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Analysis of pore-fluid pressure gradient and effective vertical-stress gradient distribution in layered hydrodynamic systems

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SUMMARY

A theoretical analysis is carried out to investigate the pore-fluid pressure gradient and effective vertical-stress gradient distribution in fluid-saturated porous rock masses in layered hydrodynamic systems. Three important concepts, namely the critical porosity of a porous medium, the intrinsic pore-fluid pressure and the intrinsic effective vertical stress of the solid matrix, are presented and discussed. Using some basic scientific principles, we derive analytical solutions and explore the conditions under which either the intrinsic pore-fluid pressure gradient or the intrinsic effective vertical-stress gradient can be maintained at the value of the lithostatic pressure gradient. Even though the intrinsic pore-fluid pressure gradient can be maintained at the value of the lithostatic pressure gradient in a single layer, it is impossible to maintain it at this value in all layers in a layered hydrodynamic system, unless all layers have the same permeability and porosity simultaneously. However, the intrinsic effective vertical-stress gradient of the solid matrix can be maintained at a value close to the lithostatic pressure gradient in all layers in any layered hydrodynamic system within the scope of this study.

Key words: analytical solution, critical porosity, intrinsic effective vertical stress, intrinsic pore-fluid pressure, lithostatic pressure gradient.

1 INTRODUCTION

Whether or not the pore-fluid pressure gradient and/or the effective vertical-stress gradient of the solid matrix can be maintained at the value of the lithostatic pressure gradient in fluid-saturated porous rock masses in the Earth's crust is a fundamental question in the geosciences. This question has not been answered clearly to date. In order to gain a better understanding of the underlying physics behind this question, it is necessary to explore the conditions under which either the pore-fluid pressure gradient or the effective vertical-stress gradient of the solid matrix can be maintained at the value of the lithostatic pressure gradient. Notice that the emphasis here is on the pore-fluid pressure gradient rather than the pore-fluid pressure itself. This is because Darcy's law says that if there is an excess pore-fluid pressure gradient in a fluid-saturated porous rock mass, there must be a pore-fluid flow, no matter how large or small the excess pore-fluid pressure itself is. This clearly indicates that an intimate relationship exists between the excess pore-fluid pressure gradient and the pore-fluid flow in fluid-saturated porous rock masses. On the other hand, a change in the pore-fluid pressure gradient usually results in a change in the effective vertical-stress gradient of the solid matrix since the total vertical-stress gradient of the porous medium (solid part plus fluid part) in a layer having constant parameters must be constant, from the point of view of vertical force equilibrium. Although numerical methods (Zhao & Valliappan 1993; Zhao, Mühlhaus & Hobbs 1997, 1998) can be used to model complicated systems in the geosciences, it is very difficult to discuss and generalize, from numerical solutions, the conditions under which either the pore-fluid pressure gradient or the effective vertical-stress gradient of the solid matrix can be maintained at the value of the lithostatic pressure gradient in fluid-saturated porous rock masses. Thus, this paper aims to answer the above question through a theoretical analysis of the pore-fluid pressure gradient and effective vertical-stress gradient distributions in layered hydrodynamic systems.

To derive analytical solutions for problems in solid and/or fluid mechanics, it is common practice to make assumptions (Zhao & Steven 1996). According to Phillips (1991), the assumptions must, however, be firmly based and clearly specified. The theoretical analysis to be carried out in this paper is based on the following assumptions.

(1) All layers in a hydrodynamic system are horizontal in geometry and composed of fluid-saturated porous rock masses.

(2) Darcy's law is valid for describing the vertical pore-fluid flow.

(3) The areal porosity is equal to the volumetric porosity (Bear 1972; Phillips 1991) and is less than $\rho_s/(2\rho_s-\rho_f)$, where ρ_s and ρ_f are constants representing the densities of the solid particles and the pore fluid, respectively.

(4) The permeability and porosity are constants within each layer. This implies that compaction and consolidation of the medium have been completed so that the system is in a steady state.

(5) The total force or stress and the intrinsic pore-fluid pressure, i.e. the pore-fluid pressure averaged over the pore area of a representative elementary area (REA), are continuous at the interface between any two layers.

(6) For a multiple-layered system, the overall vertical pore-fluid flow is controlled by the least permeable layer. We call this layer the valve of the hydrodynamic system. Thus, the Darcy velocity, which is the velocity averaged over the total area of a representative elementary area (Nield & Bejan 1992; Phillips 1991), is constant in the vertical direction throughout all layers because the total flux of the pore-fluid flow must be constant at any horizontal cross-section of the system, from the point of view of mass conservation.

(7) The intrinsic pore-fluid pressure gradient does not exceed the lithostatic pressure gradient in any layer.

The rest of the contents of this paper are arranged as follows. In Section 2, some basic scientific principles are employed to derive analytical solutions for the intrinsic pore-fluid pressure gradient and the intrinsic effective vertical-stress gradient of the solid matrix in a single-layer hydrodynamic system. Three important concepts, the critical porosity of a fluid-saturated porous medium, the intrinsic pore-fluid pressure and the intrinsic effective vertical stress of the solid matrix, are presented and discussed in this section. In Section 3, the analytical solutions derived in Section 2 are used to derive analytical solutions for the intrinsic pore-fluid pressure gradient and the intrinsic effective vertical-stress gradient of the solid matrix in a layered hydrodynamic system. In Section 4, some numerical results are presented to show the intrinsic pore-fluid pressure gradient distribution and the intrinsic effective vertical-stress gradient distribution of the solid matrix in layered hydrodynamic systems. Finally, some conclusions are summarized in Section 5.

2 THEORETICAL ANALYSIS OF A SINGLE-LAYER HYDRODYNAMIC SYSTEM

Before a multiple-layer hydrodynamic system is analysed, it is necessary to consider a single-layer hydrodynamic system, because it is the single layer that forms the basic building block of a multiple-layer system. Generally, we need to consider the following three situations: (1) the pore-fluid pressure gradient is lithostatic; (2) the pore-fluid pressure gradient is hydrostatic; (3) the pore-fluid pressure gradient is between hydrostatic and lithostatic.

For the single-layer hydrodynamic system shown in Fig. 1, considering the equilibrium of any horizontal cross-section in the vertical direction leads to the following equation:

$$\phi\rho_f gh+(1-\phi)\rho_s gh=\phi\bar{P}_f+(1-\phi)\bar{\sigma}_s^v, \tag{1}$$

where ϕ is either the areal porosity or the volumetric porosity of the system; ρ_s and ρ_f are the densities of the solid particles and the pore fluid as defined above; $\bar{P}_f$ is the intrinsic pore-fluid pressure, defined as the pressure averaged over the pore area of an REA; $\bar{\sigma}_s^v$ is the intrinsic effective vertical stress in the solid matrix, defined as the vertical stress averaged over the solid area of an REA; and g is the acceleration due to gravity.

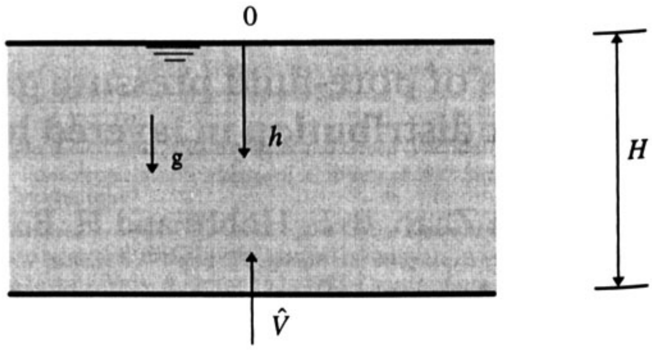

Figure 1. Single-layer hydrodynamic system with vertical throughflow.

It must be pointed out that when the vertical Darcy velocity in the system is not zero, the solid particles are subjected to drag forces from the flowing pore-fluid particles. Equally, the pore-fluid particles are subjected to resisting forces from the solid particles. Since the drag forces from the flowing pore-fluid particles and the resisting forces from the solid particles are a pair of action and reaction forces, Newton's third law says that they are equal in magnitude but opposite in direction. Thus, if a fluid-saturated porous medium (solid part and pore-fluid part) is considered as a whole system (as is the case here), these forces are internal interactive forces and, therefore, automatically cancel each other in the system. This is the reason why such forces do not appear in eq. (1).

According to Bear (1972), Phillips (1991) and Nield & Bejan (1992), the intrinsic pore-fluid pressure and the intrinsic effective vertical stress defined above can be expressed as

$$\bar{P}_f=\frac{1}{A_f}\int_{A_f}P_f\,dA,$$

$$\bar{\sigma}_s^v=\frac{1}{A_s}\int_{A_s}\sigma_s^v\,dA,$$

$$A_t=A_s+A_f,$$

$$\phi_A=\phi=\frac{A_f}{A_t}, \tag{2}$$

where A_t is the total area of an REA; A_s and A_f are the solid area and pore area, respectively, of the REA; ϕ_A is the areal porosity; P_f is the microscopic pore-fluid pressure of the pore fluid; and σ_s^v is the microscopic vertical stress of the solid particles.

In order to facilitate the analysis, we also define the following two quantities:

$$\hat{P}_f=\frac{1}{A_t}\int_{A_t}P_f\,dA,$$

$$\hat{\sigma}_s^v=\frac{1}{A_t}\int_{A_t}\sigma_s^v\,dA, \tag{3}$$

where $\hat{P}_f$ is the pore-fluid pressure averaged over the total area of the REA and $\hat{\sigma}_s^v$ is the vertical stress averaged over the total area of the REA.

From eqs (2) and (3), the following equations can be obtained:

$$\hat{P}_f = \phi \bar{P}_f,$$

$$\hat{\sigma}_s^v = (1-\phi)\bar{\sigma}_s^v. \tag{4}$$

For a porosity close to one, Terzaghi (1925, 1943, 1960) defined $\hat{\sigma}_s^v$ as the effective stress of the solid matrix. Since, however, the porosity is far from one, e.g. $\phi \le 0.614$ when the intrinsic pore-fluid (water) pressure gradient is lithostatic, it is appropriate to use the intrinsic effective stress defined in this section to represent the macroscopic vertical stress of the solid matrix. Clearly, since the porosity has to be greater than zero, the effective stress defined by Terzaghi is always less than the intrinsic effective stress defined above. This fact was addressed by Taylor (1948) and further discussed in detail by Garg & Nur (1973). Recently, Vardoulakis & Sulem (1995) used modern mixture theory to deal with the pore-fluid pressure and the contact stress of the solid. It is interesting to note that the partial stress for the pore fluid and the partial (vertical) stress for the solid defined by Vardoulakis & Sulem (1995) are exactly the same as the intrinsic pore-fluid pressure and the intrinsic effective vertical stress of the solid matrix defined here.

By adding the pore-fluid pressure $\hat{P}_f$ and the effective vertical stress $\hat{\sigma}_s^v$ together, the total vertical stress on a representative elementary area can be expressed as

$$\hat{\sigma}_t = \hat{P}_f + \hat{\sigma}_s^v, \tag{5}$$

where $\hat{\sigma}_t$ is the total vertical stress, which is the vertical stress averaged over the total area of the REA.

2.1 Pore-fluid pressure gradient is lithostatic in the layer

From modern mixture theory (Vardoulakis & Sulem 1995), a pore-fluid-saturated porous medium can be considered as a medium of two phases, a solid phase and a pore-fluid phase. Thus, it is not only convenient but also technically appropriate to use the densities of the solid particles and the pore fluid to express the lithostatic pressure ($P_{\text{lithostatic}} = \rho_s gh$) and the hydrostatic pressure ($P_{\text{hydrostatic}} = \rho_f gh$) in the layered hydrodynamic system considered here (England *et al.* 1987).

In this case, the intrinsic pore-fluid pressure is expressed as

$$\bar{P}_f = \rho_s gh. \tag{6}$$

Substituting eq. (6) into eq. (1) yields

$$\bar{\sigma}_s^v = \rho_s gh - \frac{\phi}{1-\phi}(\rho_s - \rho_f)gh. \tag{7}$$

As shown in Fig. 1, we define h to be positive in the downward direction and the vertical Darcy velocity $\hat{V}$ to be positive in the upward direction. With such definitions, Darcy's law reads (Phillips 1991)

$$\frac{\mu}{K}\hat{V} = \frac{\partial \bar{P}'}{\partial h}, \tag{8}$$

where K is the permeability of the porous medium, μ is the viscosity of the pore fluid and $\bar{P}'$ is the excess pore-fluid pressure averaged over the pore area of an REA.

For the hydrodynamic system considered here, we have

$$\bar{P}' = \bar{P}_f - \rho_f gh = (\rho_s - \rho_f)gh. \tag{9}$$

Therefore,

$$\frac{\partial \bar{P}'}{\partial h} = (\rho_s - \rho_f)g. \tag{10}$$

Substituting eq. (10) into eq. (8) yields the following equation:

$$(\rho_s - \rho_f)g = \frac{\mu}{K}\hat{V}. \tag{11}$$

Note that a non-zero vertical Darcy velocity is needed to enable both the intrinsic pore-fluid pressure and its gradient to be lithostatic in the layer. Thus, the driving force that maintains the intrinsic pore-fluid pressure gradient at the value of the lithostatic pressure gradient seems to be the non-zero vertical Darcy velocity in the layer. This implies that maintaining the pore-fluid pressure gradient at the value of the lithostatic pressure gradient requires that there be an inexhaustible fluid source underneath the layer. This can be considered as the first condition under which the intrinsic pore-fluid pressure gradient may be maintained at the value of the lithostatic pressure gradient.

It should be pointed out that although the entrapped pore-fluid pressure may reach or even exceed the lithostatic pressure in an overpressured system consisting of a permeable layer and two adjacent impermeable layers (Powley 1990; Fisher & Brantley 1992; Yardley 1997), the entrapped pore-fluid pressure gradient cannot reach or exceed the lithostatic pressure gradient, as can be seen from considering the following two possibilities. (1) If the permeable layer is bounded by two relatively impermeable layers, the permeabilities of which can be relatively very small but not equal to zero, the overpressure (i.e. the pore-fluid pressure minus the hydrostatic pressure) cannot exist in a steady state, since this overpressure must be dissipated at the completion of consolidation. (2) On the other hand, if the permeable layer is bounded by two absolutely impermeable layers, the permeabilities of which are exactly equal to zero, the entrapped pore fluid behaves just like the fluid in a pressure vessel. In this case, the intrinsic pore-fluid pressure gradient must be exactly equal to zero in the absolutely impermeable layers. This is because there is no pore fluid and, of course, the pore-fluid pressure itself must be exactly zero in these absolutely impermeable layers. Also, the intrinsic pore-fluid pressure gradient must be exactly equal to zero in the permeable layer, because Pascal's law requires that the entrapped pore-fluid pressure is constant within this closed permeable layer. Thus, no matter how the permeable layer is bounded in an overpressured system, the intrinsic pore-fluid pressure gradient must be less than the lithostatic pressure gradient unless there is a throughflow across this system.

Substituting eq. (11) into eq. (7) yields

$$\bar{\sigma}_s^v = \rho_s gh - \frac{\phi\mu}{(1-\phi)K}\hat{V}h. \tag{12}$$

It is clear that $\bar{\sigma}_s^v < \rho_s gh$, i.e. the intrinsic effective vertical stress (gradient) of the solid matrix is less than the lithostatic pressure (gradient) in this situation.

Letting the intrinsic effective vertical stress be zero, one can deduce the critical porosity ϕ_{critical} for which the intrinsic effective vertical stress of the solid matrix vanishes. From the equations above, the critical porosity for this situation can be shown to be

$$\phi_{\text{critical}} = \frac{\rho_s}{2\rho_s - \rho_f}. \tag{13}$$

If $\rho_s = 2700\ \text{kg m}^{-3}$ and $\rho_f = 1000\ \text{kg m}^{-3}$, the corresponding critical porosity is approximately 0.614. This implies that if the porosity of a porous medium exceeds its corresponding critical value (0.614 in this case), it is impossible to maintain the intrinsic pore-fluid pressure gradient at the value of the lithostatic pressure gradient unless the tensile strength of the solid matrix is greater than zero. This is the second condition under which the intrinsic pore-fluid pressure gradient may be maintained at the value of the lithostatic pressure gradient.

2.2 Pore-fluid pressure gradient is hydrostatic in the layer

In this case, the vertical Darcy velocity must be identical to zero. Thus, the relevant results can be expressed as

$$\bar{\sigma}_s^v = \rho_s g h,$$

$$\bar{P}_f = \rho_f g h. \tag{14}$$

Note that eq. (14) indicates that the intrinsic effective vertical-stress gradient of the solid matrix is lithostatic, while the intrinsic pore-fluid pressure gradient is hydrostatic.

2.3 Pore-fluid pressure gradient is between hydrostatic and lithostatic

In this situation, the intrinsic pore-fluid pressure can be expressed as

$$\bar{P}_f = \rho_c g h, \tag{15}$$

where $\rho_f < \rho_c < \rho_s$. This situation can occur in the more permeable layers in a multiple-layer hydrodynamic system.

From Darcy's law, we have

$$(\rho_c - \rho_f) g = \frac{\mu}{K} \hat{V}. \tag{16}$$

Thus,

$$\rho_c g = \rho_f g + \frac{\mu}{K} \hat{V}. \tag{17}$$

Substituting eq. (17) into eq. (15) yields

$$\bar{P}_f = \rho_f g h + \frac{\mu}{K} \hat{V} h. \tag{18}$$

Similarly, considering the equilibrium in the vertical direction leads to the following equation:

$$\bar{\sigma}_s^v = \rho_s g h - \frac{\phi \mu}{(1-\phi) K} \hat{V} h. \tag{19}$$

Clearly, eq. (18) indicates that the intrinsic pore-fluid pressure gradient is greater than the hydrostatic pressure gradient but less than the lithostatic pressure gradient. Eq. (19) indicates that the intrinsic effective vertical-stress gradient of the solid matrix is less than the lithostatic pressure gradient in this situation.

Following the same considerations as in Section 2.1, the corresponding critical porosity for this situation can be derived and expressed as

$$\phi_{\text{critical}} = \frac{\rho_s}{\rho_s + \rho_c - \rho_f}. \tag{20}$$

It is worth noting that as ρ_c tends to ρ_s, ϕ_{critical} approaches $\rho_s/(2\rho_s - \rho_f)$, which is identical to the value of the critical porosity when the intrinsic pore-fluid pressure gradient is lithostatic in the layer.

3 SOME SOLUTIONS FOR LAYERED HYDRODYNAMIC SYSTEMS

Using the analytical results in the last section, the intrinsic pore-fluid pressure and the intrinsic effective vertical stress of the solid matrix can be derived for layered hydrodynamic systems.

3.1 Two-layer hydrodynamic system

For the two-layer hydrodynamic system shown in Fig. 2, it is assumed that the top layer (layer 1) is more permeable than the bottom layer (layer 2). Layer 2 is the valve of this two-layer hydrodynamic system, which controls the overall vertical throughflow. If the pore-fluid pressure gradient in the bottom layer is lithostatic, then the pore-fluid pressure gradient in the top layer must be between hydrostatic and lithostatic, and will depend on the ratio of the permeabilities of the top layer and the bottom layer. The general solution for the intrinsic pore-fluid pressure in the system can be expressed as

$$\bar{P}_{f1}(h) = \rho_f g h + \frac{\mu_1}{K_1} \hat{V} h \quad (0 \le h \le H_1),$$

$$\bar{P}_{f2}(h) = \rho_f g h + \frac{\mu_1}{K_1} \hat{V} H_1 + \frac{\mu_2}{K_2} \hat{V}(h - H_1) \quad (H_1 \le h \le H_1 + H_2). \tag{21}$$

Accordingly, the general solution for the intrinsic pore-fluid pressure gradient can be expressed as

$$\frac{\partial \bar{P}_{f1}(h)}{\partial h} = \rho_f g + \frac{\mu_1}{K_1} \hat{V} \quad (0 \le h \le H_1),$$

$$\frac{\partial \bar{P}_{f2}(h)}{\partial h} = \rho_f g + \frac{\mu_2}{K_2} \hat{V} \quad (H_1 \le h \le H_1 + H_2). \tag{22}$$

Since the total force and stress are continuous at the interface between two layers, the general solution for the intrinsic effective vertical stress of the solid matrix can be derived, and

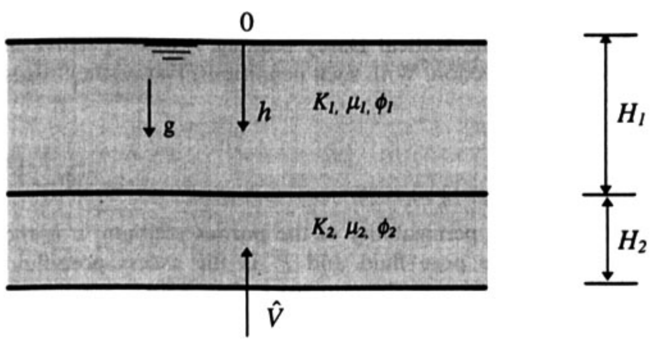

Figure 2. Two-layer hydrodynamic system with vertical throughflow.

it can be expressed as

$$\bar{\sigma}^{v}_{s1}(h)=\rho_{s}gh-\frac{\phi_{1}\mu_{1}}{(1-\phi_{1})K_{1}}\hat{V}h \quad (0\le h\le H_{1}),$$

$$\bar{\sigma}^{v}_{s2}(h)=\bar{\sigma}^{v}_{s2}(H_{1})+\rho_{s}g(h-H_{1})$$
$$-\frac{\phi_{2}\mu_{2}\hat{V}}{(1-\phi_{2})K_{2}}(h-H_{1}) \quad (H_{1}\le h\le H_{1}+H_{2}), \qquad (23)$$

where $\bar{\sigma}^{v}_{s2}(H_1)$ is the intrinsic effective vertical stress of the solid matrix for $h=H_1^{+}$, which means that h approaches the interface (H_1) from the bottom layer:

$$\bar{\sigma}^{v}_{s2}(H_{1})=\frac{\phi_{1}-\phi_{2}}{1-\phi_{2}}\bar{P}_{f1}(H_{1})+\frac{1-\phi_{1}}{1-\phi_{2}}\bar{\sigma}^{v}_{s1}(H_{1}), \qquad (24)$$

where $\bar{\sigma}^{v}_{s1}(H_1)$ is the intrinsic effective vertical stress of the solid matrix when h approaches the interface (H_1) from the top layer. Eq. (24) indicates that the intrinsic effective vertical stress is discontinuous at the interface between two layers unless the porosity of the top layer is equal to that of the bottom layer. The reason for this is a sudden change in the solid contact area, which is the direct result of a sudden change in porosity at the interface between the two layers.

From eq. (23), the general solution for the intrinsic effective vertical-stress gradient of the solid matrix can be expressed as

$$\frac{\partial\bar{\sigma}^{v}_{s1}(h)}{\partial h}=\rho_{s}g-\frac{\phi_{1}\mu_{1}}{(1-\phi_{1})K_{1}}\hat{V} \quad (0\le h\le H_{1}),$$

$$\frac{\partial\bar{\sigma}^{v}_{s2}(h)}{\partial h}=\rho_{s}g-\frac{\phi_{2}\mu_{2}}{(1-\phi_{2})K_{2}}\hat{V} \quad (H_{1}\le h\le H_{1}+H_{2}). \qquad (25)$$

The above solutions are also valid for $K_1\le K_2$ and $0\le\hat{V}\le\hat{V}_{\mathrm{control}}$, where $\hat{V}_{\mathrm{control}}$ is the vertical Darcy velocity required to maintain the intrinsic pore-fluid pressure gradient in the least permeable layer at the value of the lithostatic pressure gradient. In the case $K_1<K_2$, the top layer (layer 1) is the controlling valve of the system instead of layer 2.

Clearly, in the case $0<\hat{V}<\hat{V}_{\mathrm{control}}$, the intrinsic pore-fluid pressure gradients in both layers are greater than the hydrostatic pressure gradient but less than the lithostatic pressure gradient, while the intrinsic effective vertical-stress gradients of the solid matrix in both layers are less than the lithostatic pressure gradient. However, in the case $\hat{V}=\hat{V}_{\mathrm{control}}$, the intrinsic pore-fluid pressure gradient is equal to the lithostatic pressure gradient in the less permeable layer but less than the lithostatic pressure gradient in the more permeable layer. In addition, the intrinsic effective vertical-stress gradients of the solid matrix in both layers are also less than the lithostatic pressure gradient in this case. It turns out that in order to maintain the intrinsic effective vertical-stress gradient of the solid matrix at the value of the lithostatic pressure gradient, the vertical Darcy velocity in this hydrodynamic system must be zero. Thus, the third condition under which the intrinsic pore-fluid pressure gradient can be maintained at the value of the lithostatic pressure gradient is that the intrinsic effective vertical-stress gradient of the solid matrix must be less than the lithostatic pressure gradient.

3.2 Three-layer hydrodynamic system

The same procedure as above can be followed to derive the solutions for a three-layer hydrodynamic system (Fig. 3). In order to save space, only the final results are given below.

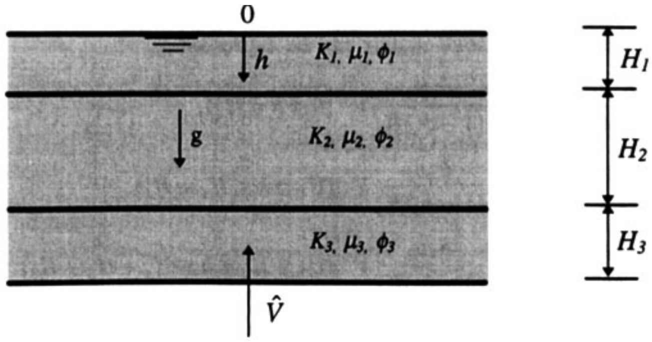

Figure 3. Three-layer hydrodynamic system with vertical throughflow.

The general solution for the intrinsic pore-fluid pressure in the three-layer hydrodynamic system is as follows:

$$\bar{P}_{f1}(h)=\rho_{f}gh+\frac{\mu_{1}}{K_{1}}\hat{V}h \quad (0\le h\le H_{1}),$$

$$\bar{P}_{f2}(h)=\rho_{f}gh+\frac{\mu_{1}\hat{V}}{K_{1}}H_{1}+\frac{\mu_{2}\hat{V}}{K_{2}}(h-H_{1}) \quad (H_{1}\le h\le H_{1}+H_{2}),$$

$$\bar{P}_{f3}(h)=\rho_{f}gh+\frac{\mu_{1}\hat{V}}{K_{1}}H_{1}+\frac{\mu_{2}\hat{V}}{K_{2}}H_{2}+\frac{\mu_{3}\hat{V}}{K_{3}}[h-(H_{1}+H_{2})]$$
$$(H_{1}+H_{2}\le h\le H_{1}+H_{2}+H_{3}). \qquad (26)$$

From eq. (26), the corresponding intrinsic pore-fluid pressure gradient can be expressed as

$$\frac{\partial\bar{P}_{f1}(h)}{\partial h}=\rho_{f}g+\frac{\mu_{1}}{K_{1}}\hat{V} \quad (0\le h\le H_{1}),$$

$$\frac{\partial\bar{P}_{f2}(h)}{\partial h}=\rho_{f}g+\frac{\mu_{2}}{K_{2}}\hat{V} \quad (H_{1}\le h\le H_{1}+H_{2}),$$

$$\frac{\partial\bar{P}_{f3}(h)}{\partial h}=\rho_{f}g+\frac{\mu_{3}}{K_{3}}\hat{V} \quad (H_{1}+H_{2}\le h\le H_{1}+H_{2}+H_{3}). \qquad (27)$$

The general solution for the intrinsic effective vertical stress of the solid matrix is

$$\bar{\sigma}^{v}_{s1}(h)=\rho_{s}gh-\frac{\phi_{1}\mu_{1}}{(1-\phi_{1})K_{1}}\hat{V}h \quad (0\le h\le H_{1}),$$

$$\bar{\sigma}^{v}_{s2}(h)=\bar{\sigma}^{v}_{s2}(H_{1})+\rho_{s}g(h-H_{1})$$
$$-\frac{\phi_{2}\mu_{2}\hat{V}}{(1-\phi_{2})K_{2}}(h-H_{1}) \quad (H_{1}\le h\le H_{1}+H_{2}),$$

$$\bar{\sigma}^{v}_{s3}(h)=\bar{\sigma}^{v}_{s3}(H_{1}+H_{2})+\rho_{s}g[h-(H_{1}+H_{2})]-\frac{\phi_{3}\mu_{3}\hat{V}}{(1-\phi_{3})K_{3}}$$
$$\times[h-(H_{1}+H_{2})] \quad (H_{1}+H_{2}\le h\le H_{1}+H_{2}+H_{3}), \qquad (28)$$

where

$$\bar{\sigma}^{v}_{s2}(H_{1})=\frac{\phi_{1}-\phi_{2}}{1-\phi_{2}}\bar{P}_{f1}(H_{1})+\frac{1-\phi_{1}}{1-\phi_{2}}\bar{\sigma}^{v}_{s1}(H_{1}),$$

$$\bar{\sigma}^{v}_{s3}(H_{1}+H_{2})=\frac{\phi_{2}-\phi_{3}}{1-\phi_{3}}\bar{P}_{f2}(H_{1}+H_{2})+\frac{1-\phi_{2}}{1-\phi_{3}}\bar{\sigma}^{v}_{s2}(H_{1}+H_{2}). \qquad (29)$$

Similarly, the corresponding intrinsic effective vertical-stress gradient of the solid matrix can be expressed as

$$\frac{\partial \bar{\sigma}^{v}_{s1}(h)}{\partial h} = \rho_s g - \frac{\phi_1 \mu_1}{(1-\phi_1)K_1}\hat{V} \quad (0 \le h \le H_1),$$

$$\frac{\partial \bar{\sigma}^{v}_{s2}(h)}{\partial h} = \rho_s g - \frac{\phi_2 \mu_2}{(1-\phi_2)K_2}\hat{V} \quad (H_1 \le h \le H_1 + H_2),$$

$$\frac{\partial \bar{\sigma}^{v}_{s3}(h)}{\partial h} = \rho_s g - \frac{\phi_3 \mu_3}{(1-\phi_3)K_3}\hat{V} \quad (H_1 + H_2 \le h \le H_1 + H_2 + H_3). \tag{30}$$

As expected, the same conclusions as drawn for the previous two-layer hydrodynamic system can be drawn for the three-layer hydrodynamic system.

4 NUMERICAL RESULTS

To produce numerical results, we need an equation to express the relationship between the permeability and the porosity. On this issue, Detournay & Cheng (1993) stated that

> 'The intrinsic permeability K is generally a function of the pore geometry. In particular, it is strongly dependent on porosity ϕ. According to the Carman Kozeny law [Scheidegger 1974] which is based on the conceptual model of packing of spheres, a power law relation of $K \propto \phi^3/(1-\phi)^2$ exists. Other models based on different pore geometry give similar power laws. Actual measurements on rocks, however, often yield power law relations with exponents for ϕ significantly larger than 3.'

Also, Nield & Bejan (1992) commented that 'The Carman–Kozeny law is widely used since it seems to be the best simple expression available.' Thus, for illustrative purposes, the Carman–Kozeny law is used here to calculate the porosity ϕ for a given permeability K.

Two hydrodynamic systems, a two-layer system and a three-layer system, were used to calculate numerical results. For both systems, the viscosity of the pore fluid was taken to be 10^{-3} Pa s; the densities of the solid particles and of the pore fluid were 2700 and 1000 kg m^{-3}, respectively; the acceleration due to gravity was 9.8 m s^{-2}; and the vertical Darcy velocity of the upward throughflow was 1.666×10^{-9} m s^{-1}. For the two-layer hydrodynamic system (Fig. 2), the following parameters were used: $H_1 = 15$ km, $H_2 = 10$ km, $K_1 = 10^{-14}$ m^2, $K_2 = 10^{-16}$ m^2 and $\phi_1 = 0.1$. Using the Carman–Kozeny law, we find $\phi_2 = 0.023$, approximately. For the three-layer hydrodynamic system (Fig. 3), we used the following parameters: $H_1 = H_3 = 5$ km, $H_2 = 15$ km, $K_1 = K_3 = 10^{-16}$ m^2, $K_2 = 10^{-14}$ m^2 and $\phi_2 = 0.1$. Similarly, ϕ_1 and ϕ_3 can be determined from the Carman–Kozeny law, giving $\phi_1 = \phi_3 = 0.023$, approximately.

Fig. 4 shows the normalized intrinsic pore-fluid pressure and effective vertical-stress distribution in the two-layer hydrodynamic system. In this figure, h is normalized by the total thickness ($H_t = H_1 + H_2$) of the system, while the intrinsic pore-fluid pressure $\bar{P}_f$ and intrinsic effective vertical stress $\bar{\sigma}^v_s$ are normalized by the hydrostatic pressure $P_b = \rho_f g H_t$. For ease of comparison, both the hydrostatic and the lithostatic pressures are shown by broken lines. It is obvious that the intrinsic pore-fluid pressure gradient is between the hydrostatic and the lithostatic pressure gradient in the top layer but is identical to the lithostatic pressure gradient in the bottom layer. The normalized intrinsic effective-stress gradients are 2.698 and 2.66 in the top and the bottom layers, respectively. These values are very close to the normalized lithostatic pressure gradient, which is 2.7 in this case. If the total vertical stress $\hat{\sigma}_t$ is normalized by P_b, then the gradients of the normalized total vertical stress $\hat{\sigma}_t/P_b$ are 2.53 and 2.661 in the top and the bottom layers, respectively; these are also very close to 2.7.

As shown in eqs (4) and (5), the total vertical stress $\hat{\sigma}_t$ of an REA can be calculated either from the intrinsic pore-fluid pressure $\bar{P}_f$ and the intrinsic effective vertical stress $\bar{\sigma}^v_s$ or from the pore-fluid pressure $\hat{P}_f$ and the effective vertical stress $\hat{\sigma}^v_s$. In the former case, the porosity of the porous media must be taken into account and, therefore, the total vertical stress of an REA is expressed as

$$\hat{\sigma}_t = \phi \bar{P}_f + (1-\phi)\bar{\sigma}^v_s. \tag{31}$$

This clearly indicates that although the total vertical stress is continuous at the interface between any two layers, the intrinsic effective vertical stress must be discontinuous unless the two layers have exactly the same porosity. The reason for this is that according to Pascal's law, the intrinsic pore-fluid pressure

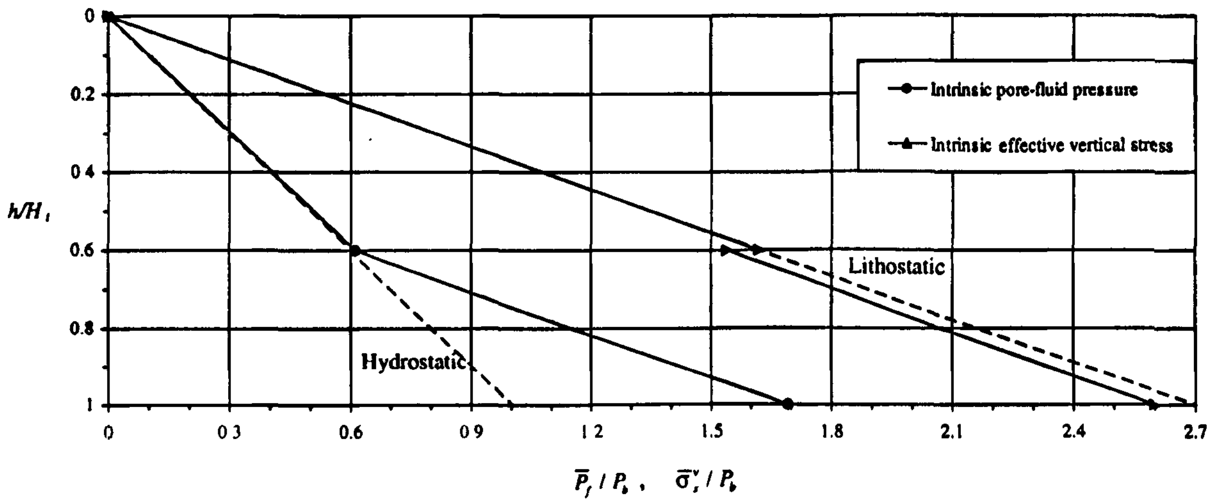

Figure 4. Normalized intrinsic pore-fluid pressure and effective vertical stress in the two-layer hydrodynamic system.

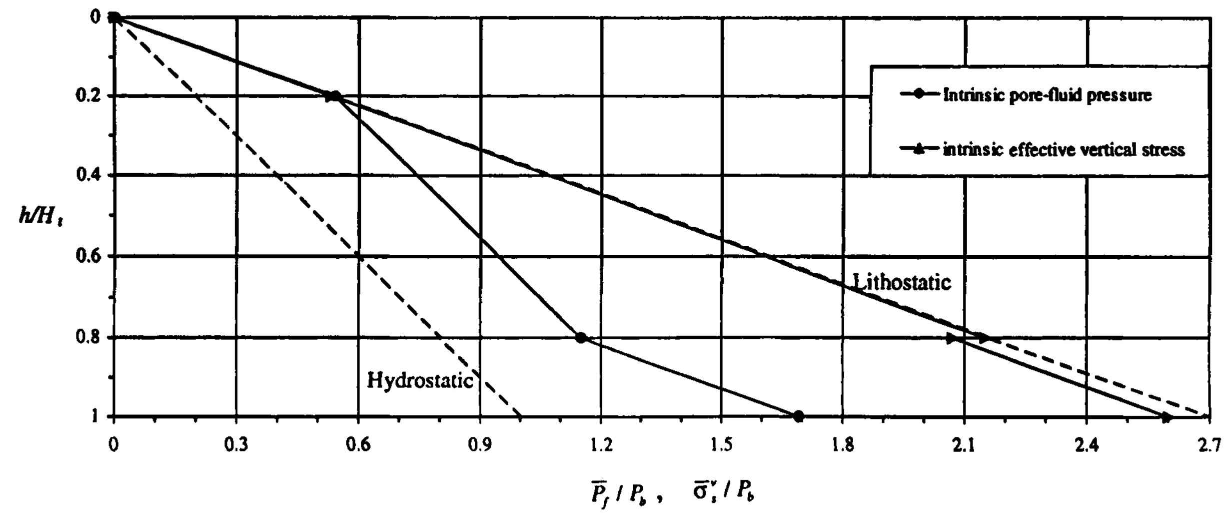

Figure 5. Normalized intrinsic pore-fluid pressure and effective vertical stress in the three-layer hydrodynamic system.

must be continuous at the interface between the two layers. Consequently, from a mathematical point of view, keeping both the total vertical stress and the intrinsic pore-fluid pressure constant at the interface between the two layers requires that the intrinsic effective vertical stress $\bar{\sigma}_s^v$ be discontinuous. This phenomenon can be observed in the numerical results.

Fig. 5 shows the normalized intrinsic pore-fluid pressure and effective vertical-stress distribution in the three-layer hydrodynamic system. Although the normalized pore-fluid pressure gradient can be maintained at the value of the lithostatic pressure gradient in the top and the bottom layers, it is equal to 1.02 in the middle (more permeable) layer. This means that the intrinsic pore-fluid pressure gradient is between the hydrostatic and the lithostatic pressure gradient. The normalized effective vertical-stress gradients are 2.66, 2.698 and 2.66 for the top, middle and bottom layers, respectively. These values are also very close to 2.7. It is interesting that both the intrinsic pore-fluid pressure and the intrinsic effective vertical stress are within the range bounded by the two broken lines. This indicates that the intrinsic pore-fluid pressure gradient is not less than the hydrostatic pressure gradient and the intrinsic effective vertical-stress gradient is not greater than the lithostatic pressure gradient, in the layered hydrodynamic systems considered here.

5 CONCLUSIONS

Within the scope of the analysis presented in this paper, some conclusions can be drawn as follows.

For a single-layer hydrodynamic system, the intrinsic pore-fluid pressure gradient can be maintained at the value of the lithostatic pressure gradient if the following three conditions are satisfied simultaneously: (1) there is an inexhaustible fluid resource underneath the layer so that the vertical Darcy velocity can be kept constant throughout the layer; (2) the porosity of the porous medium must not exceed the critical porosity of the medium unless the tensile strength of the solid matrix is greater than zero; and (3) the intrinsic effective vertical-stress gradient of the solid matrix must be less than the lithostatic pressure gradient.

Even though the intrinsic pore-fluid pressure gradient can be maintained at the value of the lithostatic pressure gradient in a single-layer system under the above conditions, it is impossible to maintain it at the value of the lithostatic pressure gradient in all layers in a layered hydrodynamic system, unless all layers have the same permeability and porosity simultaneously.

When the vertical throughflow (i.e. the vertical Darcy velocity) tends to zero, the intrinsic pore-fluid pressure gradient and the intrinsic effective vertical-stress gradient of the solid matrix approach the hydrostatic and the lithostatic pressure gradients, respectively, for all the systems considered here.

For small porosities (say $\phi \leq 0.1$), both the total vertical-stress gradient of the porous medium and the intrinsic effective vertical-stress gradient of the solid matrix are close to the lithostatic pressure gradient, within the scope of this study.

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