be nonzero to observe resonance fluorescence to the nonsqueezed modes. In our calculations presented in Figs. 1–6 we have assumed $\eta=0.9$. The features, however, degrade quickly as η decreases and disappear for $\eta\approx0.6$. This is shown in Fig. 6, where we plot the fluorescent and transmitted field spectra for different values of η.

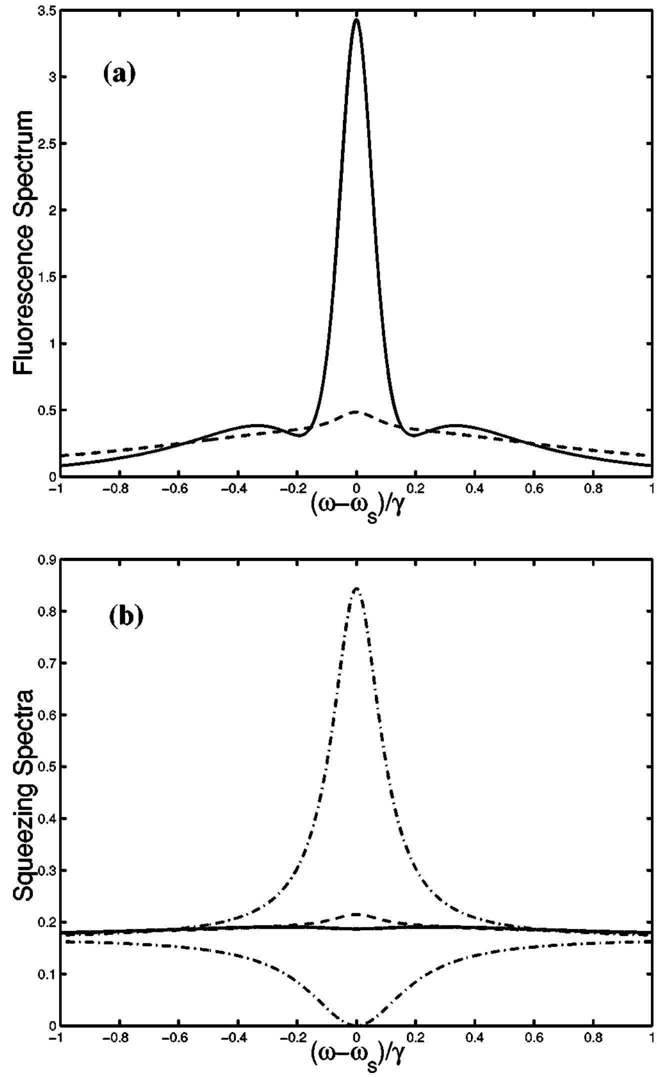

FIG. 4. (a) Fluorescence spectra for a quantum squeezed-vacuum driving field (solid line) and a classically squeezed driving field (dashed line) with $N=3.375$. The other parameters are the same as in Fig. 1. (b) Squeezing spectra $S_Y(\omega)$ (solid line) and $S_X(\omega)$ (dashed line) of the fluorescence field for a classically squeezed driving field. Dashed-dotted lines show squeezing spectra of the classically squeezed driving field. The parameters are the same as in (a).

So far we have discussed the resonant case $\delta=0$. Now we will discuss a nonresonant case. Analytical calculations of the fluorescence spectrum of a two-level atom in an off-resonance broadband squeezed vacuum show that there is a hole at the frequency $\omega=2\omega_s-\omega_A=\omega_A+2\delta$ corresponding to a three-wave-mixing process [40]. In the case of a narrow bandwidth and $\delta\neq0$ we can calculate the spectrum from the master equation (5) using the same method as for the resonant case. The results are shown in Fig. 7. For small κ the spectrum shows a narrow central peak and a small peak at frequency $\omega_A+2\delta$. There is no peak at the atomic fre-

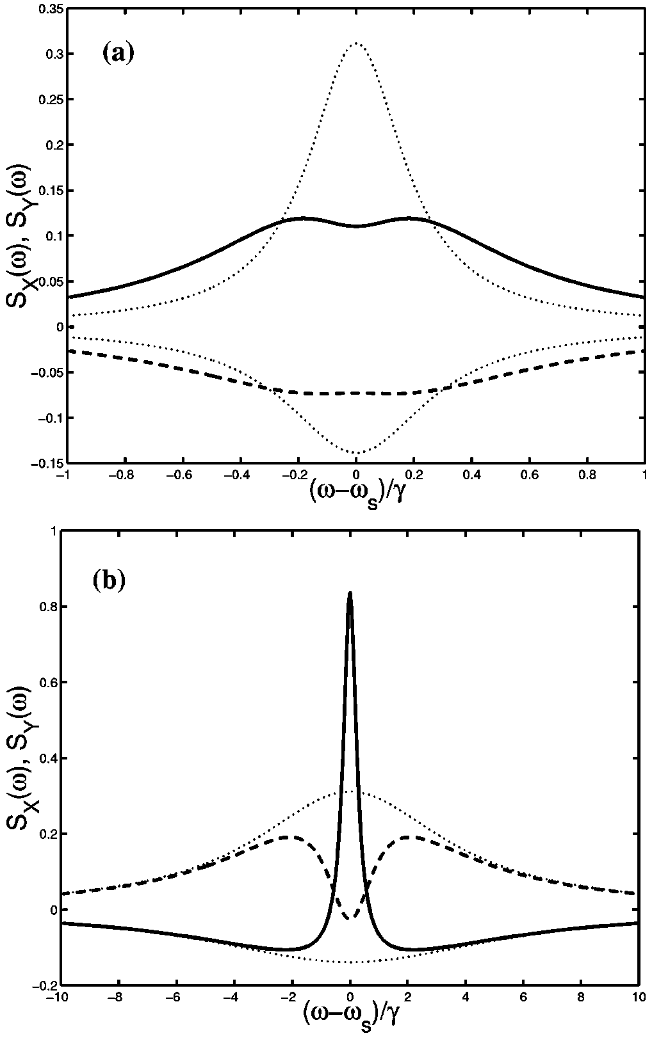

FIG. 5. Squeezing spectra $S_X(\omega)$ (dashed line) and $S_Y(\omega)$ (solid line) for the transmitted field compared to their counterparts of the DPO output field (dotted lines); (a) $\kappa=0.5$ and (b) $\kappa=10$.

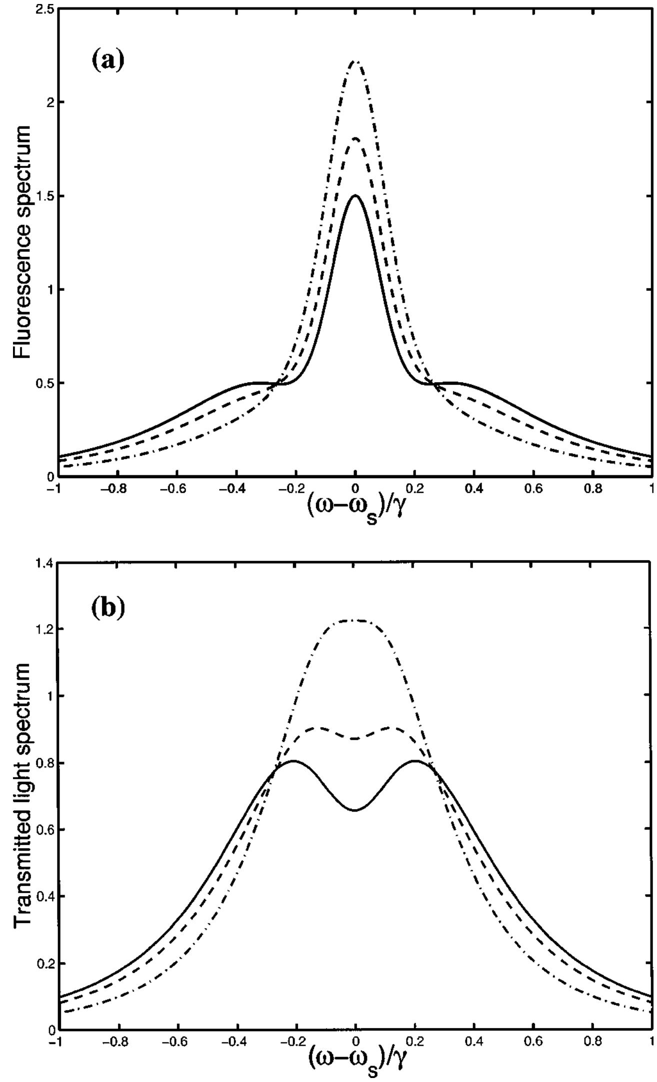

FIG. 6. Normalized (a) fluorescent and (b) transmitted light spectra for various values of η: $\eta=0.9$ (solid line), $\eta=0.8$ (dashed line), and $\eta=0.6$ (dashed-dotted line). Other parameters are: $\epsilon=\kappa/10$, and $\kappa=0.5$.

quency. The narrow peak arises from the elastic scattering of the incident squeezed field, and the small peak is due to the three-wave-mixing process induced by the two-photon correlations characteristic of the squeezed-vacuum field. As κ increases the central peak decreases and the peak at the atomic frequency grows and becomes dominant for large κ. Also, the three-wave-mixing peak at frequency $\omega_A+2\delta$ decreases with increasing κ and for very large κ (broad squeezed vacuum) the peak is replaced by a hole. Clearly, the squeezing bandwidth, given by κ, is entirely responsible for the properties of the three-wave-mixing process.

The properties of the three-wave-mixing process can be explained as follows. For small squeezing bandwidths the correlation time between two photons of frequencies $\omega_s-\delta$ and $\omega_s+\delta$ is very long. Therefore, the absorption of two correlated photons from the squeezed field is interrupted by a spontaneous emission of a photon of frequency $\omega_A+2\delta$, as it is shown in Fig. 8(a). The spontaneous emission gives the three-wave-mixing peak in the fluorescence spectrum. For large squeezing bandwidths, the correlation time between the photons is very short leading to a simultaneous absorption of two photons from the squeezed field. Absorbing these two photons the atom makes a transition to a virtual state, as is shown in Fig. 8(b). Next, the atom makes a stimulated transition to the state $|2\rangle$ emitting a photon of frequency $\omega_A+2\delta$ into the squeezed vacuum thereby partly canceling the fluorescence and burns a hole at $\omega_A+2\delta$.

We emphasize here that the origin of the hole burning for $\delta\neq 0$ is different from that of the resonant excitation. The latter is due to squeezing in the fluorescence field, whereas the former results from the strong two-photon correlations characteristic of the squeezed vacuum. Moreover, the hole burning for $\delta=0$ appears only for very small squeezing bandwidths in contrast to the hole burning for $\delta\neq 0$, which appears only in a broadband squeezed field.

V. CONCLUSIONS

We have studied the fluorescence and transmitted light spectra for a two-level atom driven by a narrow-bandwidth

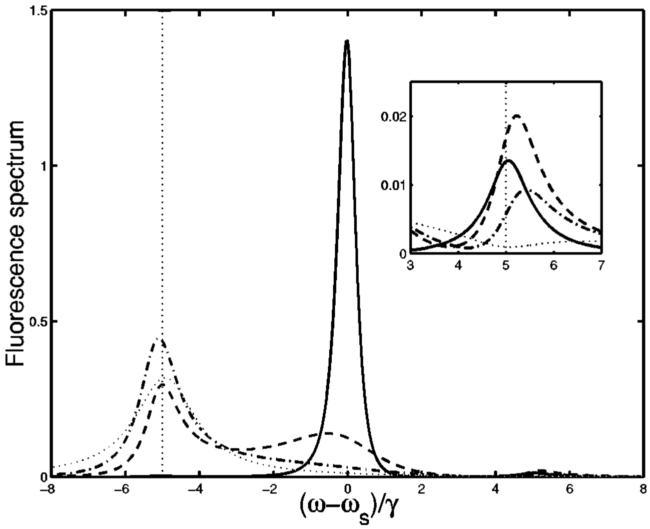

FIG. 7. Resonance fluorescence spectra for a non-resonant excitation with $\delta=5$, and $\kappa=1$ (solid line), $\kappa=5$ (dashed line), $\kappa=10$ (dashed-dotted line), and broadband squeezing (dotted line). Other parameters are: $\epsilon=\kappa/10$, and $\eta=0.9$. Vertical dotted lines mark the position of atomic resonance $\omega=\omega_A$ (main figure) and the three-photon resonance $\omega=2\omega_s-\omega_A$ (inset).

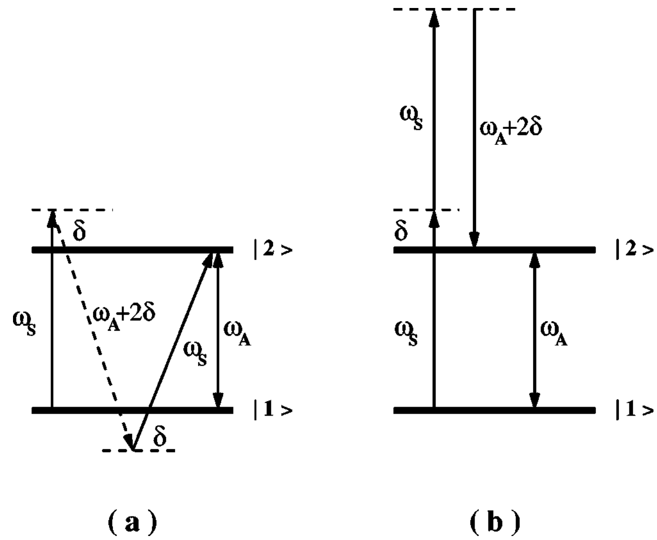

FIG. 8. Schematic diagram of a three-wave mixing process for a narrow bandwidth excitation (a), and a broadband excitation (b). The atom makes the transition $|1\rangle\rightarrow|2\rangle$ absorbing two photons from the squeezed vacuum field and emitting a photon of frequency $\omega_A+2\delta$.

squeezed-vacuum field produced by a degenerate parametric oscillator. Using the coupled-system approach we have analyzed the spectra for the case of squeezing bandwidths smaller than the atomic natural linewidth. Our results show that even for small squeezing bandwidths the spectra exhibit features that are unique for the quantum nature of squeezed light. We have found that the squeezed field of bandwidths smaller than the atomic linewidth burns a hole in the center of the spectrum or even can lead to a three-peak structure similar to the Mollow triplet. The features are strongly dependent on the squeezing bandwidth and do not extend into the regime of broadband excitation. Moreover, we have shown that the features are not present when the DPO output field is replaced by a classically squeezed field. The features arise from the quantum nature of the DPO output field and also reflect a quantum nature of the emitted fluorescence field. We have shown that the fluorescence field exhibits squeezing only in the squeezing spectrum with no squeezing in the total field. We have also calculated the spectra for an off-resonance excitation. In this case the fluorescence spectrum exhibits a hole at the three-wave-mixing frequency, which appears only for a broadband excitation. For a narrow-bandwidth excitation the hole is replaced by a peak. The three-wave-mixing structure originates from two-photon correlations characteristic of the squeezed-vacuum field.

ACKNOWLEDGMENTS

This research was partially supported by the Polish Scientific Research Committee (KBN Grant No. 2 P03B 73 13) and the Australian Research Council. We also thank Poznań Supercomputing and Networking Center for access to the computing facilities.

[1] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).
[2] B. R. Mollow, Phys. Rev. **188**, 1969 (1969).
[3] H. J. Carmichael, A. S. Lane, and D. F. Walls, J. Mod. Opt. **34**, 821 (1987).
[4] J. M. Courty and S. Reynaud, Europhys. Lett. **10**, 237 (1989).
[5] S. Swain and B. J. Dalton, Opt. Commun. **147**, 187 (1998).
[6] S. Smart and S. Swain, Phys. Rev. A **45**, 6863 (1992).
[7] S. Smart and S. Swain, Quantum Opt. **5**, 75 (1993).
[8] S. Swain, Phys. Rev. Lett. **73**, 1493 (1994).
[9] L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **l57**, 2520 (1986).
[10] N. P. Georgiades, E. S. Polzik, K. Edamatsu, H. J. Kimble, and A. S. Parkins, Phys. Rev. Lett. **75**, 3426 (1995).
[11] E. S. Polzik, J. L. Sorensen, and J. Hald, Appl. Phys. B: Lasers Opt. **66**, 759 (1998).
[12] C. W. Gardiner, A. S. Parkins, and M. J. Collett, J. Opt. Soc. Am. B **4**, 1683 (1987).
[13] A. S. Parkins and C. W. Gardiner, Phys. Rev. A **37**, 3867 (1988).
[14] H. Ritsch and P. Zoller, Phys. Rev. A **38**, 4657 (1988).
[15] A. S. Parkins, Phys. Rev. A **42**, 4352 (1990).
[16] A. S. Parkins, Phys. Rev. A **42**, 6873 (1990).
[17] G. Yeoman and S. M. Barnett, J. Mod. Opt. **43**, 2037 (1996).
[18] M. Bosticky, Z. Ficek, and B. J. Dalton, Phys. Rev. A **53**, 4439 (1996).
[19] Z. Ficek, B. J. Dalton, and M. R. B. Wahiddin, J. Mod. Opt. **44**, 1005 (1997).
[20] R. Tanaś, Z. Ficek, A. Messikh, and T. El-Shahat, J. Mod. Opt. **45**, 1859 (1998).
[21] R. Vyas and S. Singh, Phys. Rev. A **45**, 8095 (1992).

[22] Z. Ficek, J. Seke, and R. Kralicek, Phys. Rev. A **58**, 1597 (1998).
[23] C. W. Gardiner, Phys. Rev. Lett. **70**, 2269 (1993).
[24] H. J. Carmichael, Phys. Rev. Lett. **70**, 2273 (1993).
[25] C. W. Gardiner and A. S. Parkins, Phys. Rev. A **50**, 1792 (1994).
[26] W. S. Smyth, S. Swain, Z. Ficek, and M. Scott, Phys. Rev. A **57**, 585 (1998).
[27] L. A. Lugiato, Nuovo Cimento B **50**, 89 (1979).
[28] P. R. Rice and H. J. Carmichael, J. Opt. Soc. Am. B **5**, 1661 (1988).
[29] S. Swain and P. Zhou, Opt. Commun. **123**, 310 (1996).
[30] C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).
[31] A. S. Parkins and C. W. Gardiner, Phys. Rev. A **40**, 3796 (1989).
[32] Q. A. Turchetti, N. Ph. Georgiades, C. J. Hood, H. J. Kimble, and A. S. Parkins, Phys. Rev. A **58**, 4056 (1998).
[33] R. Walser, H. Ritsch, P. Zoller, and J. Cooper, Phys. Rev. A **45**, 468 (1992).
[34] P. Zhou and S. Swain, Phys. Rev. Lett. **82**, 2500 (1999).
[35] W. S. Smyth and S. Swain, Phys. Rev. A **59**, R2579 (1999).
[36] S. M. Tan, J. Opt. B: Quantum Semiclass. Opt. **1**, 424 (1999).
[37] P. Kochan and H. J. Carmichael, Phys. Rev. A **50**, 1700 (1994).
[38] M. J. Collett, D. F. Walls, and P. Zoller, Opt. Commun. **52**, 145 (1984).
[39] Z. Y. Ou, C. K. Hong, and L. Mandel, J. Opt. Soc. Am. B **4**, 1574 (1987).
[40] Z. Ficek and B. C. Sanders, J. Phys. B **27**, 809 (1994).