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# Comparative Distribution of System Losses to Market Participants using Different Loss Allocation Methods

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***Abstract*— A key part of electricity pricing is the fair and equitable allocation of system losses. This paper critically compares several existing loss allocation methods. The methods addressed include existing approaches such as *pro rata* method, proportional sharing method [1], loss formula [2], and incremental method [3], in addition to a new method proposed by the authors, which allocates losses from a loop-based representation of system behaviour. The distinct numerical allocation of losses in both the IEEE 14-bus network and a modified Nordic 41 bus system is listed for comparison. The similarity between the different loss allocations methods varies considerably, depending upon the system to which the methods are applied. This is primarily a result of the manner in which the different allocation methods address the impact of network structure. Further work is still required to determine which method encourages better system operation.**

***Index Terms*— Loss allocation, power flow tracing, transmission flow distribution**

## I. INTRODUCTION

THE pricing of electricity has always been a major concern to system participants, even before the introduction of deregulation. The previous monopolistic structure used a simple pricing scheme based on a uniform distribution of the approximated loss of 2% to 5% of generated power. This simple loss allocation, however, is not sufficient for the restructured electricity market as it does not encourage competition between market participants. Given that healthy competition should encourage lower prices, it is important to develop an electricity-pricing scheme that promotes competition.

To promote fair competition, market participants must be charged in a way that reflects their use of the system. A critical part of this is distribution of system losses to the market participant. Presently, some electricity markets such as in mainland Spain and Brazil have adopted a *pro rata* approach to loss sharing [4], while other markets such as in Australia [5] and New Zealand [6] have adopted the incremental method. Yet these present methods are not felt to be completely satisfactory leading some markets, such as Brazil, to consider implementing alternative approaches [7].

The main difficulty presented when selecting a loss allocation method is the absence of a standard means for comparing the different methods. In the absence of an electrically justifiable means of tracing power flows, one must instead assess whether the different loss allocations schemes are “fair and equitable” [2]. This involves considerations such as is the allocation method:

- simple to understand and implement;
- consistent with power flow solution;
- able to promote efficient market operation, where the losses is reflected by network usage and the relative position of the bus in the network; and
- consistent with electrical laws?

In the absence of a standard method of comparison, it is still difficult to assess the suitability of the different methods.

The objective of this paper is to critically analyze several existing methods based on the characteristics listed above, thus assessing the suitability of each method. The methods compared include: the *pro rata* method; the proportional sharing method [1]; a loss formula approach [2], and an incremental method as implement by Chowdhury et al. [3]. In considering the computational process involved in each method and weighing the results against the desired characteristics of a “fair and equitable” allocation, we will be able to comment on the type of market structure and operation promoted by each method.

An extension of the analysis includes a comparison of these established loss allocation approaches to a new method proposed by the authors [8]. In contrast to existing methods using a nodal representation of system conditions, the new approach characterizes network behaviour in terms of power flows through loops. It is believed that this makes it easier to visualise the flow of power between market participants.

The remainder of the paper is arranged as follows. Section II introduces the existing loss allocation methods and summarises their mathematical formulation. Section III introduces the loop based loss allocation method proposed by the authors. Finally, Section IV lists the results produced by the application of both the established and new methods to two distinct test systems, leading to some comments on the comparative behaviour of the different approaches.

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## II. EXISTING LOSS ALLOCATION METHODS

Most existing loss allocations methods are based broadly on either the *pro rata*, proportional sharing, incremental, circuit theory or loss formula methods. The basic mathematical formulation of each of these approaches is outlined as follows.

### A. Pro rata allocation

The *pro rata* allocation [4] method is the simplest loss allocation method. It assigns losses based on a comparison of the level of power or current injected/consumed by a specific generator or load to the total power generated or delivered in the system. Starting from a solved load flow solution, losses are systematically distributed based on the real power injected or consumed at each node, as shown in (1) and (2).

$$L_{Gi} = \frac{P_{loss}}{x}\frac{P_{Gi}}{P_G} \tag{1}$$

$$L_{Dj} = \frac{P_{loss}}{x}\frac{P_{Dj}}{P_D} \tag{2}$$

Together equations (1) and (2) represent the *pro rata* allocation of losses to the generator at bus *i* and load at bus *j*. $P_G$ is total real power generated in the system while $P_{Gi}$ is the total MW output of the generators at bus *i*. Alternatively, $P_D$ is total real power consumed and $P_{Dj}$ is the real power consumed by loads of bus *j*. $P_{loss}$ is system transmission power losses. The multiplying factor *x* can be used to weight the distribution of system losses towards either of the market participants.

It is clear from (1) and (2) that this method is totally reliant on the power injections at buses and independent of the network topology. Losses are distributed across all buses, according to their level of generation or consumption only. Two loads in different locations but with identical demands will be allocated the same level of loss, irrespective of their comparative proximity to system generation. No incentive is provided for placing generation closer to load centres, a practice which usually leads to reduced system losses. In addition, the pro rata method is also unable to trace power flows, making it difficult to justify the different allocations.

### B. Incremental allocation

The incremental allocation [4] of loss sharing is based on economic concepts and addresses how a slight change in power injections at a single bus affect system losses. The transmission system is viewed as a black box with injection points connected to it. Loss coefficients are calculated based on the change in loss due to a change in a bus injection. Losses are allocated to market participants using the loss coefficients. The losses allocated represent the losses incurred in the system when additional power is injected into the system to supply a slight change in power at that bus.

An incremental method, as described by Chowdhury [3], was implemented. It is a simple method that shows the fundamental features of the incremental method. The essence of the method is based on (3), where $P_{loss}$ is the system transmission power losses, and $P_i$ is the power injection at a particular load.

$$P_{loss} = \sum_{i=1}^{n}\frac{\partial P_{loss}}{\partial P_i} \tag{3}$$

Individual loads are incremented sequentially from zero to full load. The change in losses was determined using a series of load flow calculations rather than solving (3) directly. At each step, losses obtained are allocated to the corresponding load (and generator if contracts are specified).

The main limitation of this method is that losses are highly dependent on the incremental steps taken. It is expected then that a loss allocation would be non-unique. Furthermore, the method is also highly dependent on the choice of slack bus. Although these problems have been addressed by some researchers [8, 9], the correction techniques appear to introduce a degree of arbitrariness into the loss distribution. Finally, the method focuses on system losses produced by change in power at a node, but does consider the transmission path taken to supply any load. Together, these limitations mean that alternative loss allocation techniques are required.

### C. Proportional sharing allocation

The proportional sharing method introduced by Bialek [1] represents a fundamental shift in the process of loss allocation. Bialek introduced a topological tracing method, treating each node as an ideal mixer, such that power flowing out of a node can be considered the proportional sum of the power flowing into the node. This allows the demands of load to be traced "up" to the generators or the output of the generator to be traced "down" to the loads.

To understand the allocation method, consider the tracing of power upstream from the loads to the generating sources. Starting from a solved load flow solution, the power balance equation at node *i* considering the power inflows from "upstream" is defined by (4).

$$P_i^g = \sum_{j\in\alpha_i^u}\left|P_{ij}^g\right| + P_{Gi} \quad \text{for } I = 1,2,\ldots,n \tag{4}$$

$P_i^g$ is the unknown gross nodal power flow through node *i*, $P_{ij}^g$ is the unknown gross line flow in line *i*-*j*, $\alpha_i^u$ is the set of nodes supplying node *i*, and $P_{Gi}$ is the power generation in node *i*. The line flows $P_{ij}^g$ also can be expressed as a proportion of the flows into the upstream node *j*. By continuing this process, the contribution of system's generators to the *i*-th gross nodal power can be expressed according to (5).

$$P_i^g = \sum_{k=1}^{n}\left[A_u^{-1}\right]_{ik}P_{Gk} \quad \text{for } i = 1, 2, \ldots, n \tag{5}$$

$$\left[A_u^{-1}\right]_{ij} = \begin{cases} 1 & i = j \\ -\dfrac{\left|P_{ij}\right|}{P_j} & j \in \alpha_i^u \\ 0 & otherwise \end{cases}$$

$A_u$ is the upstream distribution matrix and $P_{Gk}$ is the generation at node *k*. In these cases, the gross nodal and line flows refer to those power flows in a lossless system. The difference between the gross and actual demand gives the loss allocated to a load.

Unlike the previous two methods, the proportional sharing method is capable of approximating the contribution of each generator to each load through tracing the flow of power. The assignment of losses to either generators or loads should encourage the market participants to take corrective actions that will reduce their share of losses. The problem with this approach, however, is that the distribution of power flows is built on the proportional sharing principle, which lacks physical and economical justification. This departure from electrical behaviour of the network may mean that proposed strategies to reduce losses may not be technically satisfactory. Additional work has been completed to improve the allocation procedure, including formalisation of the search algorithm through application of graph theory [10] as well as corroborating the principle with game theory [11]. The lack of justifiable correlation between the network's electrical behaviour and the flows tracing established using proportional sharing is still a limitation.

### *D. Loss formula allocation*

The distribution of losses using loss formula constitutes a wide range of different implementations of full and accurate calculation and distribution of electrical losses. Different implementations include the Z-bus method [2], B-loss coefficients and the representation of losses as a quadratic function of the transactions occurring within the network [12]. Given the direct correspondence of the loss formula developed using the Z-bus approach to the equations describing normal system behaviour; the following section will focus on this method as a representative example.

The Z-bus loss allocation method is based on expressing total system losses in simple manner related directly to the equations describing a solved load flow condition. Providing all generators and loads are represented as current injections into the system, total losses can be expressed according to:

$$P_{loss} = \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} Z_{ij} I_j\right)\right\} \tag{6}$$

This can be re-written in a more useful form as:

$$P_{loss} = \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} R_{ij} I_j\right)\right\} + \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} X_{ij} I_j\right)\right\} \tag{7}$$

In a network that can be represented by a symmetrical impedance matrix, the second component in (7) sum to zero. Thus total system losses can be expressed as:

$$L_i = \Re\left\{ I_i^* \left(\sum_{j=1}^{n} R_{ij} I_j\right)\right\} \tag{8}$$

or more succinctly as merely:

$$P_{loss} = \sum_{i=1}^{n} L_i \tag{9}$$

It is apparent from (8) and (9) that total system losses are now distributed to all buses in the system. This distribution is dependent upon both the size of the current injection at the bus and also the position of the bus within the network. The losses are technically justifiable and the loss formula can be used by individual market participants to adjust their operational strategies to reduce their allocated loss. In addition, as the formula also shows how losses relate to network topology, it might be possible to identify system conditions that could be adjusted to improve overall network behaviour. The focus on distribution of losses to buses, however, is at the expense of information tracing the contribution of generator to loads. Consequently, although the losses can be distributed accurately, it is not possible to know which specific power transactions are contributing most to system losses.

The previous sections have highlighted the main features of the different loss allocations methods. It is clear that each method provides different information about network operation and will encourage different forms of network operation. This suggests that it will be important not to pick the best loss allocation method, but merely one that best suits the different market structures.

A common problem with the loss distribution approach presented though is the continued absence of a technically justifiable method for tracing power flows. The lack of such an approach makes it difficult to evaluate the technical viability of bilateral contracts used in some markets. This has led the authors to pursue a new loss allocation approach based on loop flows, which will be introduced in the following section.

## III. LOOP-BASED LOSS ALLOCATION METHOD

Deviating from the conventional method of loss allocation, which commonly use a nodal based system representation, the authors have proposed a new tracing method based on loop frame of reference [8]. This method has been proposed to assess the viability of financial contracts between market participants. Power flows within the network are represented by sum of power flows around loops linking loads to active sources, which can be assigned to represent a contract path. The proposed method has the benefit of tracing load consumptions back to their originating active sources based on these assumed loops. This makes it easier to visualize and justify the allocation of losses between market participants.

A critical limitation of the loop based representation of network behaviour is the existence of multiple valid loop assignments. A formalised process of loop identification based on graph theory has been developed to address this. Starting from a 'rooted tree' that includes all active sources in the system, the "Building-up Method" [13] is used to identify a tree such that all loads will be contained within loops having at least one active source. This ensures that losses resulted from the power delivery around the loops can be readily and justifiably distributed to these active sources. Two formal search strategies, commonly applied to graph theory applications, including the Breadth First Search (BFS) and Depth First Search (DFS) can be used in the loop identification process, depending upon the desired properties of the resulting distribution. In any case, loop identification process is explained in greater detail in [8].

Once the loops used to describe the system have been assigned then system behaviour provided by a solved load flow can be formulated in the loop frame of reference. For a network with *n* nodes, *e* elements and *l* loops, a loop connection matrix, *C*, is first formed after loops are assigned. The loop connection matrix describes the structure of each loop. It is used to calculate the loop impedance matrix, $Z_{loop}$, as shown in (10), where [z] is the self-impedance matrix.

$$\boldsymbol{Z_{loop} = C^t\, [z]\, C} \tag{10}$$

The loop impedance matrix is necessary for calculating the currents flowing in each loop, $I_{loop.}$ This parameter can be determined from (11), where $E_{loop}$ is the total voltage driving current around each loop.

$$\boldsymbol{E_{loop} = Z_{loop}\, I_{loop}} \tag{11}$$

The currents flowing in each loop can be used to determine the power transfer within the loop. The real power flow around a loop can be determined by (12a) and (12b). Consider a loop containing a generator at bus *x* and a load at bus *y*. $V_x$ and $V_y$ are the voltages at the terminals of the generator and load as determined from the load flow solution. $I_{loop,xy}$ is the loop current flowing from bus *x* to bus *y*. Consequently Equation (12a) represents the real power loop flow flowing from the generator to the load, while (12b) represents real power flow delivered to the load at the end of the loop.

$$P_{loop,xy} = \Re(V_x\, I_{loop,xy}{}^*) \tag{12a}$$
$$P_{loop,yx} = \Re(V_y\, I_{loop,xy}{}^*) \tag{12b}$$

These equations are very important. For loops containing active elements they indicate an assumed transfer of power from a generator to a load in the presence of all other power flows in the system. This implies that, even though it may not be possible to totally separate the influence of a specific power transfer from the behaviour of the whole system, its effect can be visualised with the loop representation.

Losses then can be calculated from the information available from flow tracing. Calculation of each loop loss is based on the difference of real power flow at the originating bus, *x*, and ending bus, *y*, as indicated in (13).

$$P_{loop\ loss,xy} = P_{loop,xy} - P_{loop,yx} \tag{13}$$

It is then possible to allocate the losses involved in this presumed transfer to the relevant generator. This is the main benefit of the proposed flow tracing approach.

## IV. RESULTS AND DISCUSSIONS

The following results represent the distribution of losses in the IEEE 14-bus test system [14] and a form of the modified CIGRE Nordic 41 bus system [16] using the technique outlined in the preceding sections. The IEEE 14-bus system contains two generating sources, 14 loads and synchronous condensers and 12 shunt elements representing line capacitance and off-nominal transformers. It has been used regularly in other works to confirm the effectiveness of different loss allocation procedures. In contrast is 41 bus test system, based on the CIGRE Nordic 32 bus test system. The system tested contains 20 generators, 22 loads, and 52 line and transformers and 51 shunt elements representing line capacitance and off-nominal transformers. The major difference to the IEEE 14-bus system is that the Nordic system has widely distributed generating sources and loads, making system topology more important.

### *A. Results from IEEE 14-bus system*

The real power losses obtained from the different methods that the authors have implemented are listed in Table I. The *pro rata* method was carried out twice to show the losses allocated to only loads (*Pro rata* 100% to loads) and equal distribution of losses between generators and loads (*Pro rata* 50:50 gens:loads). The incremental method (IM) has been carried out assuming that all losses are the loads only. The fifth column shows the results obtained from the proportional sharing method (PS) where losses are allocated to loads. This is followed by results obtained from Z-bus method (Z-bus) which allocates losses to every bus in the system. Column six shows the results of the loop based method with losses allocated to generators after loops were identified using a Depth First Search strategy.

TABLE I
IEEE 14-BUS REAL POWER LOSS ALLOCATIONS

| **Bus no.** | **Real Power Loss (MW)** | | | | | |
|---|---|---|---|---|---|---|
| | ***Pro rata* to loads** | ***Pro rata* 50:50** | **IM** | **PS** | **Z-bus** | **Loop (DFS)** |
| 1 | 0 | 5.71 | 0 | 0 | 9.01 | 13.75 |
| 2 | 1.12 | 1.54 | 0.51 | 0.48 | 0.31 | -0.8 |
| 3 | 4.87 | 2.44 | 5.52 | 5.55 | 2.21 | 0 |
| 4 | 2.47 | 1.24 | 2.49 | 2.45 | 0.58 | 0 |
| 5 | 0.39 | 0.2 | 0.29 | 0.31 | 0.03 | 0 |
| 6 | 0.58 | 0.29 | 0.43 | 0.45 | 0.06 | 0 |
| 9 | 1.53 | 0.76 | 1.52 | 1.51 | 0.35 | 0 |
| 10 | 0.47 | 0.23 | 0.51 | 0.48 | 0.13 | 0 |
| 11 | 0.18 | 0.09 | 0.15 | 0.17 | 0.04 | 0 |
| 12 | 0.32 | 0.16 | 0.27 | 0.3 | 0.08 | 0 |
| 13 | 0.7 | 0.35 | 0.69 | 0.72 | 0.22 | 0 |
| 14 | 0.77 | 0.39 | 1.01 | 0.95 | 0.37 | 0 |
| **Total** | **13.4** | **13.4** | **13.39** | **13.37** | **13.4** | **12.95** |

To clarify the different distributions, the losses are also plotted in the following Fig. 1. Fig. 1 illustrates the percentage distribution of losses of all the different methods implemented.

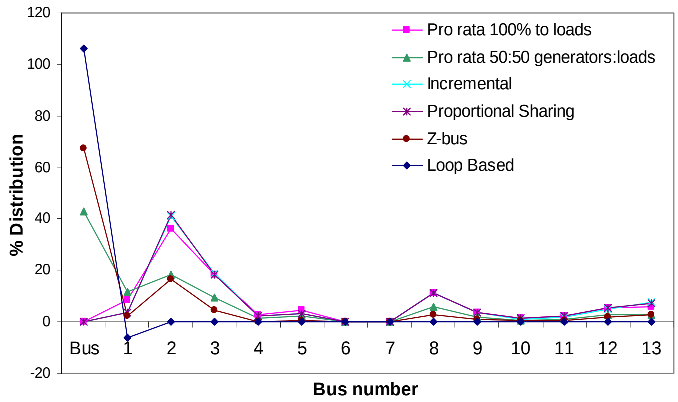

Fig. 1. Percentage distribution of losses allocated for the IEEE 14-bus system

This graph shows that the characteristics of the IEEE 14-bus system lead to a very similar distribution of losses for the different allocation methods. A near perfect correlation of 0.99 was calculated for the loss distribution produced by the *pro rata* method (100% to loads) when compared to the loss distribution produced using both the incremental and proportional sharing methods. Similarly high correspondence was observed between the *pro rata* approach (50:50 gen:load) and the Z-bus method of 0.96. These results are very interesting as they indicate that in this simple test system the complex loss allocation methods are no more informative than the basic *pro rata* method. It is believed that this is a consequence of the comparative electrical proximity of the two generators the subsequent uni-directional nature of the power flows. It does suggest that the *pro rata* method may be quite satisfactory in simple, predominantly radial systems where the generation is highly concentrated.

The only conflicting loss allocation is provided by the proposed loop based approach. Table II lists the methods where losses are allocated to generators only. Specifically, the *pro rata* method, and the loop based method. The two different results listed for the loop based method are based on the two different search strategies, DFS and BFS. The results from the DFS method represent the loop obtained directly from loop flows. The BFS results have been adjusted to account for a large amount of load demand, which although allocated to generator 2, can be traced back to generator 1. This is because generator 1 delivers a large amount of power to generator 2, which is then passed on the remaining network loads. Therefore, it seems logical to reapportion the losses.

TABLE II
LOSSES ALLOCATED TO GENERATORS

| Bus no. | Pro rata to gens | Loop Based (DFS) | Loop Based (BFS)* |
|---|---|---|---|
| 1 | 11.54 | 13.75 | 11.8 |
| 2 | 1.99 | -0.8 | 1.13 |
| Total | 13.53 | 12.95 | 12.93 |

* Based on proportionality results

Once again, results obtained from the two methods are fairly similar, especially between the *pro rata* method and the BFS loop allocation. The DFS loop based distribution has allocated negative losses to generator on bus 2. This is due to generator 2 attempting to pass power back to the first generator, and in effect oppose the more natural flows of the power within the network. This phenomenon has been termed ‘counter flow’ by other researchers, and can be used as a scheme to reward generators for lowering the overall losses in the system [9, 16].

Overall, though, it is difficult to draw significant conclusion from the results of the IEEE 14-bus network. The network is too small and the generation is highly concentrated. Yet this test system has been used regularly in earlier loss allocation studies to demonstrate different approaches. The results shown here must throw some doubt on the validity of using the test system for such a purpose.

### *B. Results from a Modified CIGRE Nordic 41 bus system*

More interesting results are found when the methods are implemented on a more realistic 41 bus system. Table III lists the losses allocated to the buses using on the different allocation methods. Greater disparity is visible between the losses allocated using the different methods.

TABLE III
NORDIC 41 REAL POWER LOSS ALLOCATIONS

| **Bus no.** | **Real Power Loss (MW)** | | | | | |
|---|---|---|---|---|---|---|
| | **Pro rata to loads** | **Pro rata 50:50** | **IM** | **PS** | **Z-bus** | **Loop** |
| 41 | 21.15 | 10.57 | 19.49 | 49.98 | 3.99 | 0 |
| 42 | 15.67 | 7.83 | 19.5 | 30.08 | 7.72 | 0 |
| 43 | 35.25 | 17.62 | 44.74 | 56.69 | 28.33 | 0 |
| 46 | 27.42 | 13.71 | 37.65 | 10.9 | 21.94 | 0 |
| 47 | 3.92 | 1.96 | 5.19 | 0 | 2.3 | 0 |
| 51 | 31.33 | 15.67 | 42.07 | 19.45 | 31.24 | 0 |
| 61 | 19.58 | 9.79 | 23.95 | 18.67 | 8.87 | 0 |
| 62 | 11.75 | 5.87 | 14.37 | 1 | 3.44 | 0 |
| 63 | 23.11 | 11.55 | 29.81 | 0 | 2.18 | 0 |
| 1011 | 7.83 | 3.92 | 0.04 | 4.07 | -24.2 | 0 |
| 1012 | 11.75 | 17.18 | 2.93 | 1.65 | 31.02 | 0.00 |
| 1013 | 3.92 | 7.61 | 1.13 | 0.3 | 22.77 | 51.58 |
| 1014 | 0 | 10.36 | 0 | 0 | 67.95 | 6.56 |
| 1021 | 0 | 7.54 | 0 | 0 | 48.61 | 20.12 |
| 1022 | 10.97 | 9.25 | 4.86 | 9.72 | -5.74 | -12.05 |
| 1041 | 23.5 | 11.75 | 30.23 | 63.77 | 34.31 | 0 |
| 1042 | 11.75 | 12.66 | 11.36 | 0 | -2.05 | 16.18 |
| 1043 | 9.01 | 7.9 | 10.14 | 18.66 | 2.86 | 13.94 |
| 1044 | 31.33 | 15.67 | 30.54 | 78.67 | 23.16 | 0 |
| 1045 | 27.42 | 13.71 | 37.12 | 61.84 | 25.94 | 0 |
| 2031 | 3.92 | 1.96 | 2.94 | 2.9 | -3.98 | 0 |
| 2032 | 7.83 | 18.05 | 7.52 | 0 | 35.3 | 39.00 |
| 4011 | 0 | 12.6 | 0 | 0 | 73.47 | -34.17 |
| 4012 | 0 | 11.31 | 0 | 0 | 64.44 | -31.29 |
| 4021 | 0 | 4.71 | 0 | 0 | 11.82 | 253.00 |
| 4031 | 0 | 5.84 | 0 | 0 | 12.32 | 27.31 |
| 4041 | 0 | 0 | 0 | 0 | -0.1 | 3.32 |
| 4042 | 0 | 11.87 | 0 | 0 | -11.68 | 2.79 |
| 4047 | 0 | 20.35 | 0 | 0 | -29.74 | 20.80 |
| 4051 | 0 | 11.31 | 0 | 0 | -23.76 | 0.04 |
| 4062 | 0 | 9.99 | 0 | 0 | -8.8 | 47.85 |
| 4063 | 0 | 19.97 | 0 | 0 | -8.93 | 3.35 |
| 4071 | 11.75 | 11.53 | 5.49 | 0.15 | -2 | 0.06 |
| 4072 | 78.33 | 76.85 | 47.4 | 0 | -13.85 | 0.00 |
| **Total** | **428.49** | **428.46** | **428.47** | **428.5** | **429.15** | **428.39** |

These differences reinforced in when plotted as shown in are can be easier visualized from the graph in Fig. 2.

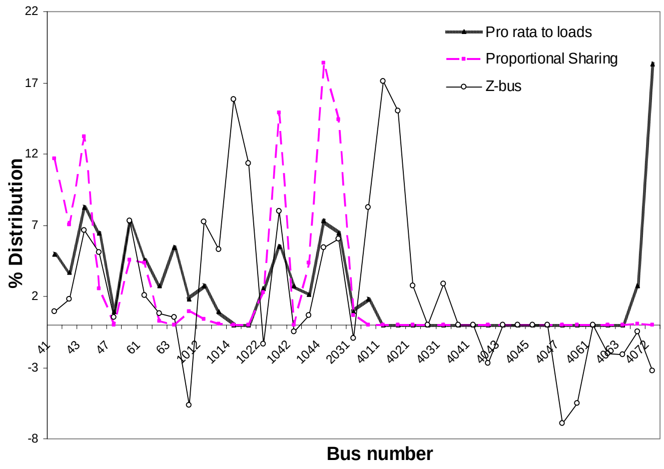

Fig. 2. Percentage distribution of losses allocated for the Nordic system

Fig 2. shows the comparative distribution of losses produced using the *pro rata* method, the proportional sharing approach and the Z-bus methods only. Losses produced using the incremental approach was not presented. This is because of the continued correlation (correlation coefficient of 0.91) between the *pro rata* (100% to loads) and incremental approaches. The similarity is not unexpected. Both methods are highly dependent on total system bus power injections. The difference between the two is, instead of distributing losses based on the power injections after obtaining the load flow solution, the incremental method looks the summation of incremental losses with respect to the power injection at each load bus. The numerical differences are probably due to the calculation process used for the incremental method. It is still interesting that the *pro rata* method, although lacking physical justification, produces comparable results to the incremental method.

The loss distribution produced using the *pro rata* (or incremental) method, however, now differs widely from the allocations produced using the proportional sharing, Z-bus or loop based approaches. These three allocations are, in turn, also significantly different.

It is believed that this is a result of the differing ways in which the proportional sharing, Z-bus or loop based approaches address the topological considerations of the network. As emphasized previously, the proportional sharing approach, although capable of considering network configuration, does so in a somewhat arbitrary fashion. This should be contrasted with the Z-bus method in which the impact of system configuration is handled in an electrically justifiable fashion, at the expense of being able to trace power flows between specific market participants. Finally, the newly proposed loop based approach is capable of producing an electrically consistent power flow tracing from the generators to the loads. This tracing or mapping of generators to loads, however, is still somewhat arbitrary, making the method most useful when it is used to justify the suitability of a financial instrument, such as a bilateral contract between the market participants. In essence, further work is still required to understand the significance of the different loss allocation produced by the proportional sharing, Z-bus or loop based approaches, especially when applied to a realistic network such as the 41 bus system.

## V. CONCLUSIONS

Different objectives of the five different loss allocation methods have been critically analysed and tested on two test systems; IEEE 14-bus system and a modified CIGRE Nordic 41-bus system. Tests carried out on the first system have highlighted a limitation of the system because generation is concentrated only at one end of the system. Thus, a fairly similar distribution of losses by all five methods is resulted. More significant results were obtained when tested on the larger Nordic system where differences were found between most methods. However, in general most methods are highly dependent on the power injection at each bus.

One of the more substantial findings is that the pro rata method exhibits fairly similar loss distribution characteristics to the incremental method for the both networks. This result questions the necessity of implementing the incremental method when the simple *pro rata* can produce fairly similar results.

Another important finding is that pro rata and incremental method do not encourage competition in the electricity market because of their simple objective, which depends only on the power injection. Network dependent alternatives such as the proportional sharing, Z-bus and loop based method would provide a better indicative measure to promote efficient network usage. Further work will focus on determining which method encourages efficient system operations.

Overall, results from this study show that different methods have their own merits and demerits. Since no method is able to allocate losses accurately, the selection of a particular method will be dependent upon the market participants and regulators level of competition they prefer in the market.

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