Interference pattern with a dark center from two atoms driven by a coherent laser field

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In a recent paper Meyer and Yeoman [Phys. Rev. Lett. **79**, 2650 (1997)] have shown that the resonance fluorescence from two atoms placed in a cavity and driven by an incoherent field can produce an interference pattern with a dark center. We study the fluorescence from two coherently driven atoms in free space and show that this system can also produce an interference pattern with a dark center. This happens when the atoms are in nonequivalent positions in the driving field, i.e., the atoms experience different intensities and phases of the driving field. We discuss the role of the interatomic interactions in this process and find that the interference pattern with a dark center results from the participation of the antisymmetric state in the dynamics of the driven two-atom system. [S1050-2947(98)10306-2]

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Young's interference-type pattern has recently been observed experimentally in the resonance fluorescence of two trapped ions [1]. In the experiment the ions were separated by a distance $r_{12}\approx 20\lambda$, where λ is the optical wavelength, a separation for which the collective effects can be ignored [2]. The ions were driven by a coherent laser field propagating at an angle of 62° to the atomic axis, so that the ions experienced different intensities and phases of the driving field. The experimental results have been explained theoretically by Wong *et al.* [3], and can be understood by treating the ions as independent radiators that are synchronized by the constant phase of the driving field. Various situations have been analyzed, including the effects of the motions of the ions in the trap [4] and the dependence of the interference pattern on the polarization as well as the intensity of the driving field. It has been shown that for a weak driving field, the fluorescence field is predominantly composed of an elastic component and, therefore, the ions behave as point sources of coherent light producing an interference pattern. Under strong excitation the fluorescence field is mostly composed of the incoherent part and consequently there is no interference pattern.

Kochan *et al.* [5] have shown that the interference pattern of the strongly driven atoms can be partially recovered by placing the atoms inside an optical cavity. The coupling of the atoms to the cavity mode induces atomic correlations, which improves the fringe visibility. Recently, Meyer and Yeoman [6] have reported an even stronger cavity-induced modification of the interference pattern that occurs when the coherent driving field is replaced by an incoherent field. They have shown that, in contrast to the coherent excitation, the incoherent field produces an interference pattern with a dark center.

In this Brief Report, we show that an interference pattern with a dark center can be observed even with a *coherent* driving field. We first consider the case in which the atoms are in free space and are separated by a distance comparable to an optical wavelength. At this distance collective effects are important. We then consider a situation where the atoms are separated by a distance $r_{12}\gg\lambda$, for which the collective effects can be ignored. We discuss the dependence of the interference pattern on the direction of propagation of the coherent field relative to the atomic axis. We also include in our calculations a detuning of the driving field from the atomic resonance and find that, depending on the detuning and the direction of propagation of the coherent field, the interference pattern can show a minimum or a maximum at line center.

Following Meyer and Yeoman [6], we calculate the fringe contrast factor

$$C=\frac{\langle S_1^+S_2^-+S_2^+S_1^-\rangle}{\langle S_1^+S_1^-+S_2^+S_2^-\rangle}, \tag{1}$$

for the steady-state fluorescence field emitted by two two-level atoms driven by a coherent field. We assume that the atomic axis lies parallel to the (x,y) plane and the fluorescence is observed in the direction of the z axis, perpendicular to the interatomic axis. The driving field is propagated in directions that lie in the (x,y) plane. In Eq. (1), S_i^+ (S_i^-) are the dipole raising (lowering) operators of the ith atom. The factor C can be positive as well as negative, and $|C|$ is simply the fringe visibility [6,7]. For positive values of C, the interference pattern exhibits a maximum at line center (bright center), whereas for negative C there is a minimum (dark center). The optimum positive (negative) value is $C=1$ $(C=-1)$, and there is no interference pattern when $C=0$.

The correlation functions appearing in Eq. (1) can be found from the Lehmberg master equation [8], which, for an arbitrary combination Q of the atomic operators, reads

$$\begin{aligned}\langle\dot{Q}\rangle=&-i\sum_{i=1}^{2}\{\Delta\langle[S_i^+S_i^-,Q]\rangle+\tfrac{1}{2}\langle[\Omega_iS_i^++\Omega_i^*S_i^-,Q]\rangle\}\\&-\sum_{i,j}\gamma_{ij}\langle S_i^+S_j^-Q+QS_i^+S_j^--2S_i^+QS_j^-\rangle\\&+i\sum_{i\neq j}\Omega_{ij}\langle[S_i^+S_j^-,Q]\rangle,\end{aligned} \tag{2}$$

where $\Delta=\omega_L-\omega_0$ is the detuning of the driving-field frequency ω_L from the atomic resonance ω_0, and Ω_i is the Rabi frequency of the driving field at the position of the ith atom. In the following, we choose the reference frame such that the atoms are at positions $\mathbf{r}_1=(0,-r_{12}/2,0)$ and $\mathbf{r}_2=(0,r_{12}/2,0)$, i.e., they are a distance r_{12} apart. In this case $\Omega_1=\Omega$, whereas

$$\Omega_2=\Omega e^{i\mathbf{k}_L\cdot\mathbf{r}_{12}}, \tag{3}$$

where $\mathbf{k}_L$ is the wave vector of the driving field and $\Omega=|\Omega_i|$. In Eq. (2), the parameter γ_{ij} $(i=j)$ is one-half the Einstein A coefficient for spontaneous emission, whereas γ_{ij} and Ω_{ij} $(i\neq j)$ describe the interatomic coupling [2,8], and are the collective damping and the dipole-dipole interaction potential defined, respectively, by

$$\gamma_{ij}=\frac{3}{2}\gamma\left\{[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{ij})^2]\frac{\sin(kr_{ij})}{kr_{ij}}+[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{ij})^2]\times\left[\frac{\cos(kr_{ij})}{(kr_{ij})^2}-\frac{\sin(kr_{ij})}{(kr_{ij})^3}\right]\right\} \tag{4}$$

and

$$\Omega_{ij}=\frac{3}{2}\gamma\left\{-[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{ij})^2]\frac{\cos(kr_{ij})}{kr_{ij}}+[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{ij})^2]\times\left[\frac{\sin(kr_{ij})}{(kr_{ij})^2}+\frac{\cos(kr_{ij})}{(kr_{ij})^3}\right]\right\}, \tag{5}$$

where $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{r}}_{ij}$ are unit vectors along the transition dipole moment and the interatomic axis, respectively.

The master equation (2) leads to a closed system of 15 equations describing the evolution of the coupled atomic correlation functions [9]. However, for a specially chosen geometry for the driving field, namely that the field is propagated perpendicularly to the interatomic axis ($\mathbf{k}_L\cdot\mathbf{r}_{12}=0$), the system of equations decouples into nine equations for symmetric and six equations for antisymmetric combinations of the atomic operators [9,10]. In this case, we can solve the system analytically, and find that the steady-state values of the correlation functions appearing in Eq. (1) are

$$\langle S_1^+S_2^-+S_2^+S_1^-\rangle=\frac{2(\gamma^2+\Delta^2)\Omega^2}{D}, \tag{6}$$

$$\langle S_1^+S_1^-+S_2^+S_2^-\rangle=\frac{[2(\gamma^2+\Delta^2)+\Omega^2]\Omega^2}{D}, \tag{7}$$

where

$$D=\Omega^4+2(\gamma^2+\Delta^2)\{\Omega^2+2[(\gamma+\gamma_{12})^2+(\Delta-\Omega_{12})^2]\}, \tag{8}$$

and $\gamma=\gamma_{11}=\gamma_{22}$;

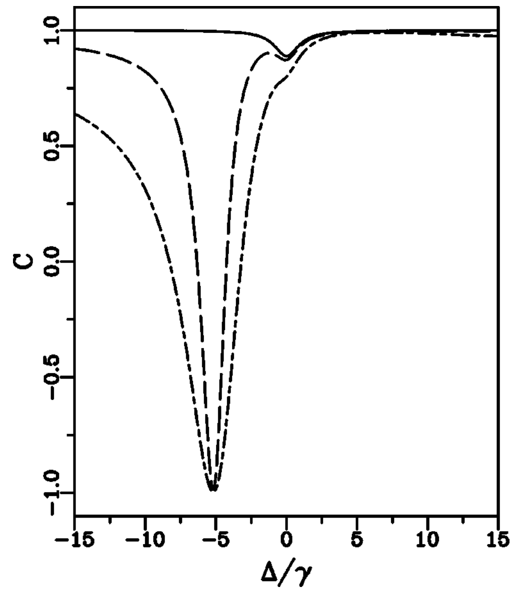

FIG. 1. The fringe contrast factor C as a function of the detuning Δ for $r_{12}/\lambda=0.1$, $\hat{\boldsymbol{\mu}}\perp\hat{\mathbf{r}}_{12}$, $\Omega=0.5\gamma$, and different directions of laser field propagation with respect to the atomic axis: $\theta=\pi/2$ (solid line), $\theta=0.4\pi$ (dashed line), and $\theta=0.25\pi$ (dashed-dotted line).

$$C=\frac{2(\gamma^2+\Delta^2)}{2(\gamma^2+\Delta^2)+\Omega^2}, \tag{9}$$

which is positive for all parameter values and is completely independent of the interatomic interactions. As has been pointed out before [3,5,11], for $\Omega\ll\gamma$ the fringe visibility $|C|\approx 1$, whereas $|C|\approx 0$ for large Ω and there is no interference pattern. For $\Omega\neq 0$ the visibility may be improved by detuning the laser field from the atomic resonance.

The fringe contrast factor exhibits interesting modifications when the driving field propagates in directions different from perpendicular to the interatomic axis. In this situation C strongly depends on the interatomic separation and the detuning Δ. This can produce the interesting modification that C may become negative. We show this by solving numerically the system of 15 equations for the atomic correlation functions. The factor C is plotted against the detuning Δ in Fig. 1 for $r_{12}/\lambda=0.1$, $\Omega=0.5\gamma$, and various angles θ between the interatomic axis and the direction of the driving-field propagation. The factor C is positive for most values of Δ, except $\Delta\approx-\Omega_{12}$. At this detuning the parameter C is negative and reaches the optimum negative value $C=-1$. In order to understand what is happening at $\Delta=-\Omega_{12}$, we refer to the collective states of the two atom system [2,7–10]:

$$\begin{aligned}|G\rangle&=|g_1\rangle|g_2\rangle, \quad E_G=-\hbar\omega_0,\\ |S\rangle&=\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle+|g_1\rangle|e_2\rangle), \quad E_S=\hbar\Omega_{12},\\ |A\rangle&=\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle-|g_1\rangle|e_2\rangle), \quad E_A=-\hbar\Omega_{12},\\ |U\rangle&=|e_1\rangle|e_2\rangle, \quad E_U=\hbar\omega_0,\end{aligned} \tag{10}$$

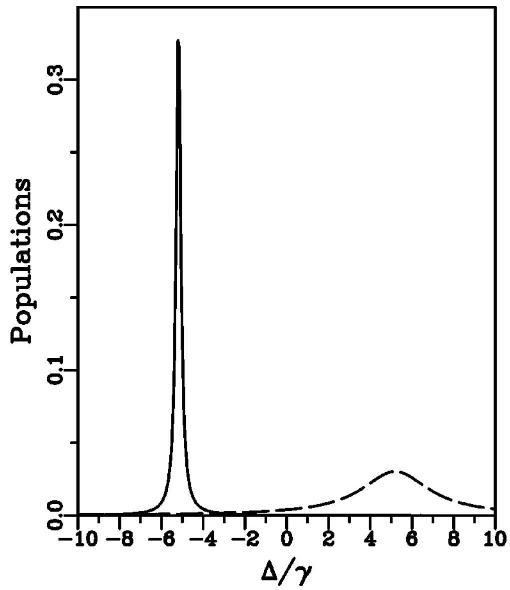

FIG. 2. Populations of the symmetric and antisymmetric states for $r_{12}/\lambda = 0.1$, $\hat{\boldsymbol{\mu}} \perp \hat{r}_{12}$, $\Omega = 0.5\gamma$, and $\theta = \pi/4$: ρ_{AA} (solid line) and ρ_{SS} (dashed line).

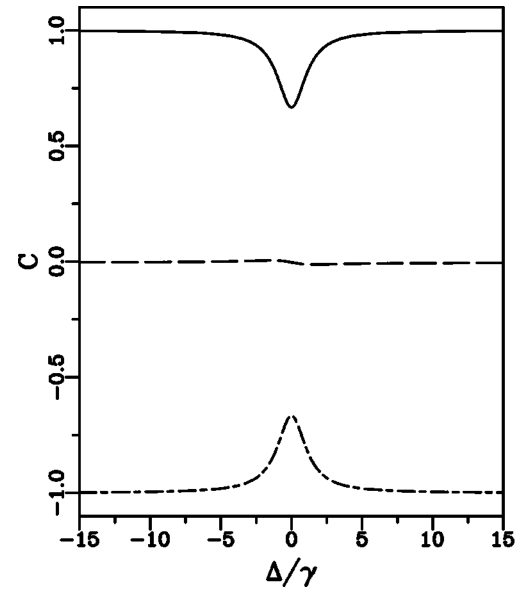

FIG. 3. The fringe contrast factor C as a function of the detuning Δ for $r_{12}/\lambda = 19$, $\hat{\boldsymbol{\mu}} \perp \hat{\mathbf{r}}_{12}$, $\Omega = \gamma$, and different θ: $\theta = 61.7°$ (solid line), $\theta = 60.9°$ (dashed line), and $\theta = 60°$ (dashed-dotted line).

where $|g_i\rangle$ ($|e_i\rangle$) is the ground (excited) state of the ith atom, and E_α ($\alpha = G,S,A,U$) are the energies of the collective states.

Within the framework of the collective states (10), the two-atom system behaves as a single four-level system with ground state $|G\rangle$, upper state $|U\rangle$, and two intermediate states: the symmetric state $|S\rangle$ and antisymmetric state $|A\rangle$. It is clear that the resonance $\Delta = -\Omega_{12}$, seen in Fig. 1, corresponds to the situation when the driving field is resonant with the $|A\rangle \rightarrow |G\rangle$ transition. Therefore, the negative values of C are related to the dynamics of the antisymmetric state. In order to demonstrate this quantitatively, we write the fringe contrast factor in the basis of the collective states (10) as

$$C = \frac{\rho_{SS} - \rho_{AA}}{\rho_{SS} + \rho_{AA} + 2\rho_{UU}}, \tag{11}$$

where ρ_{ii} ($i = S,A,U$) are the populations of the excited collective states $|i\rangle$. Equation (11) shows that the sign of C depends on the population difference between the symmetric and antisymmetric states. For $\rho_{SS} > \rho_{AA}$, the fringe contrast factor is positive, whereas it is negative when $\rho_{SS} < \rho_{AA}$. We plot the populations of the $|S\rangle$ and $|A\rangle$ states in Fig. 2, for the same parameters as in Fig. 1, with $\theta = \pi/4$. The figure shows that at $\Delta = -\Omega_{12}$ the antisymmetric state is significantly populated, whereas the population of the symmetric state is close to zero. This effect leads to negative values of C, as seen in Fig. 1.

We have shown that the fringe contrast factor can be negative for the fluorescence field emitted by two coherently driven atoms in free space. However, this requires interatomic separations smaller than or comparable to the optical wavelength. Nevertheless, we will show that interference fringes with a dark center can be observed in the fluorescence field emitted by two *independent* atoms in free space. In Fig. 3, we plot the fringe contrast factor for two atoms separated by $r_{12} = 19\lambda$, $\Omega = \gamma$ and different θ. The separation $r_{12} = 19\lambda$ corresponds to that in the experiment performed by Eichmann *et al.* [1], where the interference fringes in the light scattered by two trapped ions were observed. At this separation the parameters γ_{12} and Ω_{12} are negligibly small and the atoms radiate independently. The three values of θ have been chosen such that for $\theta = 61.7°$ both atoms experience the same phase of the driving field; for $\theta = 60.9°$ the phases are shifted by $\pi/2$, whereas for $\theta = 60°$ the atoms are in opposite phases. In practice the atoms experience opposite phases when $\mathbf{k}_L \cdot \mathbf{r}_{12} = (2n+1)\pi$, where $n = 0,1,2 \ldots$. When the atoms experience the same phase, the factor C is positive and $C = 1$ for large detunings. When the atoms experience opposite phases the factor C is negative and approaches -1 for large detunings. No interference pattern is observed for a phase difference of $\pi/2$. In the case of opposite phases the atoms are synchronized antisymmetrically by the constant phase of the driving laser field. This type of interference is not possible with an incoherent field.

In summary, we have shown that the resonance fluorescence from two coherently driven atoms can produce interference fringes with a dark center. This happens when the atoms experience different amplitudes and/or phases of the driving field. We have shown that the effect results from an excess population in the antisymmetric state over that contained in the symmetric state of the two-atom system. For small interatomic separations, where the atoms experience almost the same field amplitude and phase, the collective effects are important in producing the negative fringe contrast. At large distances, where the atoms radiate independently, the negative fringe contrast can be obtained by allowing the two atoms to experience different phases of the driving field.

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