A New Calibration Technique for Multiple-Component Stress Wave Force Balances

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Force measurement in hypervelocity expansion tubes is not possible using conventional techniques. The stress wave force balance technique can be applied in expansion tubes to measure forces despite the short test times involved. This paper presents a new calibration technique for multiple-component stress wave force balances where an impulse response created using a load distribution is required and no orthogonal surfaces on the model exist.. This new technique relies on the tensorial superposition of single-component impulse responses analogous to the vectorial superposition of the calibration loads. The example presented here is that of a scale model of the Mars Pathfinder, but the technique is applicable to any geometry and may be useful for cases where orthogonal loads cannot be applied.

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Nomenclature

A	$=$	area [m^2]
$\vec{a}_i$	$=$	calibration load on the top rearward facing conical frustum
$\vec{A}_{a_i}$	$=$	vector location of the point of application of load $\vec{a}_i$
$\vec{A}_{b_i}$	$=$	vector location of the point of application of load $\vec{b}_i$
$\vec{A}_{n_i}$	$=$	vector location of the point of application of load $\vec{n}_i$
$\vec{b}_i$	$=$	calibration load on the bottom rearward facing conical frustum
g	$=$	impulse response function
g_o	$=$	impulse response tensor element
$\vec{g}_{a_{i,k}}$	$=$	single-component impulse response relating load $\vec{a}_i$ with stress bar k
$\vec{g}_{b_{i,k}}$	$=$	single-component impulse response relating load $\vec{b}_i$ with stress bar k
$\vec{g}_{n_{i,k}}$	$=$	single-component impulse response relating load $\vec{n}_i$ with stress bar k
G_{IJ}	$=$	impulse response tensor relating load in direction J with strain in stress bar I
$\vec{n}_i$	$=$	calibration load on the forward facing surface
u	$=$	force function
U_J	$=$	discretised force vector in direction J
Y_I	$=$	discretised strain vector for bar I
y	$=$	strain function

Subscripts

$D1$	$=$	axial direction
$D2$	$=$	normal direction
m	$=$	moment direction

I.   Introduction

Planetary entry vehicles experience high velocity flows with nonequilibrium chemical kinetics. To simulate such flows experimentally it is necessary to reproduce high velocity flows of a gas with a chemical composition closely resembling that experienced by a capsule in flight. The *Pathfinder* entry into the Martian atmosphere[1], for example, was at a nominal speed of 7.35 km/s, corresponding to a total enthalpy of approximately 27 MJ/kg, in an atmosphere that is mostly carbon dioxide. Expansion tubes are currently capable of generating useful aerodynamic flows with low levels of dissociation at such high total enthalpies[2-4].

An expansion tube uses a shock wave generated by a difference in pressure across two adjoining sections of a tube to add energy to a slug of gas called the test gas. An unsteady expansion is used to accelerate the test gas further to superorbital speeds, if necessary, in a third adjoining section of the tube. Most force measurement techniques used in low speed tunnels rely on establishment of equilibrium between aerodynamic forces on the test model and the reaction forces in the balance and its support structure[5]. There is insufficient test time to establish such equilibrium in expansion tubes and alternative techniques have had to be developed. One such approach is based on the stress wave force balance, which characterizes the dynamic response of the model and its support in an impulse response function relating the force time histories on the model to strain time histories in the force balance during calibration[6-9]. Since the stress wave force balance approach accounts for the dynamic response of the model, if the model is calibrated dynamically, the inertia of the model is taken into account and relied upon to provide a characterization of the model's response to loads of differing magnitudes and time histories. The time histories of the forces acting on the model in a tunnel test are then calculated from the strain time histories recorded in the test using numerical deconvolution[10]. This enables the measurement of force coefficients under conditions of structural non-equilibrium.

II. Deconvolution

Strain time histories can be measured using strain sensors and the force time history can be calculated from the recorded strain time histories using deconvolution techniques. For a single-component linear system with an applied load, $u(t)$, and a single output signal, $y(t)$, the output and input are related via the impulse response function, $g(t)$, using the convolution integral

$$y(t) = \int_0^t g(t - \tau)u(\tau)d\tau. \quad (2)$$

One way to obtain the solution of this equation is to solve it in the time domain. If the signals are discretised with time step, Δt, Eq. 2 can be written as

$$y_i = \sum_{j=0}^{i} g_{i-j} u_j \Delta t, \quad (3)$$

which, in matrix form, is termed a Fredholm equation of the second kind. In matrix form this can be written as

$$Y = GU, \quad (4)$$

where U and Y are the discretised force and strain vectors and G is a square impulse response matrix obtained via calibration tests. G is then the lower triangular matrix of the form

$$\begin{bmatrix} g_0 & & & & \\ g_1 & g_0 & & & \\ g_2 & g_1 & g_0 & & \\ \vdots & \vdots & \vdots & \ddots & \\ g_n & g_{n-1} & g_{n-2} & \cdots & g_0 \end{bmatrix} \Delta t. \quad (5)$$

In an experiment, Y is measured and U is determined using techniques such as iterative deconvolution[10].

For a three-component force balance that is designed to measure forces in two orthogonal directions and a moment in the plane of these forces, a minimum of three independent output signals is required. If the three output signals are discretized into vectors Y_A, Y_B and Y_C, the forces in orthogonal directions are in vectors U_{D1} and U_{D2} and the moment is in vector U_m, then the relationship between the inputs and outputs can be written as

$$\begin{pmatrix} Y_A \\ Y_B \\ Y_C \end{pmatrix} = \begin{bmatrix} G_{A,D1} & G_{A,D2} & G_{A,m} \\ G_{B,D1} & G_{B,D2} & G_{B,m} \\ G_{C,D1} & G_{C,D2} & G_{C,m} \end{bmatrix} \begin{pmatrix} U_{D1} \\ U_{D2} \\ U_m \end{pmatrix} \Delta t \ . \tag{6}$$

This results in nine square impulse response submatrices, $G_{i,j}$, relating the inputs to the outputs. Coupled deconvolution techniques are then required to determine the three input signals from the three measured output signals.

The impulse response can be determined from calibration tests (see section III). When a wind-tunnel test to measure aerodynamic forces is conducted, the strain vector is recorded and the only unknown in Eq. 6 is the vector of force time histories. This is determined using a deconvolution process.

III.   Calibration

Calibration of the force balance is required to determine the impulse response matrix in Eq. 6. The linearity of the system can be exploited to use superposition of the results from a series of point load calibrations to determine the impulse response matrix for a distributed load on the model. Dynamic calibration of stress wave force balances can be performed by cutting wires attaching weights to the model or by applying loads using a calibrated impact hammer[11]. Using an impact hammer is usually preferred because of the simpler experimental arrangement required.

Calibration of multiple-component stress wave force balances in the past has used mutually orthogonal calibration loads exclusively[6-9]. For a three-component balance, by applying a pure force in direction $D1$ (see Figure

2), the three submatrices $G_{A,D1}$, $G_{B,D1}$ and $G_{C,D1}$ in Eq. 6 can be determined by using single-component deconvolution. Similarly, when a pure force in the orthogonal direction, $D2$, is applied, one is able to solve for the central column of the impulse response matrix in Eq. 6. Finally, with the application of a pure moment load, one can solve for the final column of the impulse response matrix. However, for the present case, the application of distributed orthogonal loads using an impact hammer is not possible since the hammer can only reliably be used to apply loads normal to the surface and no two surfaces on the model are perpendicular. A new method of calibration was therefore developed.

The new technique involves using an impact hammer to apply a series of point loads at different locations on the model and measuring the outputs from the strain gauges for these loads. For each test, single-component deconvolution can be used to find the impulse response relating the output signals to the applied load. The principle of superposition is then used to combine these impulse responses to determine an impulse response that would be obtained if the loads from combinations of the calibration loads were applied simultaneously, but in different proportions, to the model. By judicious selection of the locations and proportions, it is possible to determine loading combinations that produce pure loads in given directions or a pure moment on the model. It is possible also to simulate the effects of a distributed pressure load by applying calibration loads at many points on the model and using superposition.

The present balance was calibrated to resolve forces in the axial direction ($D1$) and normal direction ($D2$) as well as a pitching moment, M (see Figure 1). Twenty-seven impact-hammer hits were used to calibrate the balance (see Figure 1). All forces were applied perpendicular to the local surface. The nomenclature used for the calibration tests is shown in Table 1. Note that the subscript i indicates the location and the subscript k indicates the output number. The loads on the forward facing nose and conical frustum of the model are labeled $\vec{n}_i$ where $1 \leq i \leq 21$. The loads on the top and bottom sides of the rear conical frustum in the x-y plane (the plane in which the angle of attack varies) are labeled $\vec{a}_i$ and $\vec{b}_i$ respectively (where $1 \leq i \leq 3$). Three output signals are produced for each load - one for each bar.

Single-component impulse responses were formed between each input and the three outputs. Therefore, for each calibration load, three single-component impulse responses are produced.

Superposition was then used to combine the impulse responses from the 27 calibration tests to produce impulse responses that would be obtained for decoupled loads in the directions $D1$ and $D2$ and for a decoupled moment. Our impulse response superposition is analogous to the vectorial superposition of forces of unit magnitude (denoted as $\hat{n}_i$, $\hat{a}_i$, and $\hat{b}_i$ respectively) resulting in decoupled loads in the directions $D1$ and $D2$ and a decoupled moment M. We shall make use of this analogy for the derivation of the impulse response superposition equations.

Firstly, since the loads applied are symmetric about the direction $D1$, summation of all loads, $\sum D1 = \sum_{i=1}^{i=21} \hat{n}_i - \sum_{i=1}^{3} \hat{a}_i - \sum_{i=1}^{3} \hat{b}_i$, produces a net load in the positive $D1$ direction and zero resultants for the other components, $\sum D2 = \sum M = 0$. (See Fig. 2 for directions). The analogous impulse response operation is:

$$\vec{g}_{D1_k} = \left(\sum_{i=1}^{21} \vec{g}_{n_i,k} \right) - \left(\sum_{i=1}^{3} \vec{g}_{a_i,k} \right) - \left(\sum_{i=1}^{3} \vec{g}_{b_i,k} \right). \qquad (7)$$

Here the single-component impulse responses $\vec{g}_{n_i,k}$, $\vec{g}_{a_i,k}$ and $\vec{g}_{b_i,k}$ arise from the loads $\vec{n}_i$, $\vec{a}_i$ and $\vec{b}_j$, respectively (see Table 1). These terms are also the constituents of Eq. 9 and Eq. 11.

Secondly, we seek to superpose vectorially the loads such that the net result is a load in the $D2$ direction with $\sum D_1 = \sum M = 0$. We denote unit vectors in the direction of application of $\vec{a}_i$, $\vec{b}_i$ and $\vec{n}_i$ as $\hat{a}_i$, $\hat{b}_i$ and $\hat{n}_i$ respectively. To achieve $\sum D_1 = 0$, we first reverse the direction of the a-series loads on the rearward facing surfaces of the capsule (see Fig. 1) so that their $D1$ components cancel the $D1$ components of the b-series loads. The resultant $\sum_{i=1}^{3} \hat{b}_i - \sum_{i=1}^{3} \hat{a}_i$ still has a positive moment contribution about the centre of the balance due to a nonzero moment arm. Similarly, the reversal of the upper half of the n-series loads (see Fig. 1) on the forward facing surfaces of the capsule and superposition to the lower half of the n-series loads, according to $\sum_{\substack{i=5 \\ j=0}}^{\substack{j=2 \\ i=7}} \hat{n}_{i+7j} - \sum_{\substack{i=1 \\ j=0}}^{\substack{j=2 \\ i=3}} \hat{n}_{i+7j}$,

gives a resultant in the D_2 direction with $\sum D_1 = 0$ but also with a moment contribution (which is negative) due to a non-zero moment arm from the centre of the balance.

Since the moment contributions are opposing we can multiply the n-series loads by the ratio of magnitudes of the

moment contribution of $\sum\limits_{i=1}^{3}\hat{b}_i - \sum\limits_{i=1}^{3}\hat{a}_i$ to the moment contribution of $\sum\limits_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}}\hat{n}_{i+7\,j} - \sum\limits_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}}\hat{n}_{i+7\,j}$. This ratio is given by

$$1/k_1 = \frac{\left|\sum\limits_{i=1}^{3}\bar{A}_i\hat{b}_i - \sum\limits_{i=1}^{3}\bar{A}_i\hat{a}_i\right|}{\left|\sum\limits_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}}\left[\bar{A}_{i+7\,j}\hat{n}_{i+7\,j}\right] - \sum\limits_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}}\left[\bar{A}_{i+7\,j}\hat{n}_{i+7\,j}\right]\right|} . \qquad (8)$$

where $1/k_1$ is the ratio of moment contribution magnitudes and $\bar{A}_{a_i}$, $\bar{A}_{b_i}$ and $\bar{A}_{n_i}$ are the vector locations of the points of application of loads $\vec{a}_i$, $\vec{b}_i$ and $\vec{n}_i$, respectively (see Fig. 2). The analogous superposition of impulse responses is then given by

$$\vec{g}_{D2_k} = k_1\left(\sum\limits_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}}\left[\vec{g}_{n_{i+7\,j,k}}\right] - \sum\limits_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}}\left[\vec{g}_{n_{i+7\,j,k}}\right]\right) - \left(\sum\limits_{i=1}^{3}\vec{g}_{a_{i,k}}\right) + \left(\sum\limits_{i=1}^{3}\vec{g}_{b_{i,k}}\right), \qquad (9)$$

By examining Eq. (4), it is can be seen that the multiplication of a load, U, by any factor is analogous to the division of the impulse response, G, by that factor. Hence, Eq. 9 shows the factor k_1 which is the reciprocal of the formulation in Eq. 8.

Finally we seek to superpose the loads such that there is a resultant moment due to a distributed load but with $\sum D_1 = \sum D_2 = 0$. We reverse the direction of the b-series loads (see Fig. 1) so that their axial components

cancel the axial components of the *a*-series loads. The resultant $\sum_{i=1}^{3} \hat{b}_i - \sum_{i=1}^{3} \hat{a}_i$ is a moment but with $\sum D_1 = 0$ and

a net force in the positive D_2 direction. The summation $\sum_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}} \hat{n}_{i+7j} - \sum_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}} \hat{n}_{i+7j}$ resulting from the direction reversal

of the upper *n*-series loads with superposition of the lower *n*-series loads also has a positive moment with a negative

D2 force contribution. We now seek to cancel the *D2* force contribution of these loads. This can be achieved by

multiplying the *n*-series loads by the ratio of magnitudes of the *D2* force contribution of $\sum_{i=1}^{3} \hat{b}_i - \sum_{i=1}^{3} \hat{a}_i$ to the *D2*

force contribution of $\sum_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}} \hat{n}_{i+7j} - \sum_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}} \hat{n}_{i+7j}$. This is given by

$$1/k_2 = \frac{\left| \sum_{i=1}^{3} \hat{a}_i - \sum_{i=1}^{3} \hat{b}_i \right|}{\left| \sum_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}} \hat{n}_{i+7j} - \sum_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}} \hat{n}_{i+7j} \right|}, \qquad (10)$$

where $1/k_2$ is the ratio of *D2* force contribution magnitudes. The analogous formulation for the impulse response is

then

$$\vec{g}_{m,k} = k_2 \left(\sum_{\substack{i=1 \\ j=0}}^{\substack{i=3 \\ j=2}} \left[\vec{A}_{ni+7j} \vec{g}_{ni+7j,k} \right] - \sum_{\substack{i=5 \\ j=0}}^{\substack{i=7 \\ j=2}} \left[\vec{A}_{ni+7j} \vec{g}_{ni+7j,k} \right] \right) - \sum_{i=1}^{3} \vec{A}_{ai} \vec{g}_{ai,k} + \sum_{i=1}^{3} \vec{A}_{bi} \vec{g}_{bi,k}, \quad (11)$$

where

Note that the impulse response $\vec{g}_{mk}$ (units of 1/Nmms) relates a unit impulse of a moment to the strain response

of the model while $\vec{g}_{D1k}$ and $\vec{g}_{D2k}$ (units of 1/Ns) relate unit impulse forces to the strain response of the model. As

such, the formulation of $\vec{g}_{mk}$ differs from the other two impulse responses in that a moment arm is required. It is up to the practitioner to select an appropriate length scale for the problem at hand and this can be achieved through the selection of appropriate units for the moment arms $\vec{A}_{a_i}$, $\vec{A}_{b_i}$ and $\vec{A}_{n_i}$.

The scalar k_1 is necessary in order that moments caused by the components of the *n*-series loads in the *D*1 direction cancel the moments caused by the components in the *D*1 direction of the *a*- and *b*-series loads. The sum of forces in the *D*1 direction is also zero, leaving a decoupled force in the *D*2 direction. Similarly the scalar k_2 is necessary in order that forces in direction *D*2 of the *n*-series loads cancel the forces in the *D*2 direction of the *a*- and *b*-series loads leaving the decoupled moment *M*.

The resulting impulse responses for components *D*1, *D*2, and *M* for each bar are determined. The results from each of the three bars are combined to form one, three-component impulse response. This impulse response relates three strain time histories to the three loads (axial force, normal force and pitching moment) and is termed the global impulse response.

IV.   Assessment of Accuracy of the Balance

The performance of any force balance can be assessed by its ability to recover known loads. The recovery of the loads used to make up the global impulse response is not a particularly severe test, while the recovery of single point loads with a distributed impulse response is a more severe test. Neither have any practical application for a force balance and are recommended only as a general assessment tool for the appraisal of force balance performance. These tests were conducted for this balance and are reported elsewhere[11]. A good test for the performance of the balance is to examine its performance for load distributions and temporal variations similar to those expected in tunnel tests.

The three output signals that would be obtained if a point load of a given time history was applied at one of the calibration points can be simulated by convolving the load signal with the impulse responses obtained at that location in the calibration tests. A series of these signals can then be superposed to obtain the output signals that would be obtained for a combination of these loads applied simultaneously. Using this principle, the output signals that would be obtained for a distributed load with a given time history can be obtained. The entire set of loads on the

forward facing surfaces of the capsule were combined to give a loading distribution similar to that expected in the expansion-tunnel tests. The front face of the model was split into seven horizontal strips (see Figure 4). The loads $\vec{n}_1$ to $\vec{n}_{21}$ were weighted according to the area of the capsule they act upon (see Figure 4) and multiplied by factors such that the combination of loads produced axial forces, normal forces and moments with time histories and levels similar to those expected in the experiments.

For example, if a resultant distributed load acting on the capsule is required to act in direction D1 with no net force in direction D2 and no net moment, then loads $\vec{n}_1$ to $\vec{n}_{21}$ are multiplied by the same factor. Similarly, if the resultant distributed load is required to produce a resultant force in directions D1 and D2 with a pitching moment then the loads $\vec{n}_1$, $\vec{n}_8$ and $\vec{n}_{15}$ can be multiplied by a factor greater than that for loads $\vec{n}_7$, $\vec{n}_{14}$ and $\vec{n}_{21}$. In this manner both Newtonian load distributions (where pressure varies as a function of the square of the sine of the azimuthal angle[12]) and departures there from can be modeled using vectorial superposition of loads. This is done to simulate the application of a pressure load.

A force signal (see Figure 3) similar to that expected in tunnel tests was convolved with various weighted combinations of single-component impulse responses obtained from loads $\vec{n}_1$ to $\vec{n}_{21}$ described in Table 2. Note that the values are reported accurate to 2 decimal places rather than a constant number of significant figures. This is more suitable in this case in order to resolve the accuracy of force and moment measurement to the same fraction of a unit of measurement. From Table 2 it can be seen that forces in direction $D2$ are around two orders of magnitude smaller than those in direction $D1$. This is typical of the proportions expected in the tunnel and provides a severe test of capability of the balance to decouple the forces. The levels recovered for axial forces, normal forces and pitching moments were averaged between times 200 μs and 325 μs, the test time window in the tunnel tests, and were compared with the input levels. The errors in recovered axial forces, and normal forces were used to calculate the error in measurement for the lift and drag for the experiments while the error in the moment recovered from these tests was used directly.

Ten cases were selected to test the sensitivity of the balance. Different loading distributions were chosen to investigate the performance of the balance for different aerodynamic situations. Case C1 was chosen to simulate a zero incidence test with all loads combined to produce an axial force of 200 N and no normal forces or pitching moments. Cases C2 and C3 were chosen to simulate the performance for Newtonian pressures on the model. The net resultant axial forces acting on the capsule were similar to Newtonian levels at angles of attack of 2° and 5° respectively. Cases C4 and C5 were chosen to simulate conditions where a reduction in static stability is experienced by the capsule at 2° angle of attack. The loading was similar to that for case C2 but loads near the shoulder on the pitch-in side ($\vec{n}_7, \vec{n}_{14}$, and $\vec{n}_{21}$ for case C4 and $\vec{n}_6$, $\vec{n}_7$, $\vec{n}_{13}$, $\vec{n}_{14}$, $\vec{n}_{20}$ and $\vec{n}_{21}$ for case C5) were multiplied by a factor less than that which Newtonian theory would prescribe. This resulted in a reduced static stability in comparison to that expected from Newtonian pressures for a 2° angle of attack. Similarly cases C6 and C7 were chosen to simulate conditions where a reduction in static stability is experienced by the capsule at 5° angle of attack. Loads near the shoulder on the pitch-in side were multiplied by a factor less than that which Newtonian theory would prescribe, resulting in a reduced restoring moment compared with the Newtonian level at 5° angle of attack. Cases C8, C9 and C10 were produced with the pitch-in side moments weighted to produce an outright instability.

The input and recovered axial forces (direction *D*1), normal forces (direction *D*2) and moments are shown in Figure 5 and Figure 6. Errors in the mean axial force averaged between times of 200 μs and 325 μs were seen to be less than approximately 0.6% (approximately 1N) for all cases. Errors in the mean of the normal force were all below 0.4 N. Errors in the mean of the moment were all below 10 Nmm. However, as the craft moves through 0 angle of attack, the normal forces and moments drop to zero as the absolute measurement accuracy remains constant. Relative errors are therefore not meaningful in this situation but the balance is useful to indicate if significant normal forces and moments are present.

The balance displays very good capabilities for recovering the applied loads. However, the moment recovery shows some sensitivity to load distribution. For example, the magnitude of the input moment for case C6 was -10.15 Nmm which was half of that for case C5, yet the error in the recovered moment for case C6 is approximately seven times less than that for case C5 due to the different load distributions.

V.   Conclusion

This paper shows the mathematical formulation for the tensorial superposition of loads to allow for the use of nonorthogonal calibration loads in the formulation of three-component impulse response. Assessment of the accuracy of the balance shows that the technique is able to resolve small moments for loading distributions that represent departures from stable Newtonian loading distributions. The technique is useful where a three-component impulse response based on loads distributed over an area is required, such as for the case presented here (the Mars Pathfinder geometry), yet it is applicable to any geometry.

1.   R. L. Spencer, Braun, R.D., J. Spacecraft and Rockets **31**, 670 (1996).
2.   A. J. Neely, Morgan, R. G., Aeronaut. J. **98**, 97 (1994).
3.   C. G. Miller, AIAA Journal **13**, 1676 (1977).
4.   H. Bernstein, Journal of the Aeronautical Sciences. **20**, 790 (1953).
5.   L. Bernstein, AGARDograph **214** (1975).
6.   S. L. Tuttle, Mee, D. J., Simmons, J. M., Experiments in Fluids **19**, 336 (1995).
7.   D. J. Mee, Shock Waves **12**, 443 (2003).
8.   S. Sanderson, Simmons, J., AIAA Journal **29**, 2185 (1991).
9.   A. L. Smith, Mee, D.J., Daniel, W. J. T, Shimoda, T., Computers and Structures **79**, 1077 (2001).
10.   R. Prost, Goute, R., Signal Process **7**, 209 (1984).
11.   M. M. Abdel-jawad, PhD thesis, The University of Queensland (2005).
12.   J. D. Anderson, Hypersonic and high temperature gas dynamics (McGraw Hill, Boston MA, 1989), pp.

Figure 1 Schematic of the force balance showing location of calibration loads and directions of resolution.

Figure 2. Vector location of calibration hits.

Figure 3 Force time history used in accuracy assessment of the balance.

Figure 4 Areas acted upon by calibration loads.

Figure 5 Input and recovered distributed loads for cases C1 to C5

Figure 6 Input and recovered distributed loads for cases C6 to C10

Table 1 Nomenclature of impulse responses

Load	Impulse response	Load location identification	Strain output identification
$\vec{a}_i$	$\vec{g}_{a_{i,k}}$	$1 \leq i \leq 3$	$1 \leq k \leq 6$
$\vec{b}_i$	$\vec{g}_{b_{i,k}}$	$1 \leq i \leq 3$	$1 \leq k \leq 6$
$\vec{n}_i$	$\vec{g}_{n_{i,k}}$	$1 \leq i \leq 21$	$1 \leq k \leq 6$

Table 2 Distributed load case. Resultant axial forces, normal forces and pitching moments recovered with global impulse response and the errors in the recovery of the mean. (averaging window between 200 µs and 325 µs.)

Load Case	D1 [N]	D2 [N]	Moment [Nmm]	Error in the mean -D1 [N]	Error in the mean -D2 [N]	Error in the mean -M [Nmm]
C1	200.69	0.00	0.00	0.13	0.01	1.65
C2	200.56	0.93	-45.27	0.32	0.03	1.29
C3	199.92	2.32	-113.40	0.14	0.05	1.15
C4	198.81	0.29	-4.05	0.33	0.13	9.76
C5	199.49	0.54	-21.97	0.38	0.21	9.87
C6	195.55	0.73	-10.15	0.36	0.33	1.44
C7	196.69	1.14	-28.06	0.23	0.35	0.24
C8	198.20	0.07	14.63	1.06	0.39	8.87
C9	197.15	-0.31	36.56	0.03	0.04	9.70
C10	195.26	-1.00	82.57	0.13	0.22	9.88