Comparative Effectiveness of Loss Allocation Methods for providing Signals to affect Market Operation

V. S. C. Lim, T. K. Saha
School of Information Technology and Electrical Engineering
University of Queensland
Brisbane, Queensland, Australia
lim@itee.uq.edu.au, saha@itee.uq.edu.au

J. D. F. McDonald
Department of Electrical and Electronic Engineering,
Imperial College London, South Kensington, London, UK
j.mcdonald@imperial.ac.uk

ABSTRACT

The distribution of system losses, an integral part of electricity pricing, can play an important role in the operation of electricity markets. To date, despite the existence of many loss allocation methods, no one method is commonly used in established electricity markets. Furthermore, some markets are still considering using different methods that will provide more efficient treatment of losses and aid in improving market operations and structures. This paper compares the loss allocation methods used in existing markets in Eastern Australia and Great Britain, as well as with the pro rata and proportional sharing approaches. Through implementation of the loss allocation methods on the CIGRE Nordic 32 bus system we examine what behaviour each method encourages. Results suggest that the method used in the Australian market provides the most sophisticated signal to market participants. Similar results, however, can be obtained using the simpler approach taken in Great Britain. This reinforces that the selection of loss allocation will be a market dependent problem.

1. INTRODUCTION

Deregulation of the electricity industry has brought many new challenges into the industry, one of which is the development of a fair pricing for electricity that reflects a market participant's use of the system. More sophisticated treatment of losses, in contrast to traditional methods that arbitrarily assign losses as 2% to 5% of generated power, is critical in overcoming this problem. In addition, careful distribution of losses can also provide economic and operational signals to the market participants.

Essentially, the loss allocation chosen is a part of the design of the market itself. A carefully selected loss allocation method is able to:

- promote efficient matching of supply and demand;
- provide indicative measures for locational advantages of market participants; and
- provide information on the need and appropriate location of network expansions.

Therefore it is crucial for market operators of the deregulated market to adopt loss allocation methods that are compatible with their market structures, as well as promoting competition between market participants.

To date, no loss allocation method has been universally accepted. Different markets globally have employed different loss allocation schemes. For example; Australia's National Electricity Market (NEM), which is managed by the National Electricity Market Management Company (NEMMCO), has used a form of approximated marginal loss allocation [1]. In contrast, the Great Britain (GB) market has employed the simpler *pro rata* method [2]. Furthermore, New Zealand has adopted the full marginal loss allocation method [3]. The lack of a universally accepted loss allocation method suggests that there are deficiencies in all current methods, leading to continued research in this field.

Current loss allocations methods can be categorised in several broad groups such as *pro rata*, marginal, proportional sharing, loss formula, and circuit theory-based [4]. The simplest method is the *pro rata* method, which is based on an arbitrary division of losses between active generation and load. This method does not take account of the geographic distribution of the network.

Topological flow tracing methodologies based on proportional sharing principle [5, 6] have been proposed to overcome the limitation of *pro rata* loss allocation. This method assumes that power at nodal inflows is shared proportionally between nodal outflows. This enables tracing of power flows between generators and loads, although the proportional distribution of power is yet to be verified. A further limitation is that losses can only be allocated to either generators or loads.

In contrast, the incremental method is a more accepted loss allocation method. Incremental loss methods assign losses in relation to a slight change in bus injections. The basic approach has been refined to handle the presence of negative loss allocations, over estimation of losses [7, 8], and slack bus dependency [8, 9]. Critically, many of these refinements have only been possible from the introduction of further arbitrary assumptions.

This has prompted development of other flow tracing and/or loss allocation methods based on either circuit theory or loss formula. In [10, 11] flow distribution is determined from the bus impedance matrix. Analysis of the method proposed in [11] has shown that the better results are attained for lines that carry the majority of power flows in the network [12]. The method proposed in [13], which expresses loss as a quadratic function, can result in negative allocations. This highlights that there are limitations to all current approaches to loss allocation.

Despite the lack of a universally accepted method, electricity markets must still adopt some approach to calculate losses in the competitive environment. Consequently, different markets have selected different loss allocation approaches. This poses the question, "Which loss allocation method is more suitable, and why?" To address this, the paper critically analyses and compares the loss allocation methods used in two established markets, Australia's NEM and Great Britain's Market. This leads to an assessment of both the effectiveness of each method within the different market structures, and the type of network operation that each method is promoting.

These commercial methods were also compared with two other well-known methods, specifically the *pro rata* and proportional sharing principle (PSP) loss allocation methods. These additional comparisons will allow markets such as Brazil, who are considering implementing a marginal loss allocation approach, to assess the value of choosing complex marginal methods over the simpler pro-rata approach.

The remainder of the paper is arranged as follows: Section 2 provides background materials on the methods that are compared, as well as the numerical formulation of each method. Section 3 presents the results obtained from implementing each method on the CIGRE Nordic 32 bus system [14], followed by critical analysis of the behaviour of the different approaches.

2. BACKGROUND AND NUMERICAL FORMULATION

This section presents a brief description of the different methods considered, as well as the numerical formulation of each method as implemented by the authors on the CIGRE Nordic 32 bus system.

2.1. AUSTRALIA'S NATIONAL ELECTRICITY MARKET (NEM) METHODOLOGY [1]

The National Electricity Market (NEM) in Australia commenced its operation on 13 December 1998. For geographic reasons, the NEM comprises only of the eastern states of Australia including; Queensland, New South Wales (and the Australian Capital Territory), Victoria, and South Australia. These regions are linked through a set of AC and DC interconnectors.

The price of electricity in the NEM is determined using an approximated nodal pricing method, where the underlying method is to calculate the marginal cost of supplying a very small increment in demand at each location. In this way, the price accounts for several factors including the costs of producing electricity, as well as transmission loss and capacity limitations.

In the NEM, the allocation of transmission loss is divided into two parts; intra-regional losses and inter-regional losses. The intra-regional losses model the losses between a reference node, called the Regional Reference Node (RRN), and other nodes within the same region at which market participants are located. For each intra-regional node, static marginal loss factors (MLFs) are calculated to reflect the impact of marginal losses on nodal prices at each node within the network. These MLFs are updated yearly. To accommodate for the large and variable flows between RRNs of any two regions, dynamic inter-regional loss factors are updated every dispatch interval. Figure 1 provides a better visualisation of the two types of losses within the NEM.

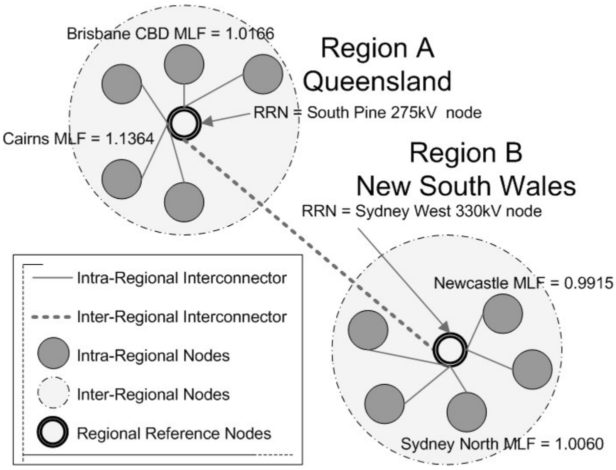

Figure 1. Nodal pricing concept

In this paper, an approximated form of the MLF calculation, outlined in [15], was implemented for one region. It follows the marginal concept where MLFs at each node is calculated from a change in loss with respect to a change in load, as shown in (1).

$$MLF = 1 + \frac{\partial loss}{\partial load\ increment} \tag{1}$$

The MLFs for all nodes within a region are calculated in reference to a swing bus, where the MLF is set to 1. In general, for nodes with an overall net injection into the system the MLFs tend to be less than 1. For nodes with an overall net demand, the MLFs tend to be more than 1. Generators and loads at these nodes are then rewarded or penalised accordingly.

In the NEM pool dispatch process, the nodal spot price at a particular location within a region is determined by multiplying the spot price at the RRN by the appropriate MLF. The resulting price is often an over-estimation and further price adjustments may be required during the settlement process.

2.2. THE GREAT BRITAIN (GB) MARKET METHODOLOGY [2]

The fully competitive British-wide wholesale electricity market was implemented on 1 April 2005. This market represents an extension of the existing England and Wales market to Scotland. Electricity trading extends across a wider geographical area, even covering the interconnector to Northern Ireland. This ensures that electricity can be traded freely across the whole of GB, thereby encouraging the development of a fully competitive electricity market.

The transmission losses in the previous England and Wales market were initially allocated to suppliers only, independent of their locations. Trading arrangement based on the "Postage Stamp" method were then introduced, where losses are allocated to both generators and suppliers based on a ratio of 45:55 respectively. Under this scheme the market participants are still charged for losses on a uniform basis.

The new GB market employs similar method as the previous England and Wales market, with future plans to incorporate transmission loss factors into the calculation. At present, the calculation of losses is based on transmission loss multipliers (TLMs), essentially a form of *pro rata* allocation. For each half hour trading period, two TLMs will be calculated for units generating and consuming electricity respectively. The relevant equations are shown in (2), and (3).

$$TLM = 1 - \left(\left\{ \frac{\alpha\left(\sum^{+} QM + \sum^{-} QM\right)}{\sum^{+} QM} \right\} \right) \quad (2)$$

$$TLM = 1 + \left(\left\{ \frac{(\alpha - 1)\left(\sum^{+} QM + \sum^{-} QM\right)}{\sum^{-} QM} \right\} \right) \quad (3)$$

QM refers to the metered MWh quantity; α is a predetermined factor that is set to 0.45; $\sum^{-}$ represents sum of all generating units; and $\sum^{+}$ refers to sum of all load units within the system.

These TLMs can be used to find the adjusted real power at each node. Subsequently, the difference between the actual power at the node and adjusted power gives the losses assigned to that bus.

2.3. PRO RATA ALLOCATION [4]

The *pro rata* allocation [16] method is the simplest loss allocation method. It assigns losses based on a comparison of the level of power or current injected/consumed by a specific generator or load to the total power generated or delivered in the system. Using a solved load flow solution, losses are systematically distributed based on the real power injected or consumed at each node, as shown in (4) and (5).

$$L_{Gi} = \frac{P_{loss}}{x} \frac{P_{Gi}}{P_G} \quad (4)$$

$$L_{Dj} = \frac{P_{loss}}{x} \frac{P_{Dj}}{P_D} \quad (5)$$

Together equations (4) and (5) represent the *pro rata* allocation of losses to the generator at bus *i* and load at bus *j*. P_G is total real power generated in the system while P_{Gi} is the total MW output of the generators at bus *i*. Alternatively, P_D is total real power consumed and P_{Dj} is the real power consumed by loads of bus *j*. P_{loss} is system transmission power losses. The multiplying factor *x* can be used to weight the distribution of system losses towards either of the market participants.

It is clear from (4) and (5) that this method is totally reliant on the power injections at buses and independent of the network topology. Losses are distributed across all buses, according to their level of generation or consumption only. Two loads in different locations but with identical demands will be allocated the same level of loss, irrespective of their comparative proximity to system generation. No incentive is provided for placing generation closer to load centres, a practice which usually leads to reduced system losses. In addition, the pro rata method is also unable to trace power flows, making it difficult to justify the different allocations.

2.4. PROPORTIONAL SHARING ALLOCATION [4]

The PSP method introduced by Bialek [5] represents a fundamental shift in the process of loss allocation. Bialek introduced a topological tracing method, treating each node as an ideal mixer, such that power flowing out of a node can be considered the proportional sum of the power flowing into the node. This allows the demands of a load to be traced "up" to the generators or the output of the generator to be traced "down" to the loads.

Consider the tracing of power upstream from the loads to the generating sources. Starting from a solved load flow solution, the power balance equation at node *i* considering the power inflows from "upstream" is defined by (6).

$$P_i^g = \sum_{j \in \alpha_i^u} \left| P_{ij}^g \right| + P_{Gi} \qquad \text{for } I = 1,2, \ldots, n \quad (6)$$

P_i^g is the unknown gross nodal power flow through node *i*, P_{ij}^g is the unknown gross line flow in line *i-j*, α_i^u is the set of nodes supplying node *i*, and P_{Gi} is the power generation in node *i*. The line flows P_{ij}^g also can be expressed as a proportion of the flows into the upstream node *j*. By continuing this process, the contribution of system's generators to the *i*-th gross nodal power can be expressed according to (7).

$$P_i^g = \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ik} P_{Gk} \qquad \text{for } i = 1, 2, \ldots, n \quad (7)$$

$$\left[A_u^{-1}\right]_{ij} = \begin{cases} 1 & i = j \\ -\frac{\left|P_{ij}\right|}{P_j} & j \in \alpha_i^u \\ 0 & otherwise \end{cases}$$

A_u is the upstream distribution matrix and P_{Gk} is the generation at node *k*. In these cases, the gross nodal and line flows refer to those power flows in a lossless system. The difference between the gross and actual demand gives the loss allocated to a load.

Unlike the *pro rata* method, the proportional sharing method is capable of defining a contribution of each generator to each load through tracing the flow of power. The assignment of losses to either generators or loads should encourage the market participants to take corrective actions that will reduce their share of losses. The problem with this approach, however, is that the distribution of power flows is built on the proportional sharing principle, which lacks physical and economical justification. This departure from electrical behaviour of the network may mean that proposed strategies to reduce losses may not be technically satisfactory. Additional work has been completed to improve the allocation procedure, including formalisation of the search algorithm through application of graph theory [17] as well as corroborating the principle with game theory [18]. The lack of justifiable correlation between the network's electrical behaviour and the flows tracing established using proportional sharing, however, still remains a limitation.

3. RESULTS AND DISCUSSION

The following results present the distribution of losses in the CIGRE Nordic 32 bus system [14]. In this system, shown in Figure 2, both the generation and loads are widely distributed. The system contains 22 generators, 22 loads, and 52 line and transformers and 51 shunt elements representing line capacitance and off-nominal transformers. The total real power load consumption amounts to 10940MW whereas the real power generation is 11368.4MW, resulting in a loss of 428.5MW.

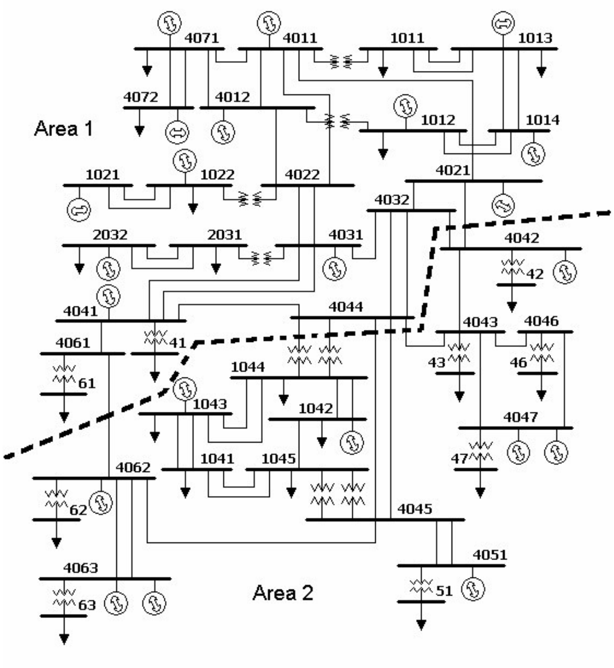

Figure 2 CIGRE Nordic 32 bus system

3.1. NEM'S METHOD (MLFS)

The MLF results obtained from implementing the approximated MLF method, outline by NEMMCO, are plotted in Figure 3. The first half of the graph shows the MLFs of the load buses, whereas the MLFs of the generator buses in the system are shown in the second half of the plot.

These static MLFs represent multiplying factors, where in this paper they are used to determine losses in the system. In NEM, generators/loads having MLF of 1 indicate that during settlement they are paid/will be paying for the volume of power they have generated/consumed. For this study, the majority of the load buses are penalised for the losses that were incurred in the system. Only two load buses, specifically buses 1011 and 4072, are not required to pay for losses and pay for only the amount of power they consumed.

On the other hand, the MLFs calculated for the generator buses indicate that while many generators were penalised, two buses were rewarded. On Figure 3 the MLFs for nodes 1014 and 1021 are above the MLF = 1 line. This suggests that increasing the output of these generators, to ensure sufficient supply to match the demand, will reduce system losses. It is believed that this is because the generation capacity of the area in close proximity to buses 1014 and 1021 is much higher than the consumption level. It would appear that the NEM's marginal loss method is capable of providing information regarding the demand and supply capability of selected localised area.

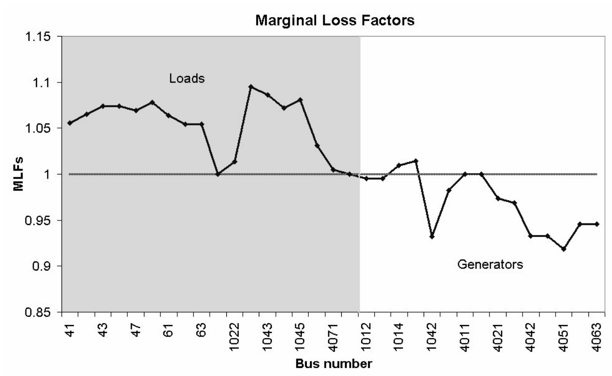

Figure 3 Marginal loss factors

It is noticed that, in general, the nodes that are more heavily penalised are located at the lower half of the Nordic system. These nodes are indicated in Figure 2, where the system is divided into two areas. It is felt that the nodes are more heavily penalised in area 2 because the generation capacity in that area is fairly low compared to the load consumption capacity in the same area; where the generation amounts to only 40% of the total system generation, and the loads amounts to 60% of the total system load.

In summary, the MLFs allocated are dependent on the ratio of generation capacity versus load consumption at the area; that is the supply and demand profile. These MLFs, however, are highly dependent on the operation point of the system. For any operating point, generators and loads that are located in areas where the generation capacity is higher than the loads are generally better off.

3.2. GB'S METHOD (TLMS)

For the base case solution of the Nordic system, the TLMs calculated from implementing the GB method are: 0.98 for all generating nodes; and 1.02 for all load nodes. A node is defined as a generating node if the net power at the node is positive, whereas a node is defined as a load node if the net power at the node is negative. All nodes within the network are categorised as either generating nodes or load loads based on the power level at each of the nodes.

The clear difference between the NEM and the GB methods is that the NEM's method is more variable and is capable of providing indicative measure to improve the supply and demand within an area of a system, as well as future expansion signals. Although only two TLMs are assigned to differentiate generating nodes from load nodes, the MW losses resulted, as shown in the next section, indicated that the simpler method employed by GB method and the volatile NEM method have fairly high correlation.

3.3. COMPARISON OF SEVERAL LOSS ALLOCATION METHODS

The loss factors for the NEM and GB method are translated to real power losses for comparison purposes. The results are listed in Table I, where the values, calculated from the base case load flow solution, are listed in MW.

Table I CIGRE Nordic 32 bus real power loss allocations

Bus number	NEM's method	GB's method	Pro rata 50:50 gens:loads	PSP
41	31.21	11.63	10.57	49.98
42	26.03	8.61	7.83	30.08
43	66.73	19.38	17.62	56.69
46	51.71	15.08	13.71	10.9
47	6.9	2.15	1.96	0
51	62.34	17.23	15.67	19.45
61	31.96	10.77	9.79	18.67
62	16.39	6.46	5.87	1
63	32.14	12.71	11.55	0
1011	0	4.31	3.92	4.07
1012	1.36	5.09	17.18	1.65
1013	0.91	3.39	7.61	0.3
1014	-5.13	9.33	10.36	0
1021	-5.6	6.78	7.54	0
1022	1.09	1.72	9.25	9.72
1041	57.03	12.92	11.75	63.77
1042	4.05	1.02	12.66	0
1043	4.29	1.08	7.9	18.66
1044	57.61	17.23	15.67	78.67
1045	56.5	15.08	13.71	61.84
2031	3.13	2.15	1.96	2.9
2032	9.81	9.33	18.05	0
4011	0	11.33	12.6	0
4012	0	10.17	11.31	0
4021	6.69	4.24	4.71	0
4022	0	0	0	0
4031	9.7	5.26	5.84	0
4032	0	0	0	0
4041	0	0	0	0
4042	42.3	10.68	11.87	0
4043	0	0	0	0
4044	0	0	0	0
4045	0	0	0	0
4046	0	0	0	0
4047	72.76	18.31	20.35	0
4051	48.67	10.17	11.31	0
4061	0	0	0	0
4062	28.78	8.99	9.99	0
4063	57.71	17.97	19.97	0
4071	0	0	11.53	0.15
4072	0	0	76.85	0
Loss	**776.06**	**290.57**	**428.46**	**428.5**

As expected, the NEM's approximated marginal method over-allocates system losses, while the GB's method under-estimates system losses. The market operators of the NEM are aware of this limitation, thus this over-allocation is readjusted in their settlement process [15]. The other two methods, *pro rata* and PSP, are essentially a systematic redistribution of system losses. Consequently, as expected, total allocated losses are equal to the total loss of the system.

Table I shows that NEM's marginal method is the only method that rewards nodes that assist in improving the overall generation and supply profile of the system. As mentioned previously, they are buses 1014 and 1021, where the losses are -5.13MW and -5.6MW respectively.

In order to compare the methods, the results have to be normalised by load flow base case losses. Figure 4 shows the comparative percentage distribution of transmission losses calculated for all four loss allocation methods. From the distribution, it is found that the NEM's method and the GB's method have quite a high correlation of 0.868. This suggests that the simple *pro rata* method, as implemented by GB, provides reasonably good indication of losses relative to NEM's marginal method.

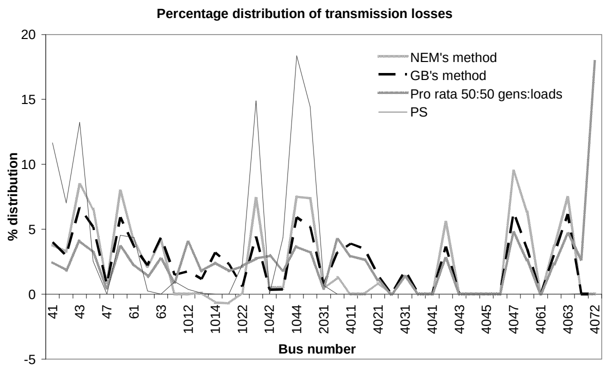

Figure 4 Percentage distribution of transmission losses

Surprisingly, although GB's method is based on the *pro rata* method, the results in Figure 4 show that the correlation between the two methods is quite low. The coefficient of correlation between the two loss allocations is only 0.244. Although GB's method is based on *pro rata*, it assigns losses uniformly based on whether the bus is generating or consuming. Thus losses are allocated based on the two predefined TLMs. The losses are then calculated from the net injection of power at that bus. On the other hand, the *pro rata* method is highly dependent on the generation and load power at each of the buses. Losses for generators and loads are calculated separately, and then summed; instead of depending on the net injection at that particular bus. These differences lead to the low correlation between the different loss allocations, although the correlation would be 0.848 when the losses for buses with zero net injection are not considered (buses 4071 and 4072).

The comparative analysis carried out also includes the PSP method. Unlike the other methods analysed, the PSP method allocates losses to either generators or loads only, where in this study they are allocated to loads. Therefore the results obtained are not readily comparable. Furthermore, the other three methods are highly dependent on the power injection at each of the buses, whereas for this method, the losses are distributed based on the proportionality principle.

When comparing only the load losses for the PSP method with both the NEM and GB method, the resultant correlation is equal to approximately 0.7, which is reasonably high. However, the correlation between the PSP and the pro rata is only 0.5. These differences are because, the PSP method allocates losses based on the sharing factor at each node. In contrast, the other three methods are dependent on the power injection.

The results from these analyses have shown that the loss allocation methods employed in the two different electricity markets, NEM and GB, produces fairly similar results. Overall, when comparing all four methods analysed, it is found that the method of losses employed will play an important role in supporting market structure and operation. For instance, the GB market adopted the TLM method as the market operators prefer a simpler and less volatile method. However, to give a better indicative measure of network structure, the market operators in the GB market are considering incorporating loss factors into the TLM calculations [2].

4. CONCLUSIONS

This paper has highlighted the importance of choosing a suitable loss allocation method in the electricity markets, where appropriate treatment of losses can not only promote competition amongst market participants, but also provide indicative signals for efficient market operations and structures.

The loss allocation methods adopted by two established markets, namely NEM and GB, was implemented and critically analysed. The results obtained from the implementation showed that the NEM's approximated method is capable of providing locational indicative measure which can assist market operators and interested market participants to plan for future expansions. Market participants that are located in the area where the immediate local power supply is sufficient or higher than demand are often better off than those located in areas where the local supply is lower than the demand. Although the GB's method is not capable of giving such measures, the losses allocated for both methods are closely correlated. This indicates that the simple *pro rata* method is sufficient if market operators are interested in only allocating losses for the present time, but that the NEM's method can give signals for future network developments.

Further comparative analysis with the basic *pro rata* and proportional sharing methods showed that at the end of the day, the method chosen is highly dependent on the requirements of the market operators.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of IEnergy Pty. Ltd. and the Australian Research Council for a Linkage research grant.

REFERENCES

[1] NEMMCO, "An Introduction to Australia's National Electricity Market," National Electricity Market Management Company Limited June 2004.

[2] Website of the ELEXON. Available: http://www.elexon.co.uk

[3] T. Alvey, D. Goodwin, X. Ma, D. Streiffert, and D. Sun, "A security-constrained bid-clearing system for the New Zealand wholesale electricity market," *IEEE Transactions on Power Systems*, vol. 13, pp. 340-346, 1998.

[4] V. S. C. Lim, J. D. F. McDonald, and T. K. Saha, "Comparative Distribution of System Losses to Market Participants using Different Loss Allocation Methods," presented at IEEE/PES T&D 2005, Asia Pacific, Dalian, China, 2005.

[5] J. Bialek, "Tracing the flow of electricity," *IEE Proceedings Generation, Transmission and Distribution*, vol. 143, pp. 313-20, 1996.

[6] D. Kirschen, R. Allan, and G. Strbac, "Contributions of Individual Generators to Loads and Flows," *IEEE Transactions on Power Systems*, vol. 12, pp. 52-60, 1997.

[7] N. Chowdhury and A. Bhuiya, "Counter-Flow in a Deregulated Power System Network and it's Effect on Transmission Loss Allocation," presented at Canadian Conference on Electrical and Computer Engineering, Chelsea Inn, Toronto, Canada, 2001.

[8] F. D. Galiana and M. Phelan, "Allocation of transmission losses to bilateral contracts in a competitive environment," *IEEE Transactions on Power Systems*, vol. 15, pp. 143-150, 2000.

[9] A. M. L. da Silva and J. G. de Carvalho Costa, "Transmission loss allocation: I. Single energy market," *IEEE Transactions on Power Systems*, vol. 18, pp. 1389-1394, 2003.

[10] A. J. Conejo, F. D. Galiana, and I. Kockar, "Z-Bus Loss Allocation," *IEEE Transactions on Power Systems*, vol. 16, pp. 105-10, 2001.

[11] Y.-C. Chang and C.-N. Lu, "An Electricity Tracing Method with Application to Power Loss Allocation," *International Journal of Electrical Power and Energy Systems*, vol. 23, pp. 13-17, 2001.

[12] V. Lim, T. K. Saha, and T. Downs, "Preliminary findings on usage allocation and loss allocation of electricity in a deregulated market," presented at Australasian Universities Power Engineering Conference, Christchurch, New Zealand, 2003.

[13] G. Gross and S. Tao, "A Physical-Flow-Based Approach to Allocating Transmission Losses in a Transaction Framework," *IEEE Transactions on Power Systems*, vol. 15, pp. 631-7, 2000.

[14] Cigre, "Long Term Dynamics Phase II," Cigre TF 38-02-08 1995.

[15] NEMMCO, "Treatment of Loss Factors in the National Electricity Market," November 1999.

[16] A. J. Conejo, J. M. Arroyo, N. Alguacil, and A. L. Guijarro, "Transmission loss allocation: a comparison of different practical algorithms," *Power Systems, IEEE Transactions on*, vol. 17, pp. 571-576, 2002.

[17] F. F. Wu, Yixin Ni, and Ping Wei, "Power transfer allocation for open access using graph theory-fundamentals and applications in systems without loopflow," *IEEE Transactions on Power Systems*, vol. 15, pp. 923-9, 2000.

[18] P. A. Kattuman, J. W. Bialek, and N. Abi-Samra, "Electricity Tracing and Co-operative Game Theory," presented at 13th Power Systems Computation Conference, Trondheim, 1999.