

Comments on “Flow, thermal, and entropy generation characteristics inside a porous channel with viscous dissipation” by S. Mahmud and R.A. Fraser

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We comment on the paper by Mahmud and Fraser [1]. The paper presents closed form solutions to the velocity and temperature distributions that would be very useful for benchmark checks of numerical computations. They are also useful for parametric studies for cases that involve large number of parameters. However, when such solutions are not free from mistakes one may no longer be able to use them. A typo in the average velocity, i.e. Eq. (10) of [1], is observed (in addition to some typos in the text for example Ref. [2] is repeated as [30]). The correct expression for the average velocity is

$$u_{av} = -Da \operatorname{Re} \left(\frac{\partial p}{\partial x} \right) \left(1 - \frac{\sqrt{Da} \sinh \left(\frac{1}{\sqrt{Da}} \right)}{\cosh \left(\frac{1}{\sqrt{Da}} \right)} \right).$$

Moreover, what [1] presents as a solution to the fully developed temperature profile in Eqns. (16) and (17) does not satisfy the boundary conditions, i.e. one observes that

$$\theta(y=1) = \frac{1}{2} \text{ and } \theta(y=-1) = \frac{-1}{2} \text{ so that one may suspect the ongoing results. The}$$

Nusselt number defined by the authors can be evaluated at one of the walls. However, the authors did not present the heat transfer rate on the opposing wall. Moreover, the

argument by Nield [2] may be applied to highlight the possibility of a singularity in Nu. Besides, in all of the equations that the authors have applied the fluid thermal conductivity k_f , should be replaced by the effective thermal conductivity k_e .

The authors have presented the temperature distribution and the Nusselt number as a function of the Darcy number only, in spite of the fact that Re, Pr, and Ec will affect the temperature profile, as noted by Eq. (16). The fully developed temperature profile, defined as the ratio of the local dimensionless temperature profile divided by that of bulk temperature, could un-couple the thermal field from the aforementioned parameters. One observes that these parameters will not affect the Nusselt number provided that Nu is defined as supposed by Nield [2], i.e. in terms of an average wall-bulk temperature difference.

We now move on to examine the second law aspects of the problem. It seems that in the denominator of Br* the term \sqrt{Da} should be replaced by Da. Following Eq. (24), Mahmud and Fraser [1] set $n = 1$ for a non-Darcy model of the porous media, leading to the disappearance of the Darcy dissipation term, so that the authors are not properly modeling a porous medium [3]. Nield [4] has stated that modeling viscous dissipation in a porous medium, one should not use just the terms involving velocity derivatives, as some authors have done in the past (see Hooman and Ejlali [5] for more details).

The authors have applied a clear-fluid compatible term in the entropy production term that has not been already considered in the thermal energy equation. On the other hand

the Darcy dissipation term is included in the energy equation but neglected in the entropy generation term. Then the following question naturally arises: on what scientific basis should each term be neglected or maintained? Based on Nield et al. [6] one knows that the added terms (by Nield [7] and Al-Hadrami et al. [8]) are $O(Da)$ compared to the Darcy dissipation term, so that one may drop these terms in the expression for the viscous dissipation term, and consequently in FFI (FFI stands for fluid friction irreversibility), just for the small Da case. Even if the authors have dropped the Darcy term assuming a relatively high Da number, the result should not be reported for small Da case (see for example Figs. 4, 5, 6, 9, 10, 11, and 12). On the other hand, in the aforementioned figures, the authors should not report results of Ns or Be for small Da since the results were obtained by dropping the added terms (in FFI) while the Darcy dissipation term was maintained.

All in all, as implied by Nield et al. [9] and stated by Nield [4], the Darcy term should not be neglected even for high Da . Viscous dissipation affects the entropy generation rate not only through FFI but also through HTI (HTI stands for heat transfer irreversibility) since viscous heating acts as a source term in the thermal energy equation and, in this way, the temperature distribution and consequently HTI is affected. This fact was also highlighted by Bejan [10,11].

Another source of inconsistency is the thermal boundary conditions, which are not clearly explained. One expects two extra boundary conditions, one in the inlet and another in the

channel outlet, for the problem statement to be completed. Moreover, the authors have not defined ΔT and T_0 so that one may not understand where Ω is coming from.

The validity of the numerical solutions is questionable because the authors have not applied the fluid temperature at the channel inlet to make their dimensionless temperature. The reader is then left in doubt about the inlet boundary condition and its possible effect on the entropy generation rate and even on the heat transfer aspects of the problem. One knows that, for symmetrical heating or cooling, when the wall is to be heated or cooled by the entering fluid, the Nusselt number behavior differs in having a singularity in the developing region, not only for flow in porous media but also for the clear fluid case (see for example [11-15]). For this problem, however, the situation is more complicated and one expects that all the three alternatives should be considered namely: $T_{in} < T_c$, $T_c < T_{in} < T_h$, and $T_{in} > T_h$; where T_{in} is the fluid inlet temperature.

The presentation of the energy streamlines in [1] is also confusing. The resulting equations, Eqns. (32, 33), are reported in dimensional space while one may not link the terms with those of dimensionless counterparts. The dimensionless parameters affecting the energy streamlines should have been clearly addressed since the Darcy number is not the only parameter affecting the problem.

Discussing Fig. 12 (c), the authors stated “the velocity magnitude increases along the axial direction in the developing region”. This statement seems nebulous since the velocity magnitude should remain constant along the channel as a result of mass

continuity (the problem is said to be steady and incompressible so that when there is no property variation and when the flow passage in each cross-section is uniform one expects that the velocity magnitude should be constant).

Moreover, the right-hand-side of both of the expressions for E_x and E_y contains both a Darcy-like dissipation term and clear flow compatible terms (added by Al-Hadrami et al. [8]) while the authors have neglected the clear flow compatible terms in solving the energy equation. Besides, the Darcy-like dissipation term should be divided by the porous medium permeability [16-20]. Also available in Nield [20] is a detailed investigation of the thermal development concept when viscous dissipation is important in a classical forced convection problem.

In [1] there is no discussion of grid independence or code validation. Hence, the reliability of the numerical results is questionable though the authors have applied a previously tested numerical scheme. Code validation remains as an essential part of a computational study.

Acknowledgments

The first author, the scholarship holder, acknowledges the support provided by The University of Queensland in terms of UQILAS, Endeavor IPRS, and School Scholarship.

References

- [1] S. Mahmud, R.A. Fraser, Flow, thermal, and entropy generation characteristics inside a porous channel with viscous dissipation, *Int. J. Thermal Sciences* 44 (2005) 21-32.

- [2] D.A. Nield, Forced convection in a parallel plates channel with asymmetric heating, *Int. J. Heat Mass Transfer* 47 (2004) 5609-5612.
- [3] D.A. Nield, *Private communication*, (2005).
- [4] D.A. Nield, Comments on ‘A new model for viscous dissipation in porous media across a range of permeability values’ by A. K. Al-Hadrami, L. Elliot and D.B. Ingham, *Transport Porous Media* 55 (2004) 253-254.
- [5] K. Hooman, A. Ejlali, Second law analysis of laminar flow in a channel filled with saturated porous media: a numerical solution, *Entropy* 7 (2005) 300-307.
- [6] D.A. Nield, A.V. Kuznetsov, M. Xiong, Thermally developing forced convection in a porous medium: parallel plate channel with walls at constant temperature, with longitudinal conduction and viscous dissipation effects, *Int. J. Heat Mass Transfer* 46 (2003) 643-651.
- [7] D.A. Nield, Resolution of a paradox involving viscous dissipation and nonlinear drag in a porous medium, *Transport Porous Media* 41 (2000) 349-357.
- [8] A.K. Al-Hadrami, L. Elliott, D.B. Ingham, A new model for viscous dissipation in porous media across a range of permeability values, *Transport Porous Media* 53 (2003) 117-122.
- [9] D.A. Nield, A.V. Kuznetsov, M. Xiong, Effects of viscous dissipation and flow work on forced convection in a channel filled by a saturated porous medium, *Transport Porous Media*, 56 (2004) 351-367.
- [10] A. Bejan, *Convection Heat Transfer*, Wiley, New York, 1984.
- [11] A. Bejan, *Entropy Generation through Heat and Fluid Flow*, Wiley, New York, 1982.

- [12] D.A. Nield, K. Hooman, Comments on "Effects of viscous dissipation on the heat transfer in forced pipe flow. Part 1: Both hydrodynamically and thermally fully developed flow [Energy Conv. Manage. 2005; 46 : 757-769] and Part 2: Thermally developing flow [Energy Conv. Manage. 2005; 46 : 3091-3202]" by O. Aydin, *Energy Conversion and Management*, 47(2006) 3501-3503
- [13] K. Hooman, M. Gorji-Bandpy, Laminar dissipative flow in a porous channel bounded by isothermal parallel plates, *Applied Mathematics and Mechanics – English Edition*, 26 (2005) 578-593.
- [14] W.J. Ou, K.C. Cheng, Viscous dissipation effects on thermal entrance region heat transfer in laminar and turbulent pipe flows with uniform wall temperature, AIAA Paper No. 74-743 or ASME Paper No. 74-HT-50, 1974, 1-7.
- [15] A.A. Ranjbar-Kani, K. Hooman, Viscous dissipation effects on thermally developing forced convection in a porous medium: circular duct with isothermal wall, *Int. Comm. Heat Mass Transfer*, 31 (2004) 897-907.
- [16] D.A. Nield, A. Bejan, *Convection in Porous Media*, 3rd edition, Springer-Verlag, New York, 2006.
- [17] K. Hooman, A. Pourshaghaghay, A. Ejlali, Effects of viscous dissipation on thermally developing forced convection in a porous saturated circular tube with an isoflux wall, *Applied Mathematics and Mechanics – English Edition*, 27 (2006) 617-626.
- [18] D.A. Nield, A.V. Kuznetsov, M. Xiong, Thermally developing forced convection in a porous medium: Circular duct with walls at constant temperature, with longitudinal conduction and viscous dissipation effects, *Transport Porous Media*, 53 (2003) 331-345.

[19] Haji-Sheikh A, Nield DA, Hooman K, Heat transfer in the thermal entrance region for flow through rectangular porous passages, *Int. J. Heat Mass Transfer*, 49 (2006) 3004-3015.

[20] D.A. Nield, A note on a Brinkman-Brinkman forced convection problem, *Transport Porous Media*, 64 (2006) 185-188.