A Near-Optimal Linear Crosstalk Precoder for Downstream VDSL

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***Abstract*— This paper presents a linear crosstalk precoder for VDSL that has a low run-time complexity. A lower bound on the data-rate of the precoder is developed and guarantees that the precoder achieves near-optimal performance in 99% of VDSL channels.**

***Index Terms*— Crosstalk cancellation, digital subscriber lines.**

I. INTRODUCTION

Crosstalk is a major problem in very-high speed digital subscriber line (VDSL) networks, limiting both the data-rate and reach of service. In the downstream direction the transmitting modems are co-located at the central office (CO). This allows crosstalk precoding to be applied, a technique where predistortion is added to each modem's signal prior to transmission. This predistortion is chosen such that it annihilates with crosstalk introduced in the binder, allowing each modem to operate over a crosstalk free channel, and achieve a much higher data-rate.

A decision feedback structure, based on the Tomlinson-Harashima precoder (THP), was shown to operate close to the single-user bound[1]. Unfortunately this structure relies on a non-linear modulus operation at the receiver side, leading to a higher run-time complexity. For example, in a standard VDSL modem operating at 4000 DMT-symbols per second, with 4096 tones, the modulus operation would require an extra 16.3 million instructions per second (MIPS). This will almost double the complexity of the customer premises (CP) modem which currently only needs to implement a frequency domain equalizer, an operation that also requires 16.3 MIPS. Since CP modems are now a commodity, cost is an extremely sensitive issue, and any technique that helps to decrease complexity is extremely beneficial.

This paper presents an alternative linear precoder which has a much lower complexity based on the channel diagonalizing criterion. The performance of the diagonalizing precoder (DP) is analyzed in a VDSL environment. This paper extends earlier work that considered the design of near-optimal linear crosstalk cancellers for *upstream* transmission[2]. There the column-wise diagonal dominance (CWDD) of the upstream VDSL channel was shown to lead to near-optimal performance for zero-forcing crosstalk cancellers. In this paper we consider crosstalk precoding for *downstream* transmission. It is shown that, due to the row-wise diagonal dominance (RWDD) of the downstream VDSL channel, the DP achieves near-optimal performance. We develop bounds that allow the performance of the DP to be predicted without explicit knowledge of the crosstalk channels, simplifying service provisioning considerably.

II. SYSTEM MODEL

Assuming that the modems are synchronized and *discrete multi-tone* (DMT) modulation is employed we can model transmission independently on each tone $\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k$. We assume perfect knowledge of the crosstalk channels. In practice these must be identified using MIMO channel identification techniques and communicated back to the transmitter side. Since the VDSL channel is slowly time-varying this can be done infrequently and requires little additional overhead. The vector $\mathbf{x}_k \triangleq \left[x_k^1, \cdots, x_k^N\right]^T$ contains transmitted signals on tone k, where the tone index k lies in the range $1 \ldots K$. There are N lines in the binder and x_k^n is the signal transmitted onto line n at tone k. The vectors $\mathbf{y}_k$ and $\mathbf{z}_k$ have similar structures. The vector $\mathbf{y}_k$ contains the received signals on tone k. The vector $\mathbf{z}_k$ contains the additive noise on tone k and is comprised of thermal noise, alien crosstalk, RFI etc. The $N \times N$ matrix $\mathbf{H}_k$ is the crosstalk channel matrix on tone k. The element $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from transmitter m to receiver n on tone k. The transmit correlation on tone k is defined $\mathbf{S}_k \triangleq \mathcal{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\}$. We denote the transmit PSD of user n on tone k as $s_k^n \triangleq \mathcal{E}\left\{|x_k^n|^2\right\}$. We assume that the transmit PSD on each line must obey a spectral mask constraint[1]

$$s_k^n \leq s_k^{\text{mask}},\ \forall k, n. \tag{1}$$

The noise power experienced by receiver n on tone k is defined $\sigma_{k,n} \triangleq \mathcal{E}\left\{|z_k^n|^2\right\}$. Since the transmitting modems are co-located, the crosstalk signal transmitted from a disturber into a victim must propagate through the full length of the victim's line. This is depicted in Fig. 1, where CO2 is the disturber and CP1 is the victim. The insulation between twisted pairs increases the attenuation. As a result, the crosstalk channel matrix $\mathbf{H}_k$ is *row-wise diagonally dominant* (RWDD), since on each row of $\mathbf{H}_k$ the diagonal element has the largest magnitude $|h_k^{n,m}| \ll |h_k^{n,n}|$, $\forall m \neq n$. RWDD implies that the crosstalk channel $h_k^{n,m}$ from a disturber m into a victim n is always weaker than $h_k^{n,n}$, which is the direct channel of the victim[2]. The degree of RWDD can be characterized with the parameter α_k

$$|h_k^{n,m}| \leq \alpha_k\, |h_k^{n,n}|,\ \forall m \neq n. \tag{2}$$

Note that crosstalk precoding requires joint processing of the signals prior to transmission, so the transmitting modems must be co-located. Hence in all channels where crosstalk precoding can be applied, the RWDD property holds. RWDD has been

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[1] The techniques described in this paper can also readily be combined with dynamic spectrum management.

[2] Contrast this with the CWDD experienced in upstream transmission, where the crosstalk channel $h_k^{n,m}$ from a disturber into a victim is always weaker than the direct channel of the disturber $h_k^{m,m}$.

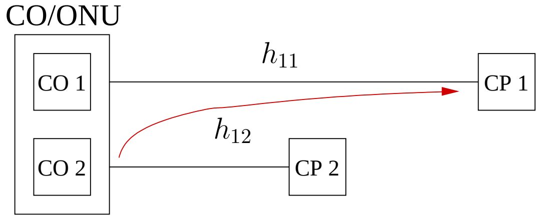

Fig. 1. Row-wise Diagonal Dominance $|h_{11}| \gg |h_{12}|$

verified through extensive measurement campaigns of real binders. In 99% of lines α_k is bounded

$$\alpha_k \leq K_{\text{xf}} \cdot f_k \cdot \sqrt{d_{\text{coupling}}}, \tag{3}$$

where $K_{\text{xf}} = -22.5$ dB and f_k is the frequency on tone k in MHz[3]. Here d_{coupling} is the coupling length between the disturber and the victim in kilometers. To find a value for α_k that is independent of the particular binder configuration, d_{coupling} can be set to 1.2 km, which is the maximum deployment length for VDSL[3]. The following sections show that RWDD ensures a well-conditioned crosstalk channel matrix. This results in the near-optimality of the DP.

III. THEORETICAL CAPACITY

We start with a bound on the capacity of the downstream VDSL channel with coordinated transmitters. This will prove useful in evaluating crosstalk precoder performance since it provides an upper bound on the achievable data-rate with any possible crosstalk precoding scheme. Denote the tone spacing with Δ_f.

Theorem 1: The achievable data-rate for user n on tone k is upper bounded

$$b_k^n \leq \Delta_f \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} s_k^{\text{mask}} \left|h_k^{n,n}\right|^2 \left[1 + (N-1)\,\alpha_k\right]^2\right). \tag{4}$$

Proof of Theorem 1: Central office modems are co-located and use co-ordinated transmission, so from an information theoretical perspective this is a broadcast channel. Consider the single-user bound, which is the capacity achieved when all transmitters (CO modems) are used to communicate to a single receiver (CP modem). In this case the received signal on the CP modem is $y_k^n = \overline{\mathbf{h}}_k^n \mathbf{x}_k + z_k^n$, where $\overline{\mathbf{h}}_k^n \triangleq [\mathbf{H}_k]_{\text{row}\, n}$. Using the single-user bound the achievable data-rate of user n on tone k is limited to

$$\begin{aligned} b_k^n &\leq \Delta_f I(\mathbf{x}_k; y_k^n), \qquad (5)\\ &= \Delta_f \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \overline{\mathbf{h}}_k^n \mathbf{S}_k \left(\overline{\mathbf{h}}_k^n\right)^H\right). \end{aligned}$$

where $I(a;b)$ denotes the mutual information between a and b. To account for the sub-optimality of practical coding schemes, we include the SNR-gap to capacity Γ[4]. Define the elements of the correlation matrix $s_k^{n,m} \triangleq [\mathbf{S}_k]_{n,m}$, and the diagonal elements $s_k^n \triangleq [\mathbf{S}_k]_{n,n}$. Since $\mathbf{S}_k$ is positive semi-definite it must be true that $s_k^{n,m} \leq \sqrt{s_k^n s_k^m}$. Hence

$$\begin{aligned} \overline{\mathbf{h}}_k^n \mathbf{S}_k \, \overline{\mathbf{h}}_k^{n\,H} &= \sum_i h_k^{n,i} \sum_j s_k^{i,j} \left(h_k^{n,j}\right)^*, \\ &\leq \sum_i \left|h_k^{n,i}\right| \sqrt{s_k^i} \sum_j \left|h_k^{n,j}\right| \sqrt{s_k^j}, \\ &= \left(\sum_i \left|h_k^{n,i}\right| \sqrt{s_k^i}\right)^2. \end{aligned}$$

Combining this with (5) and (2) yields

$$b_k^n \leq \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \left|h_k^{n,n}\right|^2 \left(\sqrt{s_k^n} + \alpha_k \sum_{m \neq n} \sqrt{s_k^m}\right)^2\right).$$

Combining this with (1) leads to (4), which completes the proof. ■

IV. DIAGONALIZING PRECODER

This section presents the diagonalizing precoder (DP), which, unlike the THP is linear, has a low complexity and requires transmitter side operations only. However, like the THP, this precoder operates close to the theoretical channel capacity. Prior to transmission the DP multiplies the true symbols $\widetilde{\mathbf{x}}_k \triangleq \left[\widetilde{x}_k^1, \ldots, \widetilde{x}_k^N\right]^T$, with a precoding matrix $\mathbf{P}_k$. Denote the transmitted symbols as $\mathbf{x}_k = \mathbf{P}_k \widetilde{\mathbf{x}}_k$. The DP is based on the channel diagonalizing criterion; after precoding, each user should see their own direct channel free from crosstalk. The DP precoding matrix is defined $\mathbf{P}_k \triangleq \beta_k^{-1} \mathbf{H}_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\}$, where $\text{diag}\{\gamma_1, \ldots, \gamma_N\}$ denotes the diagonal matrix with elements $\gamma_1, \ldots, \gamma_N$ along the main diagonal. Here the scaling factor is defined

$$\beta_k \triangleq \max_n \left\| \left[\mathbf{H}_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\}\right]_{\text{row}\, n} \right\|, \tag{6}$$

and is included to ensure compliance with the spectral masks is maintained after precoding[4]. That is, if the original signal $\widetilde{x}_k^n$ obeys the spectral mask, $\widetilde{s}_k^n \triangleq \mathcal{E}\left\{\left|\widetilde{x}_k^n\right|^2\right\} \leq s_k^{\text{mask}}$, $\forall n$, then the signal after precoding x_k^n will obey the spectral masks as well since

$$\begin{aligned} s_k^n &= \mathcal{E}\left\{\left|[\mathbf{P}_k \widetilde{\mathbf{x}}_k]_{\text{row}\, n}\right|^2\right\}, \\ &\leq \max_n \left\|[\mathbf{P}_k]_{\text{row}\, n}\right\| \cdot \max_n s_k^n, \\ &\leq s_k^{\text{mask}}. \end{aligned}$$

During transmission the predistortion introduced by the DP annihilates the crosstalk. The received vector is then

$$\begin{aligned} \mathbf{y}_k &= \mathbf{H}_k \mathbf{P}_k \widetilde{\mathbf{x}}_k + \mathbf{z}_k, \qquad (7)\\ &= \beta_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\} \widetilde{\mathbf{x}}_k + \mathbf{z}_k. \end{aligned}$$

Thus application of the DP diagonalizes the channel matrix. Each user experiences its direct channel, scaled by β_k and

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[3] Standardization groups are currently considering the deployment of VDSL2 at lengths greater than 1.2 km. However at such distances far-end crosstalk is no longer the dominant source of noise, and the benefits of far-end crosstalk precoding are reduced considerably.

[4] Note that using a zero-forcing criterion, as in [2], would lead to a highly sub-optimal design since the channel is not CWDD. In fact it can be shown that with a zero-forcing design the scaling factor causes all modems to see the channel of the worst line within the binder.

completely free from interference. RWDD in the crosstalk channel matrix implies that $\beta_k \simeq 1$. As a result, each user operates close to its single-user bound, and the DP is near-optimal[5]. This observation is made rigorous through the following theorem.

Theorem 2: If $A_{\text{min}}^{(m)} \geq \alpha_k m B_{\text{max}}^{(m)},\ m = 1 \ldots N-1$; the data-rate achieved by the DP can be lower bounded by

$$b_k^n \geq \Delta \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \widetilde{s}_k^n \left|h_k^{n,n}\right|^2 f^{-1}(N, \alpha_k)\right), \qquad (8)$$

where

$$f(N,\alpha_k) \triangleq \left(\frac{A_{\text{max}}^{(N-1)}}{A_{\text{min}}^{(N)}}\right)^2 + (N-1)\left(\frac{B_{\text{max}}^{(N-1)}}{A_{\text{min}}^{(N)}}\right)^2,$$

$$\left[\begin{array}{c} A_{\text{max}}^{(m)} \\ B_{\text{max}}^{(m)} \end{array}\right] \triangleq \left(\prod_{i=1}^{m} \left[\begin{array}{cc} 1 & (i-1)\,\alpha_k \\ \alpha_k & (i-1)\,\alpha_k \end{array}\right]\right) \left[\begin{array}{c} 1 \\ 0 \end{array}\right],$$

and

$$A_{\text{min}}^{(m)} \triangleq 1 - \sum_{i=1}^{m} \alpha_k\,(i-1)\,B_{\text{max}}^{(i-1)}.$$

Proof of Theorem 2: Equation (7) implies that after application of the DP the signal at receiver n is $y_k^n = \beta_k^{-1} h_k^{n,n} \widetilde{x}_k^n + z_k^n$. Hence the received signal power for user n on tone k is $\beta_k^{-2} \widetilde{s}_k^n \left|h_k^{n,n}\right|^2$, the received interference power is zero, and the received noise power is $\sigma_{k,n}$. So the data-rate achieved by the diagonalizing precoder is

$$b_k^n(\widetilde{s}_k^n) = \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \beta_k^{-2} \widetilde{s}_k^n \left|h_k^{n,n}\right|^2\right). \qquad (9)$$

Define $\mathbb{A}^{(N)}$, the set of $N \times N$ diagonally dominant matrices, such that for any $\mathbf{A}^{(N)} \in \mathbb{A}^{(N)}$ it holds

$$\begin{array}{rcl} |a_{n,n}| & = & 1; \\ |a_{n,m}| & \leq & \alpha_k,\ \forall n \neq m; \end{array}$$

where $a_{n,m} \triangleq \left[\mathbf{A}^{(N)}\right]_{n,m}$. Define the matrix $\overline{\mathbf{G}}_k \triangleq [\overline{g}_k^{n,m}]$, where $\overline{g}_k^{n,m} \triangleq h_k^{n,m}/h_k^{n,n}$. Now $\mathbf{H}_k = \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\} \overline{\mathbf{G}}_k$, hence $\mathbf{H}_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\} = \overline{\mathbf{G}}_k^{-1}$. Since the transmitters are co-located at the CO, the DS channel is RWDD (2). This implies that $\overline{\mathbf{G}}_k \in \mathbb{A}^{(N)}$. Theorem 3 from [5] can be applied to bound the elements of $\overline{\mathbf{G}}_k^{-1}$ as follows

$$\left|\left[\overline{\mathbf{G}}_k^{-1}\right]_{n,m}\right| \leq \begin{cases} A_{\text{max}}^{(N-1)}/A_{\text{min}}^{(N)}, & n = m; \\ B_{\text{max}}^{(N-1)}/A_{\text{min}}^{(N)}, & n \neq m. \end{cases}$$

Combining this with (6) implies $\beta_k^2 \leq f(N, \alpha_{k,})$. Combining this with (9) leads to (8), which concludes the proof. ■

Note that with the THP the achievable data-rate is difficult to predict since it depends on the magnitude of the crosstalk channels. Crosstalk channels are not well understood, and actual channels can deviate significantly from the few empirical models that exist, see for example Fig. 2, making the service provisioning difficult. Using the bound (8) allows us to overcome this problem. The bound tells us that the actual crosstalk channel gain is not important as long as RWDD is observed. RWDD is a well understood phenomenon, and models of the degree of RWDD, α_k, are available based on extensive measurement campaigns. For example, the value for α_k from (3) is based on worst 1% case models, hence for 99% of lines α_k will be smaller and a data-rate above the bound (8) is achieved. The bound is therefore a useful tool not just for theoretical analysis, but for the provisioning of services as well.

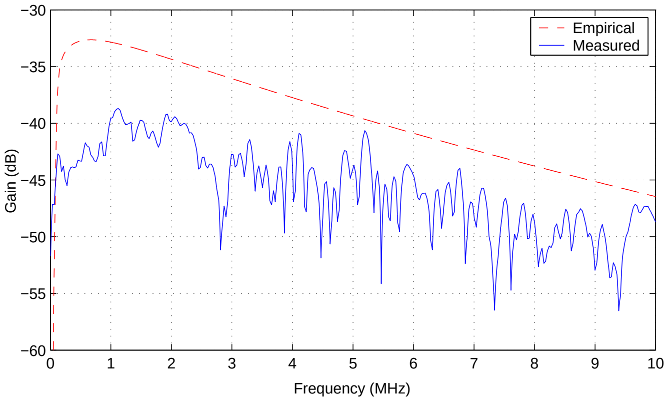

Fig. 2. Crosstalk Channel Transfer Functions (1 km cable, 0.5 mm pairs)

V. PERFORMANCE

This section evaluates the performance of the DP in a binder of 8 VDSL lines. The line lengths range from 150 m to 1200 m in 150 m increments. For all simulations the line diameter is 0.5 mm (24-AWG). Direct and crosstalk channel transfer functions are generated using semi-empirical models. The target symbol error probability is 10^{-7}, the coding gain set to 3 dB and the noise margin at 6 dB. As per the VDSL standards the tone-spacing Δ_f is set to 4.3125 kHz. The modems use 4096 tones, the 998 FDD bandplan, and a spectral mask s_k^{mask} set to -60 dBm/Hz. Background noise is generated using ETSI noise model A[3].

Fig. 3 shows the data-rate achieved on each of the lines with the different crosstalk precoding schemes. As can be seen, the DP achieves substantial gains, typically 30 Mbps or more, over conventional systems with no crosstalk precoding. The DP also achieves near-optimal performance, operating close to the single-user bound. This is a direct result of the RWDD of $\mathbf{H}_k$, which ensures that the scaling parameter β_k is always close to unity.

Fig. 4 shows the data-rate of the DP as a percentage of the single-user bound. Performance does not drop below 99% of the single-user bound. The lower bound on the performance of the DP (8) is also included for comparison. The bound is quite tight and guarantees that the DP will achieve at least 97% of the single-user bound.

VI. CONCLUSIONS

This paper investigated the design of crosstalk precoders for downstream VDSL. Existing designs have a high run-time complexity. A novel linear precoder, based on the channel

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[5] In this paper we only consider crosstalk precoding with a spectral mask, however it can also be shown that the DP is near-optimal when dynamic spectrum management is applied. Furthermore the DP decouples transmission on each line allowing the transmit spectra to be optimized with a much lower complexity.

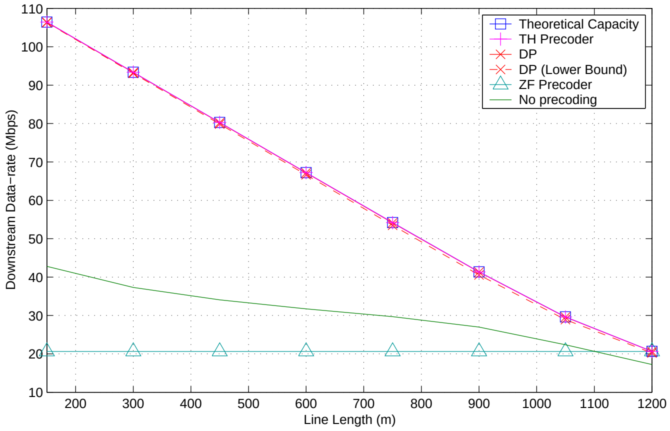

Fig. 3. Data-rate with Different Precoders

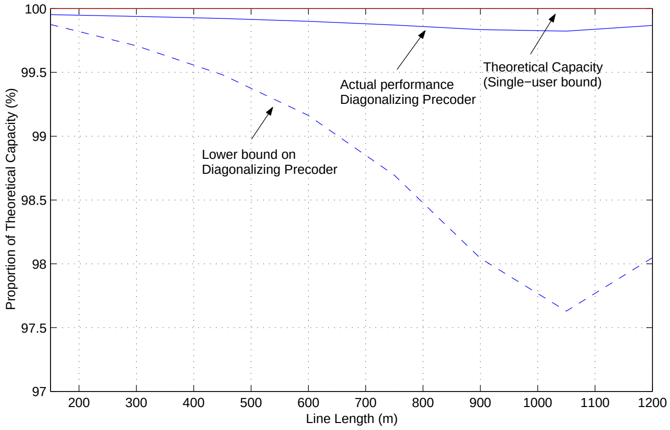

Fig. 4. Proportion of Theoretical Capacity Achieved by DP

diagonalizing criterion, is proposed. The precoder has a low complexity and does not require non-linear receiver-side operations. This is important since it helps keep the CP modem complexity as low as possible. A lower bound on the data-rate of the DP was derived which depends only on the direct channel gain and background noise. As a result the performance of the DP can be accurately predicted, simplifying service provisioning considerably. The bound also shows that the DP operates close to the theoretical channel capacity. So the DP is a low complexity design with guaranteed near-optimal performance.

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