A Nonstationary Model of Newborn EEG

Luke Rankine*, Nathan Stevenson, Mostefa Mesbah, and Boualem Boashash

***Abstract*—The detection of seizure in the newborn is a critical aspect of neurological research. Current automatic detection techniques are difficult to assess due to the problems associated with acquiring and labelling newborn electroencephalogram (EEG) data. A realistic model for newborn EEG would allow confident development, assessment and comparison of these detection techniques. This paper presents a model for newborn EEG that accounts for its self–similar and nonstationary nature. The model consists of background and seizure submodels. The newborn EEG background model is based on the short–time power spectrum with a time–varying power law. The relationship between the fractal dimension and the power law of a power spectrum is utilized for accurate estimation of the short–time power law exponent. The newborn EEG seizure model is based on a well-known time–frequency signal model. This model addresses all significant time–frequency characteristics of newborn EEG seizure which include; multiple components or harmonics, piecewise linear instantaneous frequency laws and harmonic amplitude modulation. Estimates of the parameters of both models are shown to be random and are modelled using the data from a total of 500 background epochs and 204 seizure epochs. The newborn EEG background and seizure models are validated against real newborn EEG data using the correlation coefficient. The results show that the output of the proposed models have a higher correlation with real newborn EEG than currently accepted models (a 10% and 38% improvement for background and seizure models, respectively).**

***Index Terms*—EEG, fractal dimension, modelling, neonate, nonstationary, simulation, stochastic processes, time–frequency signal processing.**

I. Introduction

The electroencephalogram (EEG) is a noninvasive tool for measuring the electrical activity in the brain. The EEG, recorded from the scalp, provides important information about the health of the central nervous system (CNS), particularly in the newborn. Abnormalities in the newborn EEG often indicate CNS disease [1]. The most notable form of EEG abnormality is seizure [2]. Seizure in the newborn is the result of an excessive discharge of neurons, caused by an imbalance between the excitatory and inhibitory processes within the brain [1]. Seizure, in general, is expressed on the EEG as a repetitive, evolving, complex, stereotyped waveform lasting a minimum of 10 s [3]. EEG seizure events may injure the fragile newborn brain [4] leading to disability or mental retardation. Therefore, a large amount of research on designing automatic detectors of newborn EEG seizure has been undertaken. The aim of these detectors is to differentiate between periods of newborn EEG seizure and newborn EEG background. In this case background is defined as EEG without any specific patterns present, [3], and is assumed to be a stochastic, $1/f$ process without any recognizable nonlinear dynamics, [5]–[7]. An accurate online automatic seizure detection system will be a valuable tool in neonatal neurology and may, indirectly, result in reducing the high mortality and morbidity rates of newborns who suffer from EEG seizure episodes [8].

A number of recent methods for the automatic detection of newborn EEG seizure have been proposed in [9]–[12]. These algorithms have been developed using signal processing techniques such as singular spectrum analysis [9], time–frequency signal processing [10], [11], and nonlinear signal processing [12]. However, the development and evaluation of these automatic newborn EEG seizure detection algorithms have used only small EEG datasets. This is due to the difficulties associated with acquiring and labelling newborn EEG data.

A realistic method for simulating newborn EEG background and seizure would allow confident development, assessment and comparison of EEG seizure detection algorithms using large signal sets.

Currently, there are two models for simulating newborn EEG background and two models for simulating newborn EEG seizure. The background model proposed by Roessgen *et al.* in [13] involved the linear parameterization of the output of the newborn brain. The Roessgen model was driven by a stationary white Gaussian input. The resultant newborn EEG background was, therefore, linear and time–invariant. This model disagrees with the findings in [14]–[16] which state that newborn EEG background exhibits nonstationary and possible nonlinear behavior. The background model proposed by Celka and Colditz in [17] is an extension of the Roessgen model, designed using a Wiener model (a linear process followed by a stationary, nonlinear shaping function) [18] to allow for the suspected nonlinear characteristics of the newborn EEG background. This Wiener model of newborn EEG background proposes a stationary (time–invariant) model, which fails to incorporate the findings of nonstationary (time–varying) behavior outlined in [14] and [15].

The first technique for modelling newborn EEG seizure was proposed by Roessgen *et al.* in [13]. This technique proposes a linear, time–invariant model driven by a stationary sawtooth waveform. The major drawback of the Roessgen seizure model was the assumption of stationarity. Recent findings in the analysis of newborn EEG have shown that newborn EEG seizures exhibit nonstationary behavior [14], [15]. Boashash

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and Mesbah in [19] proposed driving the Roessgen model with a single linear frequency modulated (LFM) signal to simulate the nonstationary behavior. However, this extension to the Roessgen model did not encapsulate all the time–varying characteristics observed in newborn EEG seizure, such as piecewise LFM components, multiple components (harmonics) [14] and harmonic amplitude modulation. The second technique for simulating newborn EEG seizure was proposed by Celka and Colditz in [17]. The authors developed another Wiener model which was driven by piecewise LFM sawtooth inputs [17]. However, this model did not make any attempt to accurately model the amplitude modulation of each harmonic: a common feature of newborn EEG seizure.

In this paper, a new model for the newborn EEG is proposed. It is composed of newborn EEG background and newborn EEG seizure submodels. Section III presents the nonstationary model for the newborn EEG background along with the estimates of the model parameters, based on 500 epochs of real newborn EEG background. An algorithm for simulating newborn EEG background is then presented. The new background model is then compared with the background Wiener model of Celka and Colditz, [17], using model outputs. Section IV details the time–frequency model for newborn EEG seizure. The model parameters are estimated using 204 epochs of real newborn EEG seizure. An algorithm for simulating EEG seizure is then presented. The new seizure model is validated and compared with the seizure Wiener model of Celka and Colditz using model outputs. Finally, the performance of the proposed newborn EEG models are discussed.

II. Data Acquisition

The EEG data were acquired at the Royal Brisbane and Women's Hospital, Brisbane, Australia, using the MEDELEC Profile System. The EEG signals used in this analysis were bandpass filtered with cutoff frequencies at 0.5 Hz and 30 Hz, and were initially sampled at 64 Hz. These specifications are standard in newborn EEG acquisition as they incorporate the spectral areas of interest in the EEG power spectrum (δ band: 0.5–4 Hz, θ band: 4–8 Hz, α band: 8–13 Hz, and β band: 13–22 Hz), [20].

A total of 12 neonates were recorded with ages ranging between 2 to 14 days and a mean age of 5.8 days. The periods of newborn EEG which exhibited seizure patterns were marked by a neurologist from the Royal Children's Hospital, Brisbane, Australia.

A total of 500 artifact free epochs of 256 samples (4 s) were selected from the database of newborn EEG background for the newborn EEG background parameter estimation. This epoch size was chosen as newborn EEG background has been reported to be quasi–stationary for periods less than 6 s [9], [14], [15]. The full spectrum is used when analysing newborn EEG background due to the assumption of a spectrum following an inverse power law.

The newborn EEG seizure data included 204 artifact free epochs of seizure. The EEG data were bandpass filtered with cutoff frequencies at 0.5 Hz and 8 Hz before resampling at 20 Hz, as the majority of spectral energy in the newborn EEG

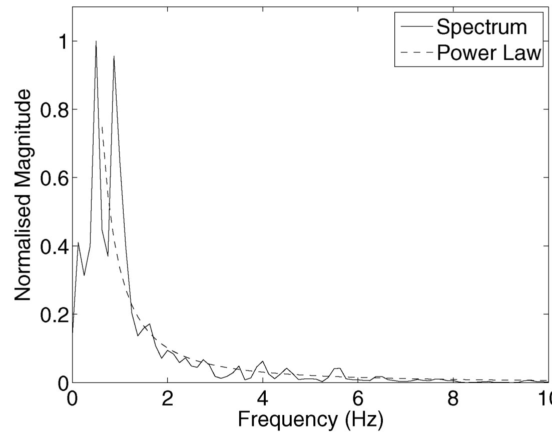

Fig. 1. Power spectrum of a newborn EEG background epoch.

(i.e., $\geq 95\%$) is concentrated in the first two frequency bands (δ and θ) [20].

III. Newborn EEG Background Model

The power spectrum is a key tool in the analysis of irregular or complex signals. The newborn EEG is an example of an irregular signal which has been analysed extensively using the power spectrum. The power spectrum of a typical 4-s newborn EEG background epoch is presented in Fig. 1.

It can be seen from Fig. 1 that the power spectrum of the newborn EEG background approximately follows a power law of the form

$$S(f) \approx \frac{c}{|f|^{\gamma}} \tag{1}$$

where c is a constant, f is frequency and γ is the power law exponent. This spectral behavior (a $1/f$ process) has also been reported in [21] and is the basis of the proposed background model.

A. Background Model Structure

Newborn EEG background has been shown to exhibit nonstationary behavior, [14], which can be modelled by a time–varying power law exponent, γ_n. Therefore, the proposed model of newborn EEG background is given as

$$S_n(f) = \frac{c}{|f|^{\gamma_n}} \tag{2}$$

where $S_n(f)$ is the power spectrum associated with nth epoch (i.e., signal block) of duration T s. This model assumes that the signal is quasi–stationary for a time period of T. That is to say, γ_n will be constant for the duration of an epoch, but will vary on an epoch by epoch basis.

The simulation of newborn EEG background from the model in (2) requires the synthesis of a time domain representation. The synthesis of a stochastic $1/f$ process, as defined by Billah and Shinozuka in [22], is described as follows.

The power spectrum $S_n(f)$ can be expressed as

$$S_n(f) = \frac{c}{|f|^{\gamma_n}} = X_n(f)X_n^*(f) \tag{3}$$

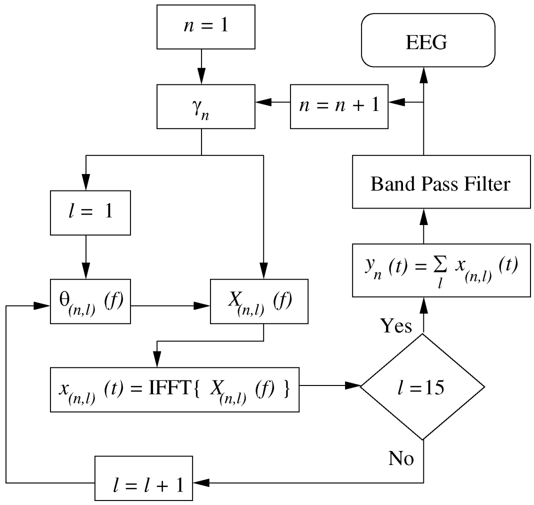

Fig. 2. Flow diagram of proposed newborn EEG background simulation.

where

$$X_n(f) = \frac{\sqrt{c}}{|f|^{\gamma_n/2}} e^{j\theta_n(f)} \tag{4}$$

is the Fourier transform of the nth epoch, $x_n(t)$, and $\theta_n(f)$ is the phase spectrum which is assumed to be a realization of a random process. Thus, the time domain signal is synthesized with a surrogate phase, [22]–[24]. Synthesis of $x_n(t)$ can then be achieved by taking the inverse Fourier transform of $X_n(f)$, expressed by

$$x_n(t) = \int_{-\infty}^{\infty} X_n(f) e^{j2\pi ft} df. \tag{5}$$

The modelled epoch, $x_n(t)$, has a power spectrum with a smooth power law. However, the power spectra of the real newborn EEG background exhibits random fluctuations around the power law. Therefore, to simulate this phenomenon, multiple subepochs with the same power law exponent, but differing random phase spectra, $\theta_{n,l}(f)$, are created. By adding these subepochs together to form one epoch, the constructive and destructive interference of the subepochs result in fluctuations around the desired power law, mimicking the power spectrum of real newborn EEG background. In the proposed simulation method, 15 subepochs were chosen to create each newborn EEG background epoch. The resultant background model becomes

$$y_n(t) = \sum_{l=1}^{15} x_{(n,l)}(t) \tag{6}$$

where $y_n(t)$ is the time domain representation of newborn EEG background for the nth epoch. The complete simulation algorithm, which includes the bandpass filtering used in data acquisition is outlined in Fig. 2.

To simulate newborn EEG background using the proposed model, estimates for the distribution of γ_n and $\theta_n(f)$ were obtained from the analysis of real newborn EEG background.

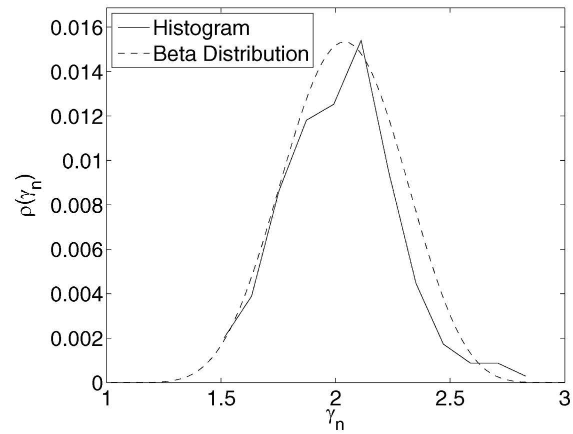

Fig. 3. The estimated pdf of the power law exponent, γ_n, of the newborn EEG background time–varying power spectrum.

B. *Background Parameter Estimation*

A measure of the complexity of a stationary signal epoch, $x_n(t)$, with a power spectrum of the form $S_n(f) \propto |f|^{-\gamma_n}$ can be given by the power law exponent, γ_n, [25]. The value of γ_n can be estimated as the negative gradient of the linear least squares fit to the log –log plot of the power spectrum [26]. However, to obtain accurate and stable estimates of the power law exponent, an ensemble average of power spectra over a long period of time is required, [27]. This method of estimating the power law exponent is not suitable for nonstationary signals such as newborn EEG background. Therefore, a more suitable technique based on fractal dimension (FD) estimation is required.

Fractal dimension is a nonlinear measure which can be used to describe the degree of complexity of irregular signals [28]. The FD has a linear relationship with the power law exponent [25] and can be expressed as

$$\mathrm{FD} = \frac{5 - \gamma_n}{2}. \tag{7}$$

A method of estimating the FD using only short time periods was developed by Higuchi in [27]. This method of FD estimation is ideally suited to nonstationary signals that can be segmented into short quasi–stationary periods like newborn EEG background. The Higuchi method of FD estimation was shown in [29] to give the most accurate estimate of FD (and, therefore, γ_n) for a wide range of $1/f$ processes.

1) Estimation of γ_n : To investigate the time–varying characteristics of γ_n, the Higuchi method of FD estimation was applied to real newborn EEG data. For fractal signals to be considered quasi–stationary the signal length must be at least twice as long as the period associated with the lowest significant frequency component (0.5 Hz) [30]. Therefore, to select a quasi–stationary period in the newborn EEG background, the EEG recordings were segmented into 4-s epochs.

Estimates of γ_n for the newborn EEG background epochs were obtained using the linear relationship between FD and γ_n given in (7). Initial analysis of the γ_n values indicated that the values were fluctuating randomly. A histogram of γ_n is shown in Fig. 3.

A Beta distribution, which can be shaped by two shaping parameters, α and β, was used to model the random fluctuations of γ_n. The probability density function (pdf) of a Beta distribution, [31], is expressed as

$$\rho(\psi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\psi^{\alpha-1}(1-\psi)^{\beta-1}, \quad \psi \in [0,1] \tag{8}$$

with a mean

$$\mu = \frac{\alpha}{\alpha+\beta}. \tag{9}$$

An estimate of the distribution of the power law exponent, γ_n, of newborn EEG background was obtained using a maximum likelihood estimate of the parameters α and β. The computed parameters for the Beta distribution were found to be $\alpha = 7.82$ and $\beta = 7.44$. The estimated Beta distribution for the power law exponent, γ_n, is also plotted in Fig. 3. The hypothesis that γ_n is a random process with the estimated Beta distribution was tested using the Kolmogorov–Smirnov test [32] and could not be rejected at the 5% significance level.

The characterization of the random behavior of γ_n results in a model for the time–varying nature of the newborn EEG background power spectrum.

2) Estimation of $\theta_n(f)$: The phase spectrum for stochastic $1/f$ processes is often assumed to be a random process with uniform distribution [22], [24]. The hypothesis of a uniform distribution was tested by analysing the Fourier transform phase spectrum of real newborn EEG background. It was found using the Kolmogorov–Smirnov test that the hypothesis of a uniform distribution for the phase spectrum could not be rejected at the 5% significance level. This result suggests that the newborn EEG background is indeed a stochastic $1/f$ process.

C. Validation of Background Simulator

The correlation coefficient, ρ, was used to validate the proposed model against real newborn EEG background. This measure of similarity was chosen because of its widespread familiarity and the fact that other distance measures were shown to provide similar results in [17]. The correlation coefficient was calculated for the time domain, frequency domain and time–frequency domain representations. The frequency domain was estimated using Welch's periodogram. The spectrogram (a two–dimensional signal representation of R rows and S columns), using a Hanning window, was used as the time–frequency representation (TFR) because of its positivity property, which results in a stable estimate of ρ.

The correlation coefficient for a one–dimensional signal representation is defined as

$$\rho = \frac{N\sum x(r)y(r) - \sum x(r)\sum y(r)}{\sqrt{N\sum x^2(r) - \left(\sum x(r)\right)^2}\sqrt{N\sum y^2(r) - \left(\sum y(r)\right)^2}} \tag{10}$$

where $x(r)$ is the original sequence, $y(r)$ is the modelled sequence of discrete length N and the summations were performed over the range $r = [1,\ldots,N]$. This formula cannot be directly applied to a two–dimensional signal representation. The calculation of the correlation coefficient of a TFR was performed by converting the $R \times S$ matrix to a vector of length $N = RS$ by concatenating matrix rows.

TABLE I
RESULTS OF THE VALIDATION OF THE NEWBORN EEG BACKGROUND MODEL

ρ	Proposed	Wiener
time	0.795 (0.081)	0.737 (0.113)
frequency	0.716 (0.131)	0.665 (0.134)
time–frequency	0.817 (0.113)	0.742 (0.158)

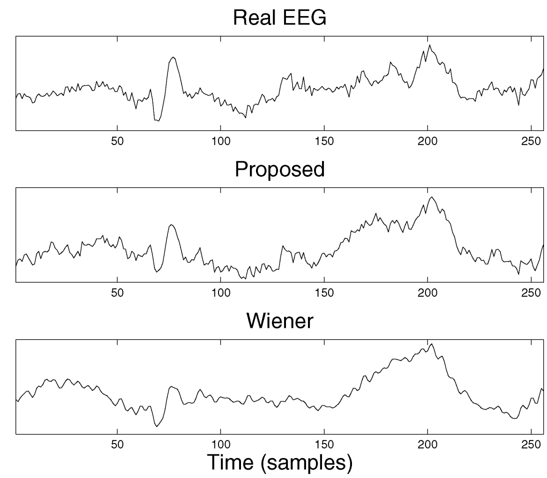

Fig. 4. Comparison between the real and simulated newborn EEG background in the time domain.

The proposed model was also compared to the Wiener background model of Celka and Colditz [17]. This model was chosen as it was the most realistic, currently accepted, model.

In the proposed background model, the time–varying power law exponent γ_n, and the phase spectra $\theta_n(f)$, were modelled as stationary random processes. Therefore, to compare the similarity of the simulated epochs with a real newborn EEG background epoch, the values γ_n and $\theta_n(f)$ obtained from the real EEG epoch were provided to the models under test and are assumed to be valid realizations of the random process.

The model validation was performed on 150 real newborn EEG background epochs. The correlation coefficient for the proposed model and the Wiener model are given in Table I, and the results are of the form, mean (standard deviation).

The results in Table I show that the proposed model provides a higher average correlation with the real newborn EEG background, and a lower standard deviation, compared to the Wiener model in all three domains. The improvement in the time–frequency correlation was 10%. The time–frequency domain was chosen for summarizing the improvement exhibited by the proposed model due to the newborn EEG background signal's nonstationarity.

The improvement of the proposed model can be attributed to the incorporation of the time–varying power law, γ_n, in the proposed simulator which models the nonstationarity of the newborn EEG background. This nonstationary behavior is neglected in the Wiener model. An example output of the proposed simulator is shown in Fig. 4. Visual inspection of the time domain representations of the real and simulated EEG indicate that the

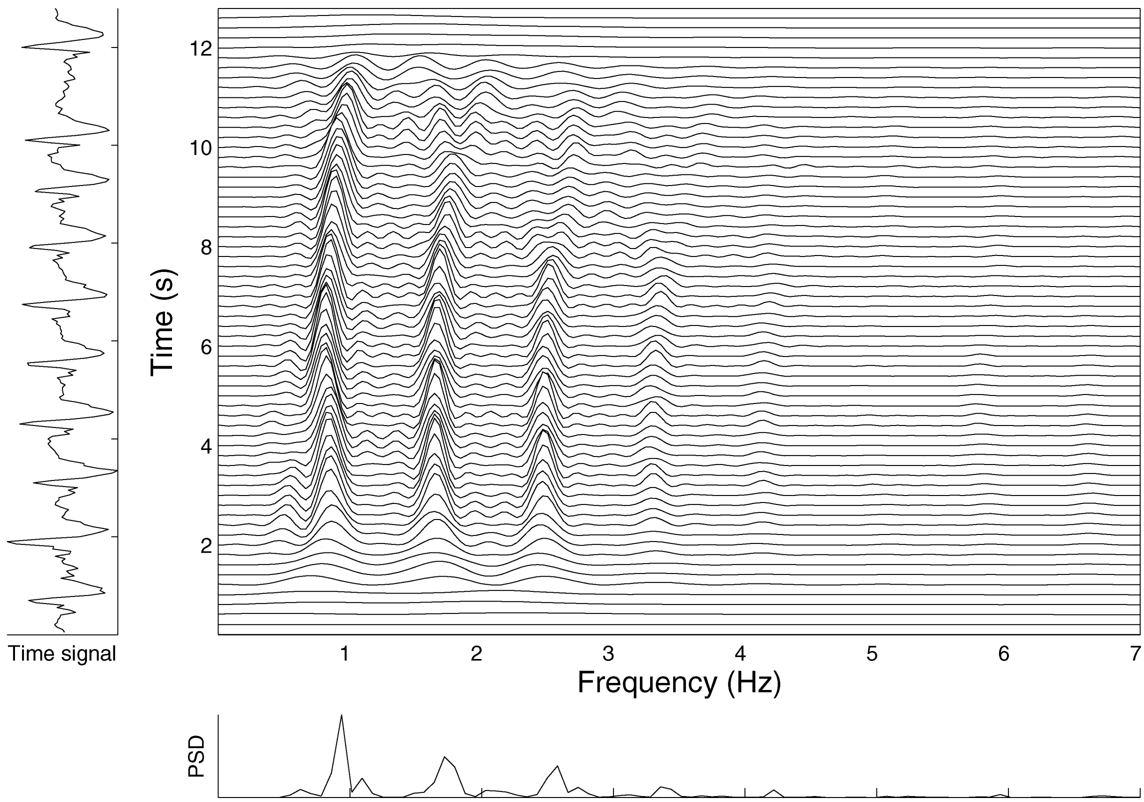

Fig. 5. TFR of a newborn EEG seizure epoch.

proposed simulator is more capable of accurately describing the complex patterns seen in newborn EEG background.

IV. Newborn EEG Seizure Model

Analysis of the newborn EEG seizure undertaken by Boashash and Mesbah in [14] and [33], using quadratic time–frequency distributions revealed that the spectral content of the newborn EEG seizure was time–varying. It was observed that newborn EEG seizure patterns can be characterized in the time–frequency domain by a main ridge (component) which follows a LFM or piecewise LFM law [33, pp. 665]. A characteristic that was further confirmed by Celka and Colditz in [17].

In addition, the TFR of newborn EEG seizure such as the one seen in Fig. 5 indicates that some newborn EEG seizures consist of multiple harmonics with time–varying amplitudes (i.e., amplitude modulation). The amplitude modulated, piecewise LFM, and multiple harmonic characteristic of newborn EEG seizure are the basis for the proposed newborn EEG seizure model.

A. Seizure Model Structure

A number of common time–frequency signal models are defined in [33, pp. 12]. One such signal model for real, nonstationary signals with multiple components is given as

$$s(t) = \sum_{k=1}^{K} a_k(t) \cos\left(2\pi \int_0^t f_k(\tau) d\tau + \theta_k\right) \tag{11}$$

where $a_k(t)$, $f_k(\tau)$, θ_k are the amplitude modulation, time–varying instantaneous frequency (IF) function, and initial phase for the kth signal component, respectively. This time–frequency signal model is used for the simulation of newborn EEG seizure.

It can be seen from (11) that estimates for the functions $a_k(t)$, $f_k(\tau)$, the phase parameter, θ_k, and the number of harmonics, K, are required for the simulation of newborn EEG seizure.

The IF function, $f_k(t)$, is modelled as a piecewise linear function based on the previous findings of [17]. The general form of a piecewise LFM function, $f(t)$, with M pieces is given by

$$f(t) = \sum_{m=1}^{M} F_m(\xi_m, C_m; t) \operatorname{rect}\left(\frac{t - 0.5(B_{m+1} - B_m)}{B_{m+1} - B_m}\right) \tag{12}$$

where

$$F_m(\xi_m, C_m; t) = \xi_m t + C_m \tag{13}$$

and

$$C_m = \begin{cases} f_{st} : & m = 1 \\ F_{m-1}(\xi_{m-1}, C_{m-1}; B_m) - \xi_m B_m : & m \geq 2 \end{cases}. \tag{14}$$

The start frequency of the LFM is given by f_{st}, $\xi = [\xi_1, \xi_2, \ldots, \xi_M]$ are the gradients in Hz/sec, $B = [B_1 = 0, B_2, B_3, \ldots, B_M, B_{M+1} = N]$ are the turning points in seconds, N is the discrete length of the seizure and C_m is the alignment intercept that ensures $f(t)$ is continuous. The multiple harmonics of the newborn EEG seizure are related to the fundamental (i.e., the component with the lowest frequency content represented by $f_1(t)$) [34]. Therefore, the IF for each harmonic can be derived from the fundamental, shown as

$$f_k(t) = kf_1(t). \tag{15}$$

The amplitude modulation function of each harmonic can be parameterized by the gain factor R_k, normalized variation, V_n, and number of turning points, P, and is given by,

$$a_k(t) = \Phi(R_k, V_n, P; t). \tag{16}$$

The gain factor, R_k, is referred to as the harmonic ratio (i.e., the ratio between the average amplitude of the harmonic and the fundamental). This infers that $R_1 = 1$.

The component amplitude modulation function is determined from a cubic spline interpolation of P randomly assigned turning points with amplitudes given by

$$a_k(q) = R_k(0.67 + V_n) \tag{17}$$

where the mean of $V_n = 0.33$. The locations of the turning points are found according to

$$q = \frac{N(p + X)}{P} \tag{18}$$

where $p = [0, \ldots, P-1]$ and X is a stationary random process, uniformly distributed between 0 and 1. The boundary conditions of the cubic spline fit are set to have a derivative of zero.

In summary, the newborn EEG seizure model can be parameterized with the following parameter vector

$$[K, R_k, V_n, P, M, \xi, B, f_{st}, \theta_k].$$

An outline of the newborn EEG seizure simulation algorithm is given in Fig. 6.

The complexity of this model was reduced by setting the discrete epoch length to 12.8 s (i.e., 256 samples), setting the number of harmonics to 5 (i.e., $K = 5$), setting the number of pieces in $f_k(t)$ to 3, (i.e., $M = 3$), assuming that B was a stationary, uniformly distributed random process ranging across the epoch, and limiting the number of turning points in $a_k(t)$ to a maximum of 8 (i.e., $P \in [1, 8]$). These assumptions were based on the analysis of newborn EEG and the literature.

Limitation of the harmonic number was based on the analysis of the newborn EEG seizure database which suggests that any higher harmonics contribute little signal energy. This fact can be observed by using the data acquisition sampling limitations and waveform prototype outlined in [13] and [17], (i.e., the sawtooth waveform). The bandpass filtering operation on the newborn EEG data limits the possible number of harmonics to 16. The energy in the first 5 harmonics of a sawtooth waveform correspond to 90% of the total signal energy.

The numbers of pieces used to simulate the LFM law was based on the results of [17]. It was demonstrated in [17] that a 3-element piecewise linear function could characterize the IF law of the signal components of a seizure with a duration of 20 s (or less).

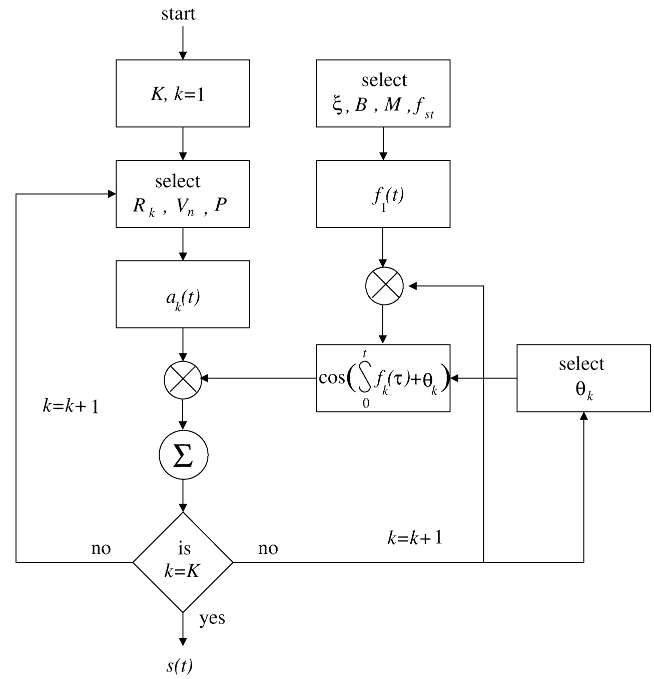

Fig. 6. Flow diagram of proposed newborn EEG seizure simulator.

TABLE II
DISTRIBUTION ESTIMATES FOR THE NEWBORN EEG SEIZURE MODEL PARAMETERS (A = accept, AND R = reject). N IS THE SAMPLE LENGTH, B IS A CONTINUOUS BETA DISTRIBUTION, B* IS A DISCRETE BETA DISTRIBUTION AND L-N IS A LOG NORMAL DISTRIBUTION

	P	R_2	R_3	R_4	R_5	V_n	ξ	f_{st}
Dist	B*	B	B	B	B	B	B	L-N
α	1.8	1.7	1.5	1.9	1.4	3.9	69.1	-0.17
β	3.0	3.2	4.1	3.6	1.2	8.0	69.8	0.55
min	1	0.2	0.2	0.2	0.2	0	-0.06	0.425
max	8	1.2	1.0	0.6	0.4	1	0.06	∞
H_0	A	A	A	A	A	A	A	A
N	204	190	90	25	3	204	204	204

The oscillation of the amplitude modulation is much slower than the IF of a seizure component using an analytic signal definition, [33, pp. 13]. This is due to Bedrosian's theorem [35] which states that the amplitude modulation of a signal component must oscillate significantly slower than the IF of the component or it will be represented as low-frequency signal energy in the time–frequency domain (i.e., a separate component). Therefore, only a relatively small number of turning points, $P \in [1, 8]$, will occur in a short seizure epoch (see Table II).

B. Seizure Parameter Estimation

To estimate the ranges and distributions of the parameters of the newborn EEG seizure signal model, real newborn EEG data were analyzed in the time–frequency domain using the spectrogram [33, pp. 38]. A total of 204, 12.8-s epochs (256 samples) were analyzed. This epoch length was chosen as it met the assumptions for a 3-element piecewise IF function and was approximately equivalent to the minimum duration of ictal discharge for a seizure event to be registered (i.e., 10 s), outlined in [2].

Estimates for all newborn EEG seizure model parameters were extracted from the TFR of each epoch. The piecewise LFM law was extracted from the TFR using a peak IF estimation algorithm with a piecewise function fit using the Levenberg-Marquardt method, [36]. The piecewise function was defined according to (12). The extracted IF law was then used to divide the TFR into harmonic sections which permitted the estimation of the harmonic amplitudes and number of turning points in the amplitude of the harmonic.

Table II shows estimates of the distribution of newborn EEG seizure model parameters resulting from the time–frequency analysis. It can be seen from Table II that each of the parameters are modelled as stationary random variables. The defined pdfs for the model parameters have been estimated with Beta (B) and Log Normal (L-N) distributions. The shaping parameters, $[\alpha, \beta]$, for the Beta distributions and the mean, α, and variance, β, of the Log Normal distribution were obtained using a maximum likelihood estimate. The range of values for each parameter, which bounds the distribution, is provided. The use of the distributions to model the data were tested with a Kolmogorov-Smirnov test at a 5% level of significance. The null hypothesis H_0 was that the data were modelled by the estimated distribution. Finally, the initial phase value, $\theta_k(f)$, was assumed to be a random variable with a stationary uniform distribution on $[-\pi, \pi)$.

C. Validation of Seizure Simulator

The correlation coefficient, ρ was used to validate the output of the newborn EEG seizure simulator. The results are compared with the output of the Wiener seizure model of Celka and Colditz, [17], which is the most realistic newborn EEG seizure model in the current literature.

The parameters for the proposed seizure model and the Wiener model were estimated from a set of seizure epochs to compare the newborn EEG seizure models. It is noted that the extracted parameters were assumed to be valid realizations from the random distributions governing the parameters of the models (i.e., the values fall with the limits of the random variable).

A total of 52 newborn EEG seizure epochs were used in the comparison. The model outputs were compared with the real newborn EEG epoch in the time domain, frequency domain and time–frequency domain. The frequency domain representation was estimated using Welch's periodogram. The time–frequency domain representation chosen for the comparison was the spectrogram with a Hanning window.

The average value and standard deviation of the correlation coefficient, over the 52 test epochs, for the time domain, frequency domain and time–frequency domain are presented in Table III.

The results presented in Table III indicate that the proposed newborn EEG seizure model provides more realistic newborn EEG seizure signals than the Wiener model, with an improvement in the average time–frequency correlation coefficient of 38%. The improvement of the proposed model is derived from more accurate harmonic modelling which, in turn, contributes to the complexity seen in newborn EEG seizure.

TABLE III
THE RESULTS OF THE VALIDATION OF THE NEWBORN EEG SEIZURE MODEL

ρ	Proposed	Wiener
time	0.345 (0.176)	0.005 (0.251)
frequency	0.799 (0.093)	0.684 (0.147)
time–frequency	0.901 (0.056)	0.654 (0.180)

The average time domain correlation coefficient in Table III for the proposed model and the Wiener model are much lower than in the frequency and time–frequency domain. The reduction is due to the fact that a slight phase offset in the time domain can result in significantly lower correlation, or negative correlation, even if the repetitive patterns are identical. The frequency and time–frequency domain representations are representations of the magnitude of the signal in these domains. Therefore, phase alignment is not an issue and negative correlation cannot occur.

Fig. 7 shows two newborn EEG seizure epochs used in the validation process. It can be observed that the proposed model can more accurately reproduce the complex repetitive patterns observed in newborn EEG seizure.

V. DISCUSSION

Short–time power spectrum analysis of the newborn EEG background indicated that the power spectrum is governed by a power law which is time–varying. This characteristic is the basis for the newborn EEG background model. It was demonstrated that the proposed newborn EEG background model can produce more realistic newborn EEG background signals than the Wiener model. The improved performance of the proposed newborn EEG background model was attributed to the modelling of the nonstationary characteristic of the background signal.

An issue which was not fully addressed in the proposed background model is that of long term amplitude/energy variations. Amplitude integrated EEG, routinely used in clinical analysis of newborn EEG, indicates that there are long term changes in the average amplitude or energy of the signal. In addition, the process of concatenating EEG epochs to generate a continuous background trace must be addressed further as there may be some underlying determinism, with respect to time, in the phase and magnitude responses. Furthermore, other techniques for simulating stochastic $1/f$ processes [24], which can cope with smoother transitions in phase and magnitude responses, may permit more realistic nonstationary modelling of the newborn EEG background.

Newborn EEG seizure has been shown to exhibit nonstationary, amplitude modulated, harmonic behavior. Although not explicitly stated in [17], by using a piecewise LFM sawtooth as an input to the autoregressive (AR) seizure model, the authors have attempted to model the harmonic nature of seizure. A piecewise LFM sawtooth will create constant amplitude harmonics in the time–frequency domain. However, it can be seen in Table II that the harmonic amplitude can vary significantly. In addition, the AR model for seizure acts as a bandpass filter

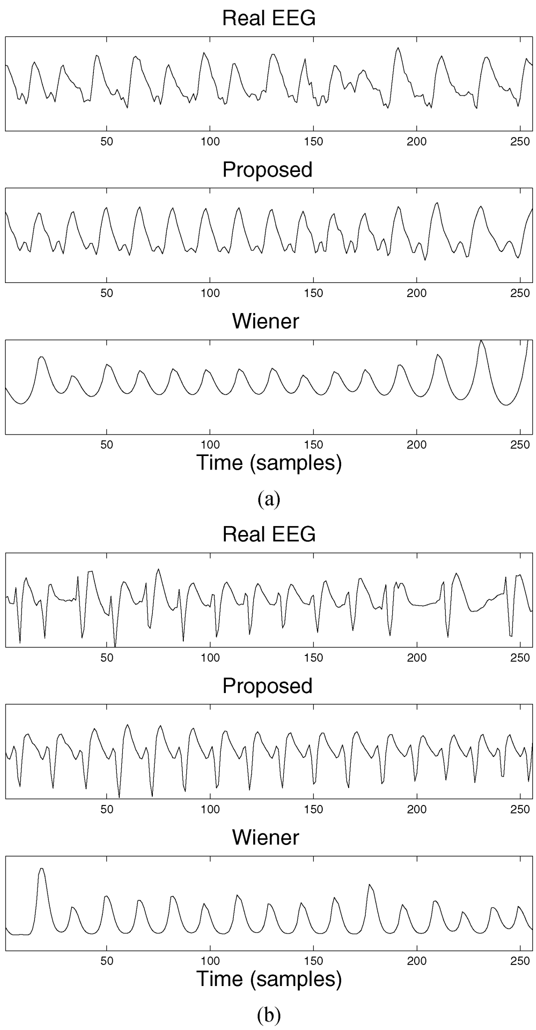

Fig. 7. A comparison between the real and simulated newborn EEG seizure in the time domain.

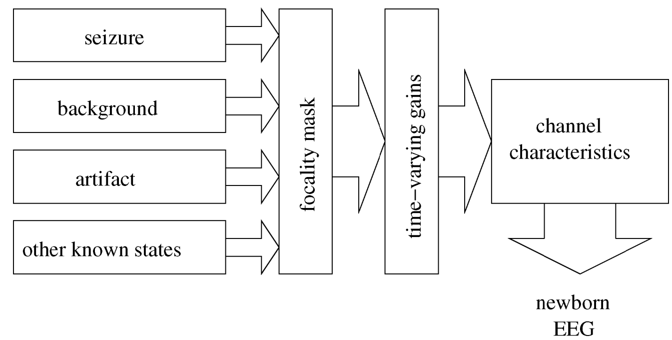

Fig. 8. A full model of newborn EEG.

which attenuates the harmonics of the piecewise LFM sawtooth input.

The proposed newborn EEG seizure model addresses the nonstationary characteristics observed in newborn EEG seizure. That is, the model incorporates the nonstationarity of individual components, the amplitude modulation of individual components, the harmonic ratio between components, and the number of signal components. As well as providing a model which encapsulates the time–frequency characteristics of the newborn EEG seizure, estimates for the range and distribution of the model parameters have been provided. This is essential for the simulation of newborn EEG using the proposed model.

The newborn EEG seizure model assumes an epoch length of 256 samples and a sampling rate of 20 Hz. The knowledge of these values permits a reduction in model complexity due to the enforcement of limits on various parameters. The newborn EEG seizure model can easily be extended to longer epochs and alternate data acquisition specifications by selecting an extended range for K, M, and P. In addition, further analysis of the phase relationship between each harmonic of a newborn EEG seizure may produce more limited, or conditioned, representations of the phase rather than an entirely random phase model. This will, potentially, remove any spurious seizure epochs that have waveforms that are not, or cannot be generated by the newborn brain.

A comparison between the proposed seizure model and the Wiener seizure model has indicated improved performance. The main improvement in the proposed newborn EEG seizure model is that it generates seizure signals with better correlation in the time domain than other models. This is due to the detailed modelling of the newborn EEG seizure harmonics which result in a model that can generate various complex nonstationary waveforms that appear more realistic in the time domain.

The modelling of newborn EEG gives insight into the fundamental signal characteristics of background and seizure. These insights can assist the design of future detection regimes. However, more research must be performed regarding the combination of seizure and background (duration, relative magnitudes), the possibility of a pre–ictal state, other known newborn EEG patterns (delta brushes, tracé alternant, burst suppression, spikes and sharp waves, [3]), artifacts, and the application to a multi–channel environment. It is envisioned that a newborn EEG model such as that outlined in Fig. 8 will result.

VI. Conclusion

The time–frequency characteristics of the newborn EEG background and seizure differ significantly. This has lead to the development of two separate models for simulating the newborn EEG background state and newborn EEG seizure events.

The two models exhibit greater correlation with real newborn EEG data than current techniques (an improvement of 10% for the background model and 38% for the seizure model). These improvements are obtained by accounting for the nonstationary behavior of newborn EEG seizure and background.

In addition, a full analysis of an existing newborn EEG database has allowed the estimation of the range and distribution of all the parameters in the two models.

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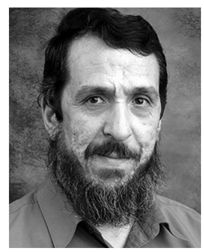

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