Effects of temperature-dependent viscosity variation on entropy generation, heat, and fluid flow through a porous-saturated duct of rectangular cross-section

K. Hooman, H. Gurgenci

School of Engineering, The University of Queensland, Brisbane, Australia

ABSTRACT

Effect of temperature-dependent viscosity on fully developed forced convection in a duct of rectangular cross-section occupied by a fluid-saturated porous medium is investigated analytically. The Darcy flow model is applied and the viscosity-temperature relation is assumed to be an inverse-linear one. The case of uniform heat flux on the walls, i.e. the **H** boundary condition in the terminology of Kays and Crawford [1], is treated. For the case of a fluid whose viscosity decreases with temperature, it is found that the effect of the variation is to increase the Nusselt number for heated walls. Having found the velocity and the temperature distribution, the second law of thermodynamics is invoked to find the local and average entropy generation rate. Expressions for the entropy generation rate, the Bejan number, the heat transfer irreversibility, and the fluid flow irreversibility are presented in terms of the Brinkman number, the Péclet number, the viscosity variation number, the dimensionless wall heat flux, and the aspect ratio (width to height ratio). These expressions let a parametric study of the problem based on which it is observed that the entropy generated due to flow in a duct of square cross-section is more than those of rectangular counterparts while increasing the aspect ratio decreases the entropy generation rate similar to what previously reported for the clear flow case [2].

Keywords: entropy generation rate, forced convection, porous medium, rectangular duct, temperature-dependent viscosity

Nomenclature

a	aspect ratio
A	coefficient defined by Eq. (17)
Be	Bejan number defined by Eq. (32)
Br	Brinkman number defined by Eq. (29)
c_P	specific heat at constant pressure
D_H	hydraulic diameter
D_n	coefficient defined by Eq. (14)
G	Negative of the applied pressure gradient
H	duct height
k	thermal conductivity
K	permeability
m	coefficient defined by Eq. (14)
N	viscosity variation number
N_{FFI}	fluid friction irreversibility
N_{HTI}	heat transfer irreversibility
N_S	dimensionless entropy generation number defined by Eq. (31)
Nu	Nusselt number defined by Eq. (20)
P^2	viscosity variation parameter defined by Eq. (11)
Pe	Péclet number defined by Eq. (27-a)
q	dimensionless wall heat flux by Eq. (27-b)
q''	wall heat flux
R	dimensionless parameter defined by Eq. (10)
S_1,S_2	series defined by Eq. (19-b,c)
$\dot{S}_{gen}$	entropy generation rate per unit volume
T*	temperature
T_w	wall temperature
T_m	bulk mean temperature
$u*$	filtration velocity
$\bar{u}$	mean velocity

$\hat{u}$ normalized velocity $u/\bar{u}$

x,y,z dimensionless coordinates

x^*,y^*,z^* Cartesian coordinates

Greek symbols

θ $k\dfrac{T_w - T^*}{q''H}$

μ fluid viscosity

λ_n Eigenvalues of the problem

ρ fluid density

Subscripts

cp constant property

w wall

1. INTRODUCTION

Because of applications to the cooling of electronic equipment there has been in recent years an increased interest in forced convection in channels and ducts filled with porous media. A substantial amount of literature on this topic is available (see, for example, the surveys in Nield and Bejan [3], Lauriat and Ghafir [4] and, for early work, Haji-Sheikh and Vafai [5]). For circular ducts or parallel plate channels the simplicity of the geometry allows analytical solutions of closed form. Analytical solutions are useful for benchmark checks on numerical computations. They are also useful for parametric studies when a large number of parameters are involved. Thus the question naturally arises as to whether analytical solutions for ducts of other cross-sections are possible.

One way to handle the differential equation problem is to use the method of weighted residuals. This method was exploited by Haji-Sheikh and Vafai [5] in their study of thermally developing convection (the Graetz problem) in ducts of various shapes, including elliptical ones. This method is especially convenient when the boundary conditions are homogeneous, as in the forced convection problem with uniform temperature imposed on the walls. For the Graetz problem it has the additional advantage that in conjunction with standard computing packages it allows the computation of all the required eigenvalues at one go rather than having to get them one at a time.

As a pioneering analytical work, Haji-Sheikh [6] has applied a Fourier series method to investigate forced convection in a duct of rectangular cross-section based on the Brinkman flow model and showed that the Nusselt number increases monotonically with the aspect ratio. Implied in his work were high Prandtl number and constant property suggestion as well as negligible axial conduction and viscous dissipation. Similar results were reported by Hooman and Merrikh [7] where the authors also reported the friction factor versus the porous media shape factor. Haji-Sheikh et al. [8] have applied a solution that uses the Green's function to investigate the thermally developing forced convection considering both the isothermal and isoflux boundary conditions.

Regarding to ducts of arbitrary cross-section, Hooman [9-10] has reported closed form solutions for the fully developed temperature distribution and the Nusselt number as well as the local entropy generation rate by applying the Darcy momentum equation for flow in a duct of elliptical cross-section. It was found that heat transfer is conduction-dominated as a result of very slow (creeping) velocities in such low porosity media. However, when the fluid viscosity is a temperature-dependent one the problem becomes more complicated and one can no longer obtain such simple expressions for the temperature profile and the Nusselt number even for ducts of circular tube cross-section or parallel plate channel, as noted by Hooman [11].

Being highly viscous in nature, for most fluid of engineering application the viscosity is strongly dependent on the temperature; while the thermal diffusivity remains relatively constant. For example, the viscosity of glycerin has a threefold decrease in magnitude for a 10^{o}C rise in temperature [12]. This trend is not only observed in such viscous liquids but also in other liquids such as water; where the viscosity of the water decreases by about 240 percent when temperature increases from 10^{o}C to 50^{o}C, as reported by Ling and Dybbs [13]. As a result, the constant property solutions most often given in the literature need to be modified. Viscosity variation with temperature in case of fluids clear of solid material was the subject of many studies so far and a complete literature survey may be found in [1].

On the other hand, the groundbreaking work by Bejan [14] was an initiation of entropy generation analysis for the constant viscosity case where a list of recently-published relevant articles may be found in [2]. Having considered the two problems simultaneously, effects of variable viscosity on entropy generation have been investigated by Sahin [15] and Al-Zahranah and Yilbas [16] for the clear flow case. However, the similar problem in porous medium case has not been yet considered in spite of the ever-increasing use of porous materials in industrial application.

Until recently, the majority of the work done to investigate the effects of temperature-dependent viscosity in forced convection in porous medium is limited to the first law analysis based on the Darcy model by Ling and Dybbs [13], Nield et al. [17], and Hooman [18] and the Hazen-Dupuit-Darcy model [19-22] for flow in a parallel plate channel. On the other hand, several articles (a complete list of them may be found in [23] and for brevity the list is not repeated here) dealt with the second law analysis for flow in a porous medium where none of them considered the effects of property variation. The aim of this paper is to consider the effects of temperature-dependent viscosity variation on flow, thermal and entropy generation characteristic inside a porous saturated duct of rectangular cross-section.

2. ANALYSIS

2.1 Heat and Fluid Flow

Fully developed forced convection in a rectangular duct occupied by a porous medium is considered as illustrated by figure 1. It is assumed that the Péclet number is sufficiently large for the longitudinal conduction (that in the x^*-direction) to be neglected.

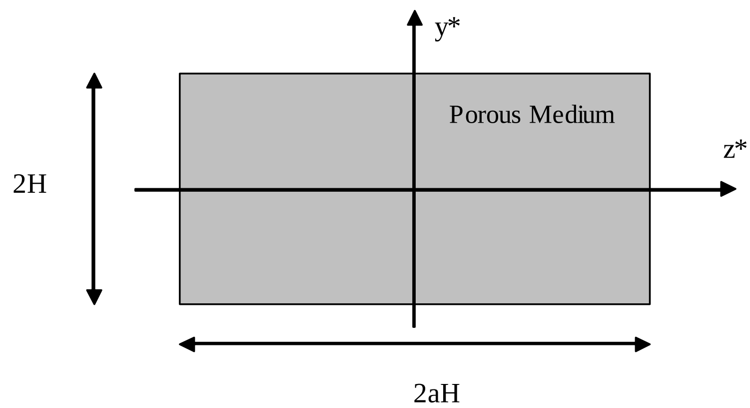

Figure 1 Definition sketch.

The Darcy momentum equation is (the flow is in the x^* direction)

$$u^* = \frac{KG}{\mu}, \tag{1}$$

In the above equation K is the permeability, G is the negative of the applied pressure gradient, and μ is the fluid viscosity related to the fluid viscosity at wall temperature (μ_w) by an inverse-linear model. Assume the viscosity-temperature relation to be as follows

$$\frac{1}{\mu} = \frac{1}{\mu_w}\left(1 - \frac{\partial \mu}{\partial T^*}\Big|_{T_w} \frac{(T^* - T_w)}{\mu_w}\right) \tag{2}$$

Applying the above model for the viscosity variation with temperature is justified when one observes that most of the fluids used in porous journal bearings and similar devices (SAE oils, PAOs, etc), together with water and alcohol, have a reciprocal viscosity closely fitted by a linear function of temperature within a limited range of temperature, of the order of tens of degrees C [22].

To be able to compare the results with those in the literature for parallel plate channel a viscosity variation number, similar to [17], is defined as

$$N = \frac{\frac{\partial \mu}{\partial T^*}\Big|_{T_w}}{\mu_w} \frac{Hq''}{k} \tag{3}$$

where k is the thermal conductivity, q'' is the wall heat flux, T_w is the wall temperature, and T^* is the temperature.

Now one rearranges the viscosity-temperature relation to find that

$$\frac{1}{\mu}=\frac{1}{\mu_w}\left(1-\frac{Nk}{Hq''}(T^*-T_w)\right) \tag{4}$$

This leads to following form for the Darcy momentum equation

$$u^*=\frac{GK}{\mu_w}\left(1-\frac{Nk}{Hq''}(T^*-T_w)\right) \tag{5}$$

With thermal conduction in the x^*-directions neglected, the thermal energy equation is

$$u^*\frac{\partial T^*}{\partial x^*}=\frac{k}{\rho c_P}\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right). \tag{6}$$

Here ρ is the density and C_P is the specific heat at constant pressure of the fluid.

The first law of thermodynamics leads to

$$\frac{\partial T^*}{\partial x^*}=\frac{q''}{\rho c_P H\bar{u}}\left(\frac{a+1}{a}\right). \tag{7}$$

Applying equations (5-7), one finds that

$$\frac{GK}{\mu_w}\left(1-\frac{Nk}{Hq''}(T^*-T_w)\right)\frac{q''}{kH\bar{u}}\left(\frac{a+1}{a}\right)=\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}} \tag{8}$$

In dimensionless form one writes the energy equation as

$$\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}-p^2\theta+\frac{1}{R}=0 \tag{9}$$

where the following dimensionless variables are defined

$$y=\frac{y^*}{H},\quad z=\frac{z^*}{H},\quad \theta=k\frac{T_w-T^*}{q''H},\ R=\frac{\bar{u}\mu_w}{GK}\frac{a}{a+1}. \tag{10}$$

For a fluid whose viscosity decreases with temperature (like most liquids) the viscosity variation number should be a negative one so that one emphasizes the point by defining

$$p^2=-\frac{N}{R} \tag{11}$$

The appropriate boundary conditions are

$\theta=0$ at $y=\pm 1$ and at $z=\pm a$. (12)

The well known eigenfunction expansion approach leads to the following solution for the dimensionless temperature

$$\theta=\sum_{n=1}^{\infty}D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y) \tag{13}$$

where

$$m = \left[p^2 + \lambda_n^2\right]^{1/2},$$
$$D_n = \frac{2(-1)^{n-1}}{\lambda_n R m^2}, \qquad (14a\text{-}b\text{-}c)$$
$$\lambda_n = \frac{(2n-1)\pi}{2}.$$

Having found the temperature distribution one finds the mean velocity as

$$\bar{u} = \frac{GK}{\mu_w} \frac{1 + \sqrt{1 + N \frac{8(a+1)}{a} \sum_{n=1}^{\infty} \frac{\left(1 - \frac{\tanh ma}{ma}\right)}{\lambda_n{}^2 m^2}}}{2} \qquad (15)$$

and the normalized velocity is found to be

$$\hat{u} = \frac{u*}{\bar{u}} = A(1 + N\theta), \qquad (16)$$

where

$$A = \frac{2}{1 + \sqrt{1 + \frac{8(a+1)N}{a} \sum_{n=1}^{\infty} \frac{1}{\lambda_n{}^2 m^2} \left[1 - \frac{\tanh ma}{ma}\right]}} \qquad (17)$$

One remembers that *A* is related to *R* as one can write $A = \frac{1}{R}\frac{a}{a+1}$.

The dimensionless mixing cup temperature defined by the following integral

$$\theta_b = \frac{\int_0^a \int_0^1 \theta \hat{u} dy dz}{a} \qquad (18)$$

is found to be

$$\theta_b = s_1 + s_2 \qquad (19\text{-a})$$

where s_1 and s_2 are

$$S_1 = \frac{2A}{R} \sum_{n=1}^{\infty} \frac{\left[1 - \frac{\tanh ma}{ma}\right]}{m^2 \lambda_n{}^2}, \qquad (19\text{-b,c})$$
$$S_2 = N \frac{A}{R^2} \sum_{n=1}^{\infty} \frac{\left(2 - \frac{3\tanh(ma)}{ma} + \sec h^2(ma)\right)}{\lambda_n{}^2 m^4}.$$

Having found the dimensionless mixing cup temperature, one readily obtains *Nu* as

$$\mathrm{N}u=\frac{q''D_H}{k(T_w-T_m)}=\frac{D_H}{H}\frac{1}{\theta_b} \tag{20}$$

Here D_H is the hydraulic diameter, here given by [7]

$$D_H=4H\frac{a}{a+1}. \tag{21}$$

In particular from equations (19-21) one obtains the Nusselt number as

$$\mathrm{N}u=\left(\frac{a}{a+1}\right)^2\frac{2}{A^2}\frac{1}{\sum_{n=1}^{\infty}\frac{\left[1-\frac{\tanh ma}{ma}\right]}{m^2\lambda_n^2}+AN\frac{a+1}{a}\sum_{n=1}^{\infty}\frac{\left(1-\frac{3\tanh(ma)}{2ma}+\frac{\sec h^2(ma)}{2}\right)}{\lambda_n^2 m^4}} \tag{22}$$

It is also worth noting that the $A=\frac{1}{R}\frac{a}{a+1}$ relation is also adopted in driving the above expression for the Nusselt number.

In the limit as a tends to infinity (parallel plate channel) one recovers the analytical solutions corresponding to both constant and the temperature-dependent viscosity case. As N tends to zero (the constant property case) one observes that equation (22) results in Nu=12 which is in complete agreement with the known results of the Darcy flow model. Simplifying equation (22), based on perturbation techniques for small N, one obtains $Nu=12\left(1-\frac{2N}{15}\right)$ that is in complete agreement with [17] where the mathematical details are neglected for brevity. The reader may note that Nield et al. [17] have defined Nu in terms of the channel width but here Nu is defined in terms of the hydraulic diameter so that this Nu is twice greater than that of [17]. One also notes that the results are intentionally truncated to check the solution within a finite range of error being O(N^2). In fact equation (22) leads to a Nusselt number being different from the aforementioned value provided that one invoked more terms of the series in equation (22). A detailed verification of the present work is presented in table 1 where one can observe that the results of this study and those reported by Haji-Sheikh [6] are the same within three significant figures. One also notes that one should not expect that the result be the same for the model applied in [6] is the Brinkman flow model and the limiting results of his study are taken for comparison purposes.

Table 1 Comparison between present Nusselt number (Nu) and those of [6] for constant property case

a	Nu (Present Study)	Haji-Sheikh (2005)

1	7.1131	7.1136
4	9.1159	9.1165
8	10.2917	10.292
10	10.5838	10.584
100	11.8375	11.838
∞	12	12

2.2 Entropy Generation

It is known that entropy is generated through heat and fluid flow and the amount of entropy generation per unit volume may be found in terms of heat transfer irreversibility (N_{HTI}) and fluid friction irreversibility (N_{FFI}) as follows

$$\dot{S}_{gen} = N_{HTI} + N_{FFI} \tag{23}$$

where N_{HTI} may be found as

$$N_{HTI} = k\frac{\nabla T^*.\nabla T^*}{T^{*2}} = \frac{k}{T^{*2}}\left(\left(\frac{\partial T^*}{\partial x^*}\right)^2 + \left(\frac{\partial T^*}{\partial y^*}\right)^2 + \left(\frac{\partial T^*}{\partial z^*}\right)^2\right) \tag{24}$$

and also N_{FFI} may be obtained as

$$N_{FFI} = \frac{\mu u^{*2}}{T^*\mathrm{K}} \tag{25}$$

In terms of dimensionless variables N_{HTI} becomes

$$N_{HTI} = \left(\frac{k}{H^2}\right)\frac{\left(\left(\frac{a+1}{aPe}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2 + \left(\frac{\partial \theta}{\partial z}\right)^2\right)}{(q-\theta)^2} \tag{26}$$

where the Péclet number, Pe, and a dimensionless wall heat flux are defined as

$$Pe = \frac{\rho c_p H\bar{u}}{k}, \qquad q = \frac{T_w k}{q'' H}. \tag{27-a,b}$$

For dimensionless N_{FFI} one obtains

$$N_{FFI} = qBr\frac{k}{H^2}\frac{(1+N\theta)}{(q-\theta)}, \tag{28}$$

with the Brinkman number, Br, defined as

$$Br = \frac{G^2 K H^2}{\mu_w T_w k}. \quad (29)$$

In dimensionless form the entropy generation number, N_S, becomes

$$N_S = \frac{H^2}{k}\dot{S}_{gen} = \frac{\left(\left(\frac{a+1}{aPe}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 + \left(\frac{\partial\theta}{\partial z}\right)^2\right)}{(q-\theta)^2} + qBr\frac{(1+N\theta)}{(q-\theta)} \quad (30)$$

In particular from equations (12-14) the entropy generation becomes

$$N_S = \frac{\left(\frac{a+1}{aPe}\right)^2 + \left(\sum_{n=1}^{\infty} D_n\lambda_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\sin(\lambda_n y)\right)^2 + \left(\sum_{n=1}^{\infty} D_n \frac{m\sinh mz}{\cosh ma}\cos(\lambda_n y)\right)^2}{\left(q-\sum_{n=1}^{\infty} D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y)\right)^2}$$
$$+qBr\frac{\left(1+N\sum_{n=1}^{\infty} D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y)\right)}{\left(q-\sum_{n=1}^{\infty} D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y)\right)} \quad (31)$$

One also obtains the Bejan number, the ratio of N_{HTI} to the total entropy generation rate, as

$$Be = \left(1+qBr\frac{\left(1+N\sum_{n=1}^{\infty} D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y)\right)\left(q-\sum_{n=1}^{\infty} D_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\cos(\lambda_n y)\right)}{\left(\frac{a+1}{aPe}\right)^2 + \left(\sum_{n=1}^{\infty} D_n\lambda_n\left[1-\frac{\cosh mz}{\cosh ma}\right]\sin(\lambda_n y)\right)^2 + \left(\sum_{n=1}^{\infty} D_n \frac{m\sinh mz}{\cosh ma}\cos(\lambda_n y)\right)^2}\right)^{-1} \quad (32)$$

For the case of negligible N_{FFI}, i.e. Br=0, the Bejan number tends to unity and one verifies that the only means of entropy generation is N_{HTI}. According to Bejan [24] one must consider N_{FFI} in entropy generation analysis even if one has already neglected the viscous dissipation term in the thermal energy equation which is the present case.

3. RESULTS AND DISCUSSION

In this problem one can apply the closed form solutions to show the velocity, isotherms, and N_S throughout the flow region, but in the interests of brevity we just present average values of N_S and *Nu*.

Fig. 2 shows N_S divided by that of constant property case versus *Pe* for some values of *N* (*a*=1). The Péclet number affects the entropy generation rate through the axial (longitudinal) temperature gradient in N_{HTI}. Observe that increasing *Pe* decreases N_S but the decline is not that severe particularly when *Pe* changes from 5 to 10.

Being linearly relevant to, N_{FFI} and consequently, the entropy generation rate, a raise in *Br* increases N_S values as illustrated in Fig. 3. It is also understood that viscosity decrease results in a reduction in entropy generation rate compared to the constant property case, as expected.

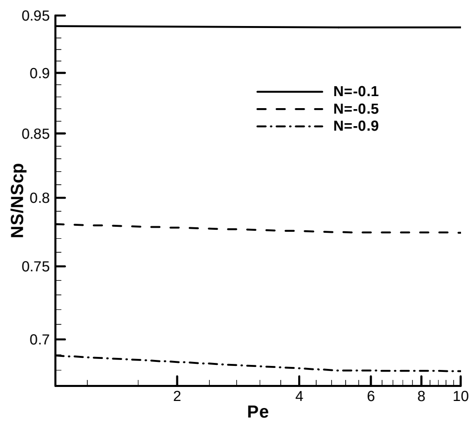

Figure 2 The dimensionless average entropy generation number divided by that of constant property case versus *Pe* for some values of *N* (*Br*=1, *q*=1, and *a*=1)

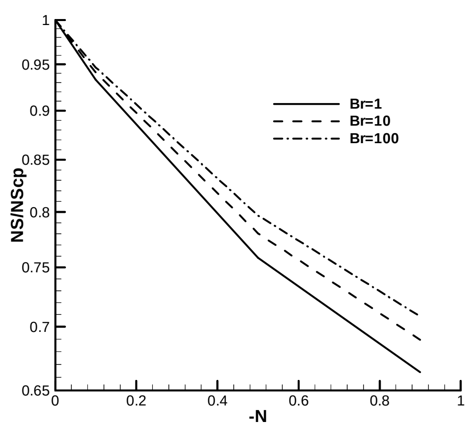

Figure 3 The dimensionless average entropy generation number divided by that of constant property case versus *N* for some values of *Br* (*a*=1, *q*=1, and *Pe*=1)

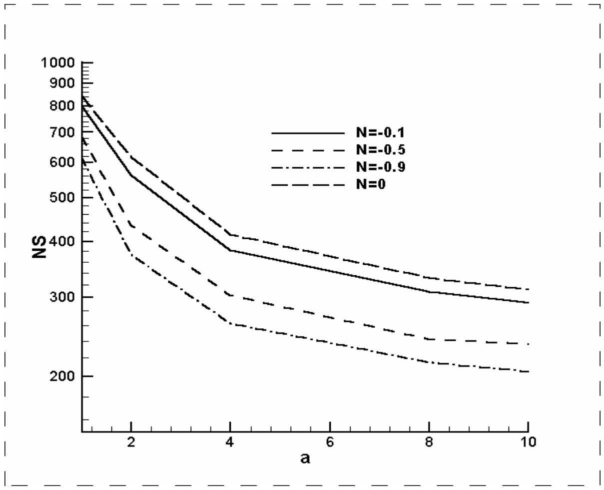

Figure 4 Dimensionless entropy generation number divided by that of constant property case versus the aspect ratio for some values of the viscosity variation number (*Pe*=1, *q*=1, and *Br*=10).

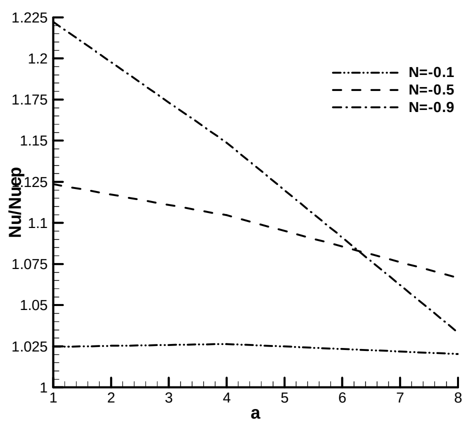

Figure 5 The Nusselt number divided by that of constant property case versus the aspect ratio for some values of the viscosity variation number.

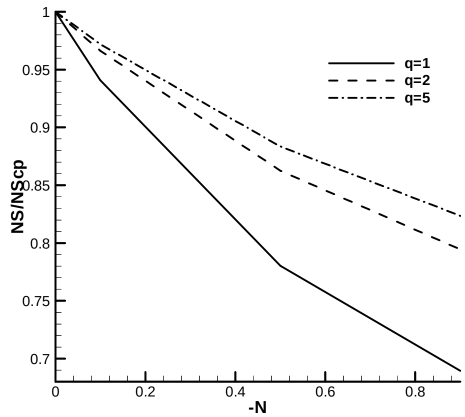

Figure 6 Dimensionless entropy generation number divided by that of constant property case versus the viscosity variation number for some values of *q*.

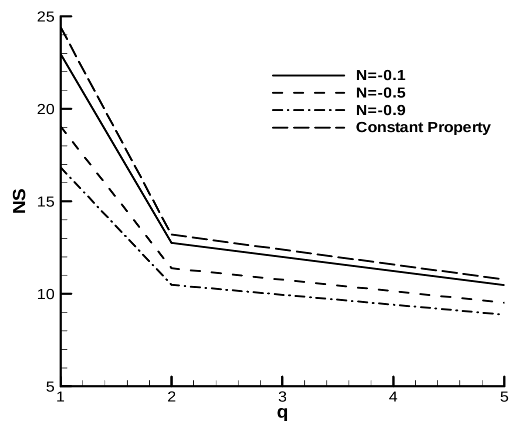

Figure 7 Dimensionless entropy generation versus the dimensionless wall heat flux *q* for some values of the viscosity variation number

Fig. 4 presents N_S versus the aspect ratio for some values of the viscosity variation number. It is concluded that the constant property case is associated with the maximum entropy generation rate compared to variable viscosity counterparts. Based on this figure one concludes that for the square cross-section N_S reaches it's maximum while moving to parallel plate channel the entropy generation decreases.

Fig. 5 shows the Nusselt number divided by the constant property counterpart, i.e. Nu/Nu_{cp} versus the aspect ratio. It is clear that increasing the *a* value, decreases the variable property effects on Nu/Nu_{cp}. It is also interesting that with $a>6$, the Nusselt ratio for N=-0.5 is higher than those of

N=-0.9 and N=-0.1. However, with smaller values of a, moving from N=-0.1 to -0.9 increases the ratio.

Another influential parameter is the dimensionless wall heat flux q. Increasing q will decrease the entropy generation but as shown in Fig. 6, when the viscosity changes with the temperature, the ratio of N_S divided by N_{Scp} increases with an increase in q. Preventing disingenuous, Fig. 7 is presented to emphasis on the fact that increasing q will decrease the entropy generation number.

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