GUALTIERI, C., and CHANSON, H. (2007). "Clustering Process Analysis in a Large-Size Dropshaft and in a Hydraulic Jump." *Proc. 32nd IAHR Biennial Congress*, Venice, Italy, G. DI SILVIO and S. LANZONI Editors, Topic C1.b, 11 pages (CD-ROM).

CLUSTERING PROCESS ANALYSIS IN A LARGE-SIZE DROPSHAFT AND IN A HYDRAULIC JUMP

Carlo Gualtieri[(1)], Hubert Chanson[(2)]

[(1)] Hydraulic and Environmental Engineering Department "Girolamo Ippolito", University of Napoli "Federico II", Via Claudio, 21 - 80125, Napoli (Italy) e-mail carlo.gualtieri@unina.it
[(2)] Division of Civil Engineering, The University of Queensland, Brisbane QLD 4072, (Australia), E-mail h.chanson@uq.edu.au

ABSTRACT

Recent efforts in the characterization of air-water flows properties have included some clustering process analysis. A cluster of bubbles is defined as a group of two or more bubbles, with a distinct separation from other bubbles before and after the cluster. The present paper compares the results of clustering processes two hydraulic structures. That is, a large-size dropshaft and a hydraulic jump in a rectangular horizontal channel. The comparison highlighted some significant differences in clustering production and structures. Both dropshaft and hydraulic jump flows are complex turbulent shear flows, and some clustering index may provide some measure of the bubble-turbulence interactions and associated energy dissipation.

Keywords: physical modelling, air-water flows, hydraulic structures, clustering process.

1 INTRODUCTION

The study of air-water flows properties is of paramount importance in hydraulic structures. Two common energy dissipators, the dropshaft and the hydraulic jump, are characterised by significant air bubble entrainment. A dropshaft is a vertical structure connecting two channels with different invert elevations. This type of structure is commonly used in sewers and storm water systems (Merlein et al., 2002). Small dropshafts are also used upstream and downstream of culverts (Apelt, 1984), while large spillway shafts were built (Vischer and Hager, 1998). The dropshaft is an ancient design used in the Roman aqueducts, although there is some controversy if its purpose was solely for energy dissipation or in combination with some flow re-aeration (Chanson, 1998, 2002a). However the hydraulics of dropshafts has not been systematically documented (Rajaratnam et al., 1997; Merlein et al., 2002). Recent works (Chanson, 2002b; Gualtieri and Chanson, 2004b) studied the hydraulics and the air-water flow properties.

A hydraulic jump is a sudden transition from a supercritical flow into a subcritical flow. It is characterized by a sharp rise of the free-surface elevation associated with strong energy dissipation and air entrainment. Basic studies of air entrainment in hydraulic jumps included Rajaratnam (1962) and Thandaverwara (1974), while a landmark work was the study of Resch and Leutheusser (1972) who showed first that the air entrainment process is strongly affected by the inflow conditions. Recent studies included Chanson (1995, 2006), Mossa and Tolve (1998), Chanson and Brattberg (2000) and Murzyn et al. (2005). Gualtieri and Chanson (2007) carried out experimental works on air-water flow properties in the hydraulic jump with inflow Froude numbers Fr_1 from 5.2 to 14.3. Chanson (2006) and Chanson and Gualtieri (2007) performed measurements in two channels in which similar experiments were conducted with a geometric scaling ratio of 2:1. The data indicated some scale effects with comparatively greater detrainment and lower dimensionless bubble count rates at low Reynolds numbers in the smaller model.

This paper presents the comparative results of clustering analyses in a large-size dropshaft and in a hydraulic jump. The findings included the average number of clusters, the percentage of clustered bubbles, the ratio between the number of clusters and the number of detected bubbles, and the percentage of clusters with two bubbles only. The comparison highlighted some significant difference in terms of cluster production.

2 EXPERIMENTAL SETUPS.

The first group of experiments was performed in a large-size rectangular dropshaft built in marine plywood and perspex at the Hydraulics Laboratory of the University of Queensland (Australia). The facility was previously used by Chanson (2002b). The dropshaft was 3.1 m high, 0.76 m wide and 0.75 long. The drop in invert was 1.7 m and the shaft pool was 1.0 m deep. The inflow and outflow channels were both horizontal, 0.5 m wide and 0.30 m deep. The upstream channel was open while the downstream conduit was covered and ended with a free overfall (Fig. 1 and 2). For a flow rate of 12 L/s, the free-falling jet impacted into the shaft pool (Fig. 2), also called R1 regime (Chanson, 2002a). Further details on the experimental data were presented in Gualtieri and Chanson (2004b).

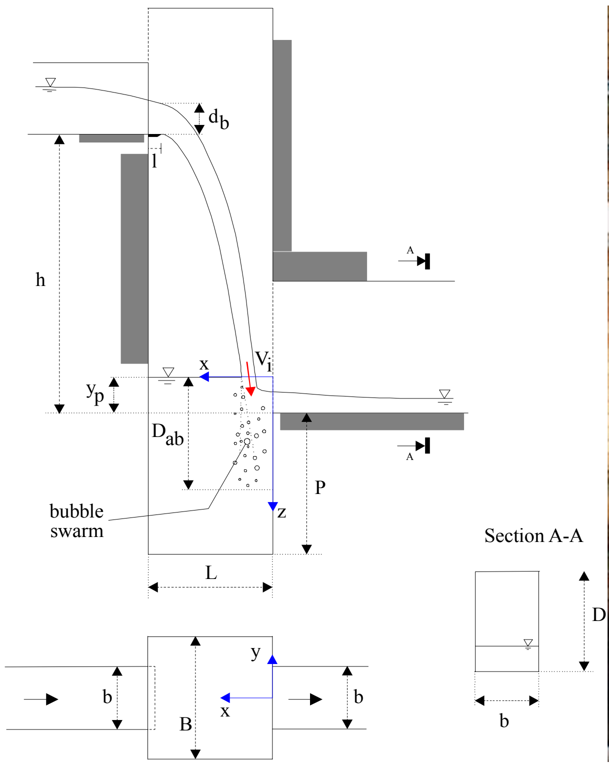

Fig. 1 – Sketch of a rectangular dropshaft   Fig. 2 – Dropshaft in operation with Q=12 L/s

Table 1 – Dropshaft experiments : position of the measurement points

Depth z – mm	x – mm
30	60-205
50	85-505
80	80-205
110	75-200
150	70-205
200	75-205
250	60-170

The second group of experiments were performed at the University of Queensland in a horizontal channel, 3.2 m long and 0.25 m wide (Fig. 3). Both bottom and sidewalls were made of 3.2 m long glass panels. This channel was previously used by Chanson (1995) and Chanson and Brattberg (2000). Preliminary clear water velocity measurements were performed in the flume using a Prandtl-Pitot tube (Ø=3.3 mm) (Chanson, 1995). The results showed that the supercritical inflow was partially-developed for all investigated flow conditions (Table 2). In Table 2, W is the channel width, d_1 is the inflow depth, V_1 is the inflow velocity, whereas *P/D* means partially developed inflow conditions. The waters were fed by a constant head tank and the flow rate in the flume was measured with a 90º V-notch weir which was calibrated on-site with a volume-per-time technique. The relative boundary layer thickness δ/d_1 was about from 0.5 to 0.6 depending upon the inflow conditions (Fig. 4). Further details about the air-water flow properties were provided by Gualtieri and Chanson (2007).

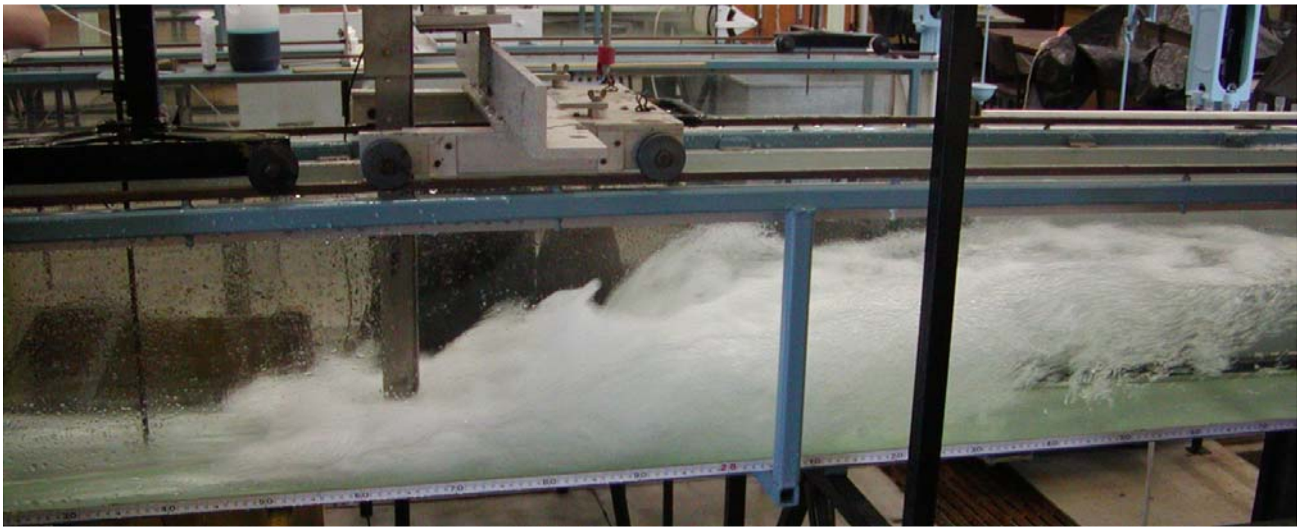

Fig. 3 – The hydraulic jump at Fr_1=14.3

Table 2 – Experimental flow conditions in the hydraulic jump

Reference	W – m	d_1 – m	V_1 – m/s	Fr_1	Comments
Present study	0.25	0.012 to 0.0138	2.23 to 4.87	6.5 to 14.3	Conductivity probe (single tip, 0.35 mm inner electrode), PD inflow conditions

In both studies, the air-water flow properties were measured with a single-tip conductivity probe (needle probe design). The probe consisted of a sharpened rod (platinum wire Ø=0.35 mm) which was insulated except for its tip and set into a metal supporting tube (stainless steel surgical needle Ø=1.42 mm) acting as the second electrode. The measurement principle of conductivity probes is based upon the difference in electrical resistivity between air and water (Jones and Delhaye 1976; Chanson 2002c). The basic data processing yielded the air concentration or void fraction C, the bubble count rate F and the bubble chord time t_{ch}, The void fraction C is the proportion of time that the probe tip is in the air. Past experience showed that the probe orientation with the flow direction has little effect on the void fraction accuracy provided that the probe support does not affect the flow past the tip (Chanson, 1988). In the present study, the probe tip was aligned with the flow direction. The bubble count rate F is the number of bubbles impacting the probe tip. The measurement is sensitive to the probe tip size, bubble sizes, velocity and discrimination technique, particularly when the sensor size is larger than the smallest bubble sizes. The bubble chord time t_{ch} is defined as the time spent by the bubble on the probe tip.

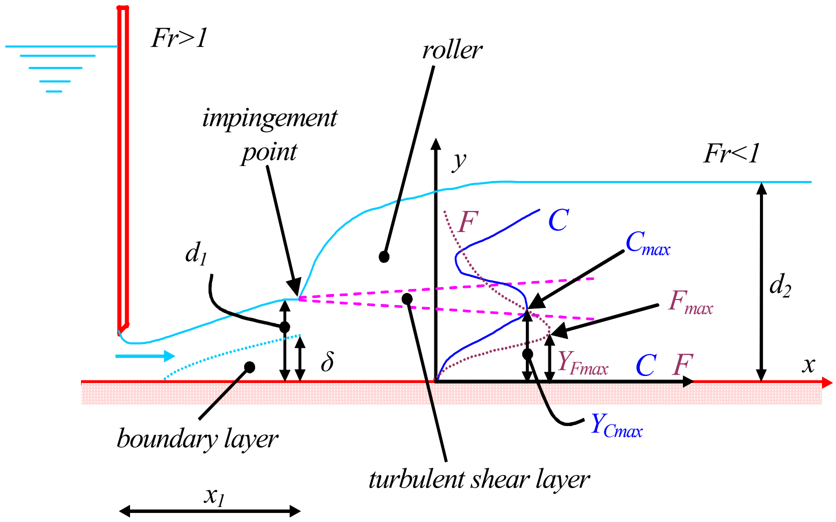

Fig. 4 – Sketch of hydraulic jump flow with partially-developed inflow conditions

The probe was excited by an electronic system designed with a response time less than 10 μs and calibrated with a square wave generator. In the dropshaft, the probe signal output was scanned at 25 kHz for 100 s. Measurements were conducted at several cross-sections along the shaft centreline beneath the nappe impingement with depths ranging from 0.03 m to 0.25 m (Table 1). In the hydraulic jump, the probe sensor was sampled at 20 kHz for 45 s. The vertical position was controlled by a fine adjustment system with an accuracy of 0.1 mm. All the measurements were conducted on the channel centreline.

For some clustering analysis, the sampling rate must be at high-frequency : i.e., at least 10 to 20 kHz. Further the scan duration must be long: that is, almost one order or magnitude longer than that required for basic air water flow measurements (e.g. void fraction and bubble count rate) (Chanson and Toombes 2002). The present experience in dropshaft and hydraulic jump flows suggested that a sampling duration of 45 s was a minimum and a sampling period of 100 s was preferable.

3 CLUSTERING ANALYSIS. RESULTS AND DISCUSSION

3.1 CLUSTER ANALYSIS

A cluster of bubbles is defined as a group of two or more bubbles with a distinct separation from other bubbles before and after the cluster. In a cluster, the bubbles are close together and the packet is surrounded by a sizeable volume of water. A concentration of bubbles within some relatively short intervals of time may indicate some clustering or it may be the consequence of a random occurrence. A study of clustering events may be useful to infer if the formation frequency responds to some particular frequencies of the flow (Luong and Sojka, 1999; Noymer, 2000; Martinez-Bazan et al., 2002). One approach is based upon the analysis of water chord between two adjacent air particles. If two bubbles are closer than a characteristic length scale, they can be considered a group of bubbles that is a cluster (Chanson and Toombes 2002). The characteristic water length scale may be related to the water chord statistics, such as the mean or the median water chord. It may be also related to the bubble size itself, since bubbles within that distance are in the near-wake and may be influenced by the leading particle (Noymer, 2000; Chanson, 2002b).

Herein the instantaneous air and water chord times were recorded in the bubbly flow region of the shaft pool and of the hydraulic jump, in addition of the void fraction and bubble count rate data. The bubble chord time is proportional to the bubble chord length and inversely proportional to the velocity. Small bubble chord times correspond to small bubbles passing rapidly in front the probe sensor, while large chord times imply large air packet flowing slowly past the probe sensor. For intermediate chord times, there are a wide range of possibilities in terms of bubble sizes depending upon the air-water interfacial velocity. The data were post-processed to study the air-water flow structure, including the existence (or not) of bubble clusters. However a clear difference between the dropshaft and the hydraulic jump must be remarked. In the dropshaft, it is possible to consider with an acceptable degree of approximation that the flow velocity is about the jet impingement velocity. Thus, the chord time could be expressed in terms of a pseudo-bubble chord length ch_{ab} defined as:

$$ch_{ab} = V_i \, t_{ch} \qquad (1)$$

where V_i is the jet impingement velocity and t_{ch} is the measured bubble chord time. Chanson et al. (2002) compared Equation (1) with true chord length measurements by Chanson and Brattberg (1996). They showed that Equation (1) predicts the exact shape of bubble size probability distribution functions although it overestimates the bubble chord lengths by about 10 to 30%.

In the hydraulic jump, the streamlines followed a more complicated pattern. Flow reversal and recirculation existed, and the phase-detection conductivity probe could not discriminate the direction nor magnitude of the velocity. Most single- and dual-tip probes are designed to measure positive velocities only and the probe sensor is affected by wake effects during flow reversal. Therefore, only air/water chord time data are presented for the hydraulic jump.

In the present study, the presence of a cluster was identified using water chord statistics. A slightly different criterion was selected for the dropshaft and for the hydraulic jump. In the dropshaft, a cluster was identified when the water pseudo-chord length was smaller than one-tenth of the mean water chord size at the measurement point. Chanson and Toombes (2002) used the same criterion in skimming flows on a stepped chute. In the hydraulic jump, the criterion for cluster existence was a water chord time being less than 10% of the median water chord time. The use of a median value is justified by the smaller number of particles, whereas a *mean* value would be biased by the few large-size particles (Chanson, 2006). For both data sets, the clustering analyses were however restricted to the streamwise distribution of bubbles and they did not include bubbles travelling side by side.

3.2 RESULTS

Figure 5 shows some typical results in the dropshaft facility. It presents some horizontal distributions of numbers of clusters at different depths z below the free-surface in the shaft pool, where x is the horizontal co-ordinate (Fig. 1), L is the dropshaft length (L=0.755 m), N_c is the number of clusters per second, d_c and V_c are the critical flow depth and velocity at the inflow channel brink. In Figure 5, the vertical axis is the dimensionless number of clusters per seconds defined as $N_c \times d_c/V_c$. Some basic statistics on clustering are summarised in Table 3. The data showed that the dimensionless number of cluster per second was maximum in the shear flow. The results indicated further that, with increasing depths z beneath the free-surface, the location where the number of clusters was maximum tended to follow that of the jet trajectory. The average number of clusters was maximum at about 0.05 m beneath the free-surface and decreased with increasing depths (Table 3, column 2). The percentage of bubbles that were associated with clusters ranged from 14% to 60% for all considered depths, although it was in average about one third (Table 3).

Table 3 – Clustering analysis data in the dropshaft (Gualtieri and Chanson, 2004a)

		Clustered bubbles – %		
Depth – z – mm	Average N_c	Minimum	Mean	Maximum
30	3.38	18.73	35.44	47.07
50	**4.46**	15.79	34.12	57.06
80	4.01	14.03	33.68	47.43
110	4.10	21.62	36.29	60.04
150	3.91	22.83	37.41	55.94
200	3.35	23.73	35.50	51.95
250	2.55	30.23	37.77	50.23

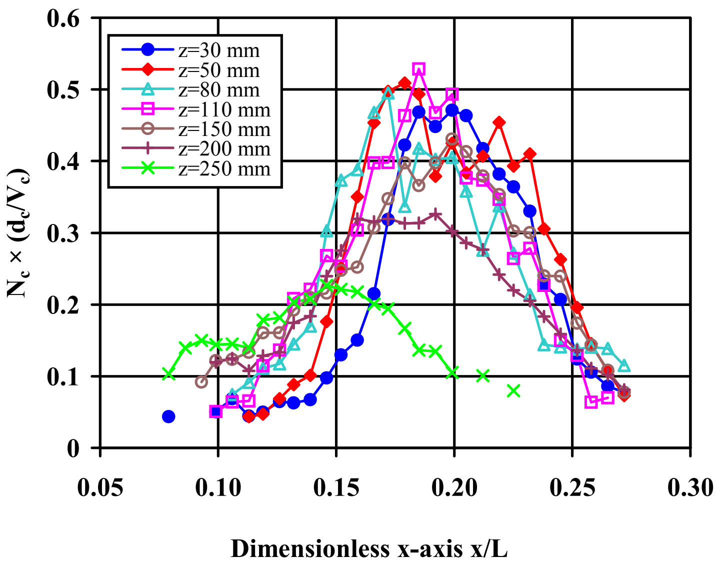

Fig. 5 – Horizontal distribution of the number of clusters per second N_c at several depths z beneath the free-surface in the dropshaft (after Gualtieri and Chanson 2004a)

Figure 6 presents some vertical distributions of the number of clusters per second N_c in the hydraulic jump for Fr_1=6.5, 10.8 and 14.3, where the dimensionless water depth y is vertical elevation above the invert and d_1 is the inflow depth. In Figure 6, the horizontal axis is the dimensionless number of clusters per seconds defined as $N_c \times d_1/V_1$ where d_1 and V_1 are the inflow depth and velocity respectively The clustering analysis regrouped 269 records from 18 vertical profiles (Table 4). Earlier studies demonstrated that an advective diffusion region, or air diffusion layer, exists in which the void fraction distributions follow an analytical solution of the classical advection-diffusion equation (Chanson, 1995, 1997; Chanson and Brattberg, 2000). Above this air diffusion layer, for $y > Y^*$, there is the boiling flow region, where void fraction increases rapidly to the unity (Fig. 4). Careful inspection of the data demonstrated that, at each dimensionless distance from the jump toe $(x-x_1)/d_1$, the maximum number of clusters per second N_{c-max} was always located within the air diffusion layer : i.e., $Y_{Nc-max}/d_1 < Y^*/d_1$ (Fig. 6, Table 4). Further the location of the maximum number of clusters Y_{Nc-max} was close to the location of the maximum void fraction and maximum bubble count rate in the shear region : i.e., Y_{Cmax}/d_1 and to Y_{Fmax}/d_1, respectively. In average, the mean number of clusters per second N_{c-avg} was about 54, 102 and 141 for Fr_1=6.5, 10.8 and 14.3, respectively.

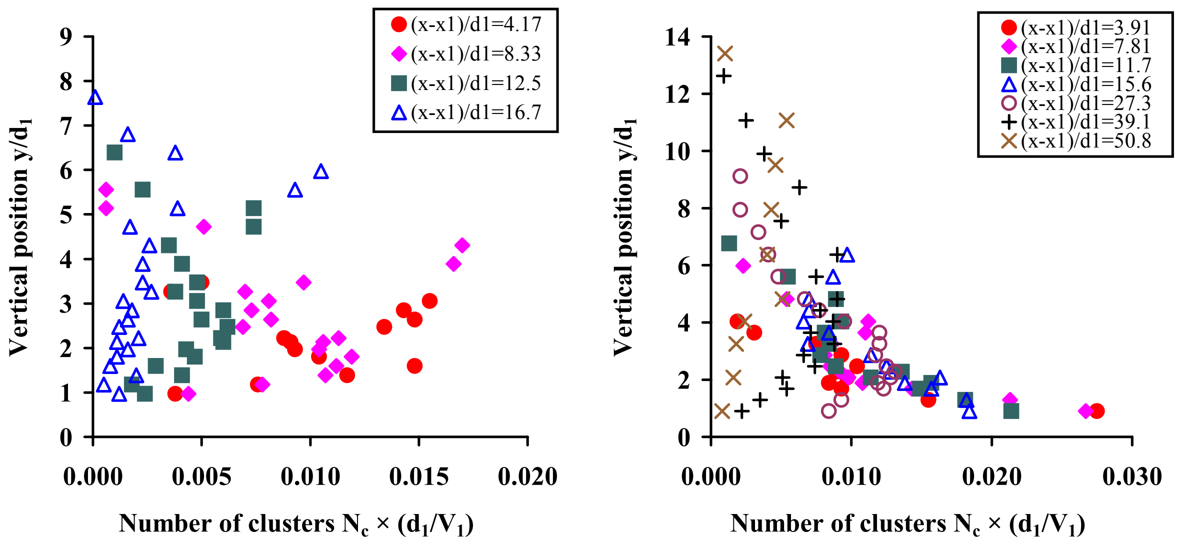

Fig. 6 – Number of clusters for $Fr_1=6.51$ (Fig. 6a) and $Fr_1=10.8$ (Fig. 6b)

Table 4 – Comparison between Y_{Nc-max}, Y_{Cmax}/d_1 and Y^*/d_1 (Hydraulic jump data)

						Clustered bubbles – %
Fr_1	$(x-x_1)/d_1$	Y_{Cmax}/d_1	Y^*/d_1	Y_{Fmax}/d_1	Y_{Nc-max}/d_1	Mean
6.51	4.17	---	---	1.60	3.05	31.5
	8.33	2.85	3.47	3.89	4.30	33.4
	12.5	2.85	4.30	5.14	4.72	31.7
	16.7	3.26	4.72	5.97	5.97	31.3
10.8	3.91	---	---	0.91	0.91	24.5
	7.81	1.95	2.86	1.30	0.91	19.4
	11.7	2.15	3.25	1.30	0.91	31.7
	15.6	2.73	4.43	1.69	0.91	15.3
	27.3	3.52	6.38	3.25	2.28	18.4
	39.1	4.30	8.72	4.82	6.38	21.8
	50.8	---	9.50	11.1	11.1	25.5
14.3	4.20	1.26	2.24	0.98	0.98	10.9
	8.40	1.26	3.50	1.40	0.98	9.2
	16.8	2.10	4.76	1.82	0.98	10.2
	29.4	3.78	7.28	2.24	1.82	12.3
	42.0	5.88	9.38	3.50	1.82	15.8
	54.6	5.46	9.38	5.60	4.34	17.7
	67.2	---	13.6	11.1	11.1	20.9

Figure 7 shows the dimensionless longitudinal profiles of the maximum number of clusters N_{c-max} in the hydraulic jump flows. In Figure 7, the vertical axis is the dimensionless maximum number of clusters per second, defined as $N_{c-max} \times d_1/V_1$. This parameter decreased with increasing distance from the jump toe and it decreased with decreasing inflow Froude number Fr_1 at a given dimensionless distance $(x-x_1)/d_1$. Overall the results showed that the percentage of clustered bubbles was typically within the range 9% to 38%, with an average value of about 21%. The averaged percentage of clustered bubbles was 32%, 22% and 14% for Fr_1=6.5, 10.8 and 14.3 respectively. The result suggested some effects of the inflow Froude number on the clustering structure, with the percentage of clustered bubbles decreasing with increasing Froude numbers.

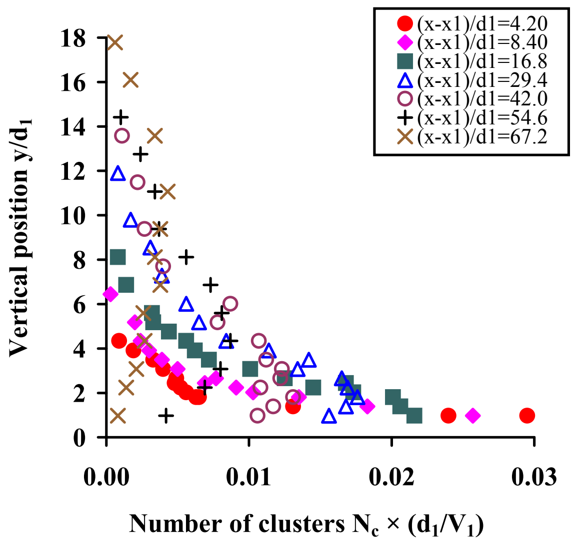

Fig. 6c – Number of clusters for Fr_1=14.3

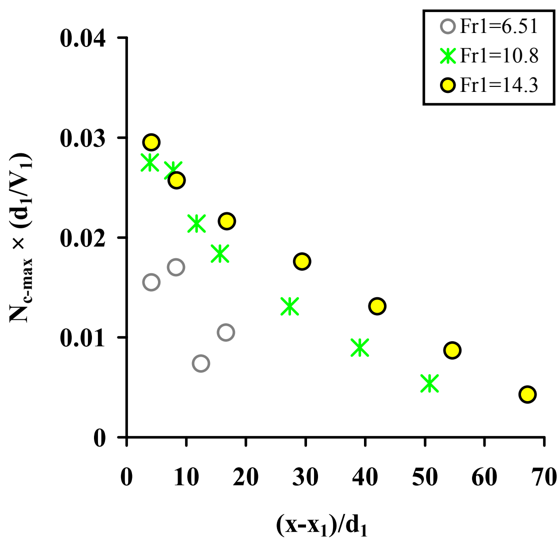

Fig. 7 – N_{c-max} for Fr_1=6.51 10.8 & 14.3

3.3 DISCUSSION

Overall the results were comparable with those observed in plunging jet flows (Chanson et al., 2002) and skimming flow on stepped chutes (Chanson and Toombes, 2002; Chanson and Gonzales, 2004). For these studies, an average of 20% to 30% of entrained bubbles were parts of cluster structures. The average number of bubbles per cluster was about 2.5 for all depths in the dropshaft and was about 2.3 in average in the hydraulic jump flows. That is, a large majority of bubble clusters consisted of two bubbles only. Note that the average number of bubbles per cluster was 2.5, 2.3 and 2.1 for Fr_1=6.5, 10.8 and 14.3, respectively. The percentage of clusters made of two bubbles was estimated. In the dropshaft, about 70% to 95% of all clusters comprised two bubbles only. In the hydraulic jump, this percentage was about 80 to 94% with an overall average value of 88%. Lower values were observed at lower Fr_1, whereas, for Fr_1=14.3, the percentage of clusters made of two bubbles ranged from 90 to 94% These results were consistent with the results obtained in stepped chutes. For skimming flows and transition flows, the clusters made of two bubbles accounted for nearly 68% and 78% of all clusters respectively (Chanson and Toombes, 2002).

It is believed that bubble trapping in large-scale vortices was a dominant clustering mechanism in the studies. Both the dropshaft and hydraulic jump flows were complex turbulent shear flows. New informations on bubble clusters may give some measure of the vorticity production rate, of the level of bubble-turbulence interactions and of the associated energy dissipation. A detailed comparison between the dropshaft and hydraulic jump data showed that the dimensionless number of clusters per second was larger in the dropshaft than in the hydraulic jump (Fig. 5 and 6). Also, the percentage of entrained bubbles involved in cluster structures was in average lower in the hydraulic jump than in the shaft pool of the dropshaft (Tables 3 and 4). These subtle differences reflected some distinction in the interactions between entrained air bubbles and turbulence in these two energy dissipation systems. Some greater degree of clustering was observed in the dropshaft, suggesting possibly a higher level of interactions in the confined shaft geometry.

4 CONCLUSION

Some new characterization of the air-water flows properties yielded some information on the clustering processes. A cluster of bubbles is defined as a group of two or more bubbles, with a distinct separation from other bubbles before and after the cluster. A study of clustering events may be useful to infer if the formation frequency responds to some particular frequencies of the flow. Herein the presence of a cluster was identified using the water chord statistics.

The clustering analyses were conducted in two large-size energy dissipators that are characterised both by some substantial air entrainment: a rectangular dropshaft and a hydraulic jump. In the hydraulic jump, the maximum number of clusters was always located within the air diffusion layer and often close the locations where the maximum void fraction and maximum bubble count rate were detected. The clustering results in the hydraulic jump were compared with those previously reported in a large-size dropshaft. The dimensionless number of clusters per second was remarkably higher in the dropshaft than in the hydraulic jump. Despite some differences, the number of bubbles associated with cluster structures was in average about 21% in the hydraulic jump and 33% in the dropshaft. In the hydraulic jump, the percentage was decreasing with the increasing Froude number. These results were comparable earlier studies in plunging jet flows and skimming flow on stepped chutes, where an average of 20 to 30% of entrained bubbles were observed forming parts of cluster structures.

The average number of bubbles per cluster was about 2.5 in the dropshaft and about 2.3 in the hydraulic jump. Most of the detected clusters comprised of two bubbles only. The number of clusters made of two bubbles was in range from 70 to 95% in the dropshaft and from 80 to 94% in the hydraulic jump, where the highest values were observed at the largest Froude number. These results were consistent with the results obtained in stepped chutes with skimming flow and transition flow in which the percentage of clusters made of two bubbles were 68% and a78%, respectively.

It is suggested that bubble trapping in large-scale vortical structures was a dominant cluster mechanism in the bubbly shear region. Both dropshaft and hydraulic jump flows were complex turbulent shear flows. Some clustering index may provide a measure of the vorticity production rate, of the level of bubble-turbulence interactions and of the associated energy dissipation.

AKNOWLEDGMENTS

The experimental works described in this paper have been conducted when the first author was *Visiting Academic* at the School of Civil Engineering of the University of Queensland. The first author acknowledges that his visits were funded by the Exchange Program for Professors and Researchers of the University of Napoli *Federico II* for the years 2003 and 2006.

REFERENCES

Apelt, C.J., (1984) Goonyella Railway Duplication Drop Structures and Energy Dissipators at Culvert Outlets. Model Studies. *Report CH27/84*, Dept. of Civil Engineering, University of Queensland, Australia, Feb., 10 pages, 11 figures & 37 plates

Chanson H., (1988), A Study of Air Entrainment and Aeration Devices on a Spillway Model, *Ph.D. thesis*, Ref. 88-8, Dept. of Civil Engrg., University of Canterbury, New Zealand

Chanson, H., (1995), Air entrainment in two-dimensional turbulent shear flows with partially developed inflow conditions, *Intl. Jl. of Multiphase Flow*, 21(6), 1107-1121

Chanson, H., (1997), *Air bubble entrainment in free-surface turbulent shear flows*, Academic Press, London, U.K., pp.401

Chanson, H. (1998), The Hydraulics of Roman Aqueducts : Steep Chutes, Cascades and Dropshafts, *Research Report CE156*, Dept. of Civil Engineering, University of Queensland, Australia, 97 pages, 1998

Chanson, H., (2002a), An Experimental Study of Roman Dropshaft Hydraulics, *Jl of Hyd. Res.*, IAHR, 40(1), 3-12

Chanson, H., (2002b), An experimental study of roman dropshaft operation: hydraulics two-phase flow, acoustics, *Research Report CH50/02*, Dept. of Civil Engineering, University of Queensland, Australia

Chanson, H. (2002c), Air-Water Flow Measurements with Intrusive Phase-Detection Probes. Can we Improve their Interpretation ?, *Jl of Hyd. Engrg.*, ASCE, 128, 3, 252-255

Chanson, H. (2006), Air Bubble Entrainment in Hydraulic Jumps. Similitude and Scale Effects, *Research Report CH57/05,* Div. of Civil Engineering, University of Queensland, Australia, 119 pages

Chanson, H., Aoki, S. & Hoque, A., (2002), Similitude of Air Bubble Entrainment and Dispersion in Vertical Circular Plunging Jet Flows. An Experimental Study with Freshwater, Salty Freshwater and Seawater, *Coastal/Ocean Engineering Report No. COE02-1*, Dept. of Architecture and Civil Eng., Toyohashi University of Technology, Japan, 94 pages

Chanson H. and Brattberg T., (1996), Air-Water Bubbly Flow in Free-Shear Layers Proc.*1996 ASME Fluids Eng. Conf.*, San Diego, USA, ASME-FED Vol. 236, Vol.1, 357-364

Chanson, H., and Brattberg, T., (2000), Experimental study of the air-water shear flow in a hydraulic jump, *Intl. Jl. of Multiphase Flow*, 26(4), 583-607

Chanson H. and Toombes L., (2002), Air-water flows down stepped chutes: turbulence and flow structure observations, *Intl. Jl. of Multiphase Flow*, 28, 1737-1761

Chanson, H., and Gonzales, C.A. (2004), Interactions between Free-surface, Free-stream Turbulence and Cavity Recirculation in Open Channel Flows: Measurements and Turbulence Manipulation, *Proc.5th Intl Conf. on Multiphase Flow*, Yokohama, Japan, Y. Matsumoto, K. Hishida, A. Tomiyama, K. Mishima and S. Hosokawa editors, Paper 104, 14 pages (CD-ROM)

Chanson and Gualtieri C., (in press) Similitude and scale effects of air entrainment in hydraulic jumps, *Journal of Hydraulic Research, IAHR*, in press

Gualtieri C. and Chanson H., (2004a), Clustering process and interfacial area analysis in a large-size dropshaft, *5th International Conference on Advances in Fluid Mechanics (AFM 2004)*, Lisbon (Portugal), March, 22/24, 2004, 415-424

Gualtieri C. and Chanson H., (2004b), Analysis air bubbles probability distribution functions in a large-size dropshaft*, 3rd International Symposium on Two-Phase Flow Modelling and Experimentation (ISTP 2004),* Pisa (Italy), September 22-24, 2004

Gualtieri C. and Chanson H., (2007), Experimental analysis of Froude number effect on air entrainment in hydraulic jumps, *Environmental Fluid Mechanics*, 7, 3, June 2007

Jones, O.C., and Delhaye, J.M., (1976), Transient and Statistical Measurement Techniques for two-Phase Flows : a Critical Review, *Intl Jl of Multiphase Flow*, 3, 89-116

Luong, J.T.K., and Sojka, P.E., (1999), Unsteadiness in Effervescent Sprays, *Atomization & Sprays*, 9, 87-109

Martinez-Bazan, C. Montanes, J.L., and Lasheras, J.C. (2002). Statistical Description of the Bubble Cloud Resulting from the Injection of Air into a Turbulent Water Jet, *Intl Jl of Multiphase Flow*, 28, 597-615

Noymer, P.D., (2000), The Use of Single-Point Measurements to Characterise Dynamic Behaviours in Spray, *Experiments in Fluids*, 29, 228-237

Merlein, J., Kleinschroth, A. and Valentin, F., (2002), Systematisierung von Absturzbauwerken. *Mitterilung No. 69*, Lehrstuhls fur Hydraulic und Gewasserkunde, Technishen Universitat Munchen, Germany, 206 pages

Mossa, M., and Tolve, U., (1998), Flow visualization in bubbly two-phase hydraulics jump, *Jl. Fluids Engineering*, ASME, 120, March, 160-165

Murzyn, F., Mouaze, D., and Chaplin, J.R., (2005), Optical fibre probe measurements of bubbly flow in hydraulic jumps, *Intl. Jl. of Multiphase Flow*, 31(1), 141-154

Rajaratnam, N., (1962), An experimental study of air entrainment characteristics of the hydraulic jump, *Jl. of Instn. Eng*. India, 42(7), 247-273

Rajaratnam, N., Mainali, A. and Hsung, C.Y., (1997), Observations on Flow in Vertical Dropshafts in Urban Drainage Systems, *Jl of Env. Engrg.*, ASCE, 123(5), 486-491

Resch, F.J., and Leutheusser, H.J., (1972), Le ressaut hydraulique: mesure de turbulence dans la

region diphasique, *La Houille Blanche*, n.4, 279-293 (in French)
Thandaverwara, B.S., (1974), Self aerated flow characteristics in developing zones and in hydraulic jumps, *Ph.D Thesis*, Dept. of Civil Engineering, Indian Institute of Science, Bangalore, India, pp.399
Vischer, D. and Hager, W.H., (1998), *Dam Hydraulics*, John Wiley, Chichester, UK, 316 pages