

Contradiction of Quantum Mechanics with Local Hidden Variables for Quadrature Phase Amplitude Measurements

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(Received 19 September 1997)

We demonstrate a contradiction of quantum mechanics with local hidden variable theories for continuous quadrature phase amplitude (“position” and “momentum”) measurements. For any quantum state, this contradiction is lost for situations where the quadrature phase amplitude results are always macroscopically distinct. We show that for optical realizations of this experiment, where one uses homodyne detection techniques to perform the quadrature phase amplitude measurement, one has an amplification prior to detection, so that macroscopic fields are incident on photodiode detectors. The high efficiencies of such detectors may open a way for a loophole-free test of local hidden variable theories. [S0031-9007(98)05671-3]

PACS numbers: 03.65.Bz, 42.50.Gy

In 1935 Einstein, Podolsky, and Rosen [1] presented an argument for the incompleteness of quantum mechanics. The argument was based on the validity of two premises: no action at a distance (locality) and realism. Bell [2] later showed that the predictions of quantum mechanics are incompatible with the premises of local realism (or local hidden variable theories). Experiments [3] based on Bell’s result support quantum mechanics, indicating the failure of local hidden variable theories.

One feature appears characteristic of all the contradictions of quantum mechanics with local hidden variables studied to date. The measurements considered have discrete outcomes, for example, being measurements of spin or photon number. By this we mean specifically that the eigenvalues of the appropriate system Hermitian operator, which represents the measurement in quantum mechanics, are discrete.

In this paper we show how the predictions of quantum mechanics are in disagreement with those of local hidden variable theories for a situation involving continuous quadrature phase amplitude (“position” and “momentum”) measurements. By this we mean that the quantum predictions for the probability of obtaining results x and p for position and momentum (and various linear combinations of these coordinates) cannot be predicted by any local hidden variable theory. This is of fundamental interest since the original argument [1] of Einstein, Podolsky, and Rosen was given in terms of position and momentum measurements. The original state considered by Einstein, Podolsky, and Rosen, and that produced experimentally in the realization by Ou *et al.* [4] of this argument, gives probability distributions for x and p completely compatible with a local hidden variable theory.

Second we suggest a new macroscopic aspect to the proposed failure of local hidden variable theories for the

case where one uses optical homodyne detection to realize the quadrature phase amplitude measurement [4,5]. The homodyne detection method employs a second “local-oscillator” field which combines with the original field to provide an amplification prior to photodetection. In these experiments then large field fluxes fall incident on highly efficient photodiode detectors, in dramatic contrast to the former photon-counting experiments. A microscopic resolution (in absolute terms) of this incident photon number is not necessary to obtain the violations with local hidden variables. This is in contrast to many previously cited macroscopic proposals [6] for which it appears necessary to resolve the incident photon number to absolute precision in order to show a contradiction with local hidden variable theories.

The high efficiency of detectors available in this more macroscopic detection regime may provide a way to test local hidden variables without the use of auxiliary assumptions [2,7] which have weakened the conclusions of the former photon-counting measurements. This high intensity limit has not been indicated by previous works [8] which showed contradiction of quantum mechanics with local hidden variables using homodyne detection, since these analyses were restricted to a very low intensity of local-oscillator field.

We consider the following two-mode entangled quantum superposition state [9,10]:

$$|\Psi\rangle = N \int_0^{2\pi} |r_0 e^{i\varsigma}\rangle_A |r_0 e^{-i\varsigma}\rangle_B d\varsigma. \quad (1)$$

Here N is a normalization coefficient. The $|\alpha\rangle_A$, where $\alpha = r_0 e^{i\varsigma}$, is a coherent state of amplitude $r_0 = |\alpha|$ and phase ς , for a system at a location A . Similarly $|\beta\rangle_B$, where $\beta = r_0 e^{-i\varsigma}$ and $r_0 = |\beta|$, is a coherent state for a second system at a location B , spatially separated from A . The

The calculation of the quantum prediction for S for the quantum state (1) is straightforward. We note certain properties of the distribution $P_{+++}^{AB}(\theta, \phi)$: it is a function only of the angle sum $\chi = \theta + \phi$ so we can abbreviate $P_{+++}^{AB}(\theta, \phi) = P_{+++}^{AB}(\chi)$; $P_{+++}^{AB}(\chi) = P_{+++}^{AB}(-\chi)$; and the marginals satisfy $P_{+}^A(\theta) = P_{+}^B(\phi) = 0.5$. Results for S are shown in Fig. 2, for the choice of measurement angles $\theta + \phi = \theta' + \phi' = -(\theta' + \phi) = \pi/4$, $\theta + \phi' = 3\pi/4$ (for example, put $\theta = 0$, $\phi = -\pi/4$, $\theta' = \pi/2$, and $\phi' = -3\pi/4$). This choice allows the simplification $S = 3P_{+++}^{AB}(\pi/4) - P_{+++}^{AB}(3\pi/4)$. It can be shown that for small r_0 (less than about 1.5) this angle choice maximizes S .

Violations of the Bell inequality, and hence contradiction with the predictions of local hidden variables, are indicated for $0.96 \leq r_0 \leq 1.41$, the maximum violation of $S \approx 1.0157 \pm 0.001$ being around $r_0 \approx 1.1$. This is a substantially smaller violation than obtained in the discrete case (where $S \approx 1.2$) of spin measurements, considered originally by Bell. The choice of Bell inequality and quantum state to give a violation may not be optimal, but nevertheless the possibility of a contradiction of quantum mechanics with local hidden variables is established.

We note that the violations are lost at large coherent amplitudes r_0 . In this limit the quantum probability distributions for x and y show two widely separated peaks (as indicated by Fig. 3), the $+1$ and -1 results of the measurement then corresponding to macroscopically distinct outcomes, resembling the ‘‘alive’’ and ‘‘dead’’ states of the Schrodinger cat [11]. We obtain asymptotic (large r_0) analytical forms for the probability distributions which allow a complete search for all angles. Results indicate no violations of the Bell inequality (5) possible.

In fact it can be demonstrated that, for any quantum state, there is no incompatibility with local hidden variables for the case where the quadrature phase amplitude results x and y only take on values which are macroscopi-

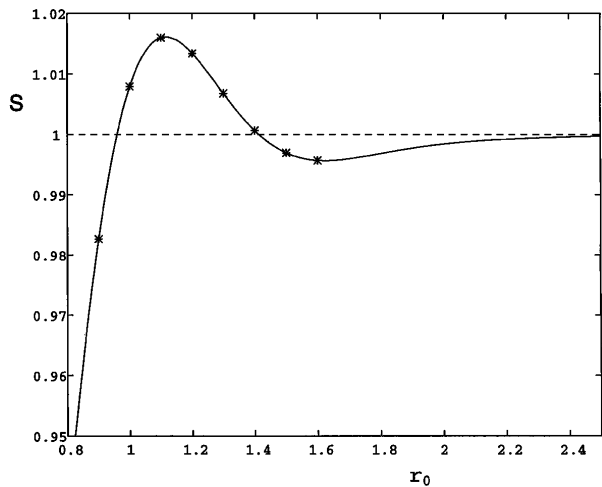


FIG. 2. Plot of S versus r_0 , for the angle values indicated in the text.

cally distinct. In this case, the addition of a noise term of order the standard quantum limit (this corresponds to a variance $\Delta^2 x = 1$) to the result of quadrature phase amplitude measurement will not alter the $+1$ or -1 classification of the result. Yet it can be shown that the quantum predictions for the results of such a noisy experiment are given by the quantum Wigner function $W(x_0^A, x_{\pi/2}^A, x_0^B, x_{\pi/2}^B)$ for the state (1), convoluted by the Gaussian noise term $(1/4\pi^2) \exp(-[(x_0^A)^2 + (x_{\pi/2}^A)^2 + (x_0^B)^2 + (x_{\pi/2}^B)^2]/2)$. This new Wigner function is always positive [14] and can then act as a local hidden variable theory which gives all the predictions in the truly macroscopic dead or alive classification limit.

An examination, however, of the homodyne method of measurement of the quadrature phase amplitudes reveals a macroscopic aspect to the experiment proposed here for

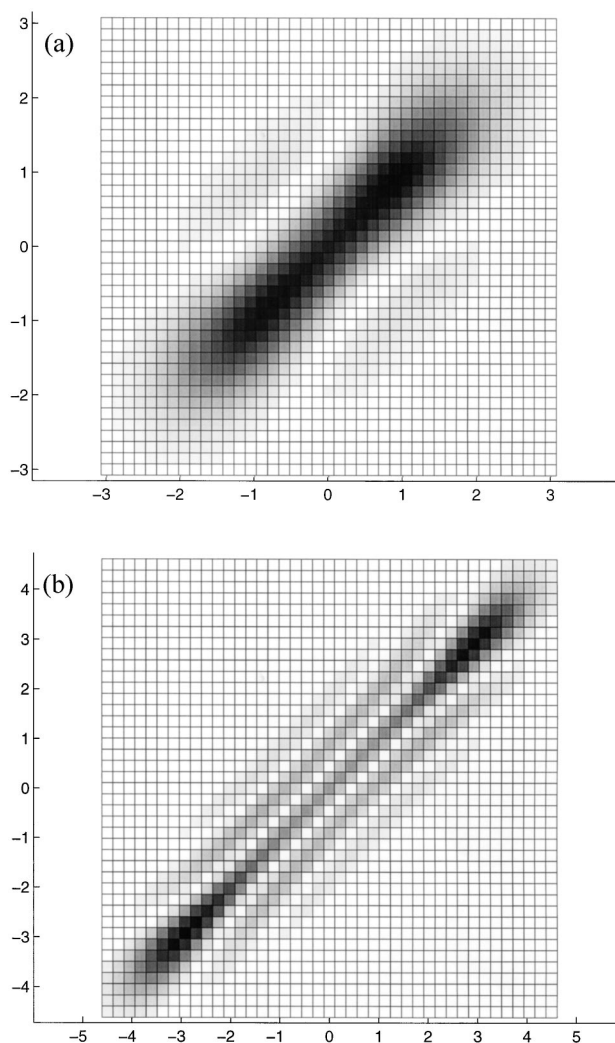


FIG. 3. Representation of the quantum prediction for the probability $P_{\theta, \phi}(x, y)$ of getting a result x (horizontal axis) and y (vertical axis), respectively, upon the simultaneous measurements X_{θ}^A and X_{ϕ}^B , where $\theta = \phi$: (a) $r_0 = 1.1$; (b) $r_0 = 2.5$ showing the increasing separation of peaks and the interference fringes characteristic of quantum superposition states.

optical fields. The optical realization [4,5] of the quadrature phase amplitude measurement (see Fig. 1) involves local oscillator fields at A and B , which we designate by the boson operators \hat{a}_2 and \hat{b}_2 , respectively. The measurement of X_θ^A proceeds when the local oscillator field at A is combined with the field \hat{a}_1 using a beam splitter to form two combined fields $\hat{c}_\pm = [\hat{a}_2 \pm \hat{a}_1 \exp(-i\theta)]/\sqrt{2}$. A variable phase shift θ allows choice of the particular observable to be measured. Direct detection, using two photodetectors, of the intensities $\hat{c}_\pm^\dagger \hat{c}_\pm$ of the combined fields and subtraction of the two resulting photocurrents results in measurement of $I_D = \hat{c}_+^\dagger \hat{c}_+ - \hat{c}_-^\dagger \hat{c}_- = s_\theta^A$, where $s_\theta^A = \hat{a}_2^\dagger \hat{a}_1 \exp(-i\theta) + \hat{a}_2 \hat{a}_1^\dagger \exp(i\theta)$. In the limit where the local oscillator fields are very intense one may replace the boson operators \hat{a}_2 and \hat{b}_2 by classical amplitudes E_A and E_B , respectively. Assuming $E_A = E_B = E$, where E is real, we see that $s_\theta^A = EX_\theta^A$. The X_ϕ^B are measured similarly to X_θ^A using a second beam splitter [to give fields $\hat{d}_\pm = [\hat{b}_2 \pm \hat{b}_1 \exp(-i\phi)]/\sqrt{2}$] and a pair of photodetectors, at location B .

The important point is that the local oscillator acts as an amplifier prior to detection, the operators s_θ^A , $\hat{c}_\pm^\dagger \hat{c}_\pm$ and $\hat{d}_\pm^\dagger \hat{d}_\pm$ being photon number operators which have a macroscopic scaling in the very intense local oscillator limit [15]. Thus in these experiments large intensities fall incident on the photodetectors, and it is not necessary to determine these photon numbers with a microscopic uncertainty in order to arrive at the conclusion that local hidden variable theories are invalid [16]. This is in contrast with the previous photon-counting experiments, and also many previous macroscopic proposals, for which it appears that an absolute resolution of the incident photon number is necessary in order to show failure of local hidden variables. Our result then opens possibilities for testing quantum mechanics against local hidden variable theories in a loophole-free way using very efficient photodiode detectors.

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 [13] As previously discussed in the literature [see A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982); A. Zeilinger, Phys. Lett. A **118**, 1 (1986)] a stricter test is provided by ensuring measurement events at A and B are causally separated and that there is a sufficiently fast random switching between alternative choices of phase shift θ and ϕ , so that the locality condition follows from Einstein's causality.
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 [16] Calculations reveal the violation of the Bell inequality to be sustained with only a very small ($\Delta^2 x \approx 0.1$) amount of noise (lack of resolution) added to the result x of the quadrature phase amplitude measurement of X_θ^A (or X_ϕ^B). However, because of the amplification, prior to photodetection, due to the local oscillator E this small amount of noise in X_θ^A can, in principle, for sufficiently large E , translate to a large amount of noise (lack of resolution) in the detected photon number s_θ^A .