Performance of Crosstalk Cancellation in VDSL

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***Abstract* — Crosstalk in DSL leads to significant performance degradation and large loses in data-rate. Several crosstalk cancellation techniques have been proposed to address this problem, however, in the existing literature the analysis of these approaches is based on SNR calculations and the SNR-gap approximation. Furthermore, for crosstalk cancellation techniques based on decision-feedback, the effect of error propagation is completely ignored. This makes it hard to predict the performance of crosstalk cancellation in real life, and to see if the significant potential gains can actually be realized. To address this problem, this paper uses Monte-Carlo simulation to investigate the performance of the various crosstalk cancellation techniques that have been proposed. The effect of noise-enhancement in zero-forcing crosstalk cancellers and error-propagation in decision-feedback cancellers is examined. The results confirm that a very simple crosstalk cancellation structure can achieve near-optimal performance.**

I. Introduction

Modern Digital Subscriber Line (DSL) systems use frequencies up to 12 MHz in transition. This leads to electromagnetic coupling between nearby twisted-pairs within the cable binder, an effect known as *crosstalk*. Crosstalk is a major source of performance degradation and significantly limits the data-rate and reach at which a DSL service may be provided[1].

Crosstalk between modems on the same end of the loop is referred to as near-end crosstalk (NEXT) whilst crosstalk on different ends of the loop is referred to as far-end crosstalk (FEXT). NEXT is typically avoided by using frequency division duplexing (FDD); however FEXT is still a major problem in most DSL systems. This is particularly true when one of the transmitters is located much closer to the receiving modems than all other transmitters. The crosstalk from this transmitter can often be stronger than the signals of interest on the other lines, leading to a total loss of service. This so-called *near-far problem* is particularly evident in upstream VDSL transmission when a customer premises (CP) modem is located further upstream of the other modems in the network.

Crosstalk cancellation has been proposed as one way of addressing the crosstalk problem. This technique is based on the concept of jointly processing the received signals of all lines in order to filter out the crosstalk whilst preserving the signal of interest[2].

In previous literature a decision-feedback crosstalk canceller was shown to achieve near-optimal performance[2]. However this analysis was based on the assumption of error-free detection and hence the effects of error propagation were not accounted for. More recent work showed that a simple linear ZF crosstalk canceller could achieve near-optimal performance[4]. However this work was based on an SNR-gap approximation, which may not accurately reflect real-life performance.

This paper uses Monte-Carlo simulation to investigate the performance, in terms of symbol-error rate, of the different proposed crosstalk cancellers. In particular, we are looking to confirm the conclusions made in previous analytical work, and make a specific study on the effects of noise-enhancement and error-propagation on crosstalk canceller performance.

The remainder of the paper is arranged as follows. Section II gives an overview of the system model. Section III presents the different crosstalk cancellation techniques. Section IV describes the performance, in terms of symbol-error rate, of the different crosstalk cancellers as we vary the disturbing modem's line length. Section V draws conclusions.

II. Channel Model

The channel model considered here is of the form

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k . \tag{1}$$

Here $\mathbf{x}_k = [x_k^1 \cdots x_k^N]^T$ is the vector of symbols transmitted on tone k, where x_k^n is the symbol transmitted by user n on tone k. Similarly $\mathbf{y}_k = [y_k^1 \cdots y_k^N]^T$ is the vector of symbols received on tone k. The vector $\mathbf{z}_k = [z_k^1 \cdots z_k^N]^T$ is the additive noise experienced by the receivers on tone k and incorporates radio frequency interference (RFI), thermal noise and alien crosstalk. We assume that $\mathbf{z}_k$ is white and Gaussian.

The crosstalk channel matrix is denoted $\mathbf{H}_k = [h_k^{n,m}]$. The diagonal element $h_k^{n,n}$ is the direct channel from transmitter n to receiver n, whilst the off-diagonal element $h_k^{n,m}$ is the crosstalk channel from transmitter m to receiver n.

In upstream transmission the receiving modems are co-located at a common central office. As a result the diagonal element of any column of the channel matrix $\mathbf{H}_k$ will have a much larger magnitude than the off-diagonal elements of that column, that is

$$\left|h_k^{n,n}\right| >> \left|h_k^{m,n}\right|, \forall m.$$

We refer to this as column-wise diagonal dominance (CWDD)[4].

III. Equalization Techniques

IIIA. Single User Detection

We begin by examining the performance of a conventional DSL modem which does not employ crosstalk cancellation. Each receiver first equalizes the received signal by dividing it by the direct channel gain

$$\hat{x}_k^n = y_k^n / h_k^{n,n}.$$

Since no cancellation is applied, the user will suffer the full effects of crosstalk.

IIIB. Zero Forcing Canceller

The zero forcing canceller estimates the transmitted symbols by multiplying the received symbol vector with the inverse of the channel matrix, hence

$$\hat{\mathbf{x}}_k = (\mathbf{H}_k)^{-1}\mathbf{y}_k.$$

The zero-forcing canceller has a linear design, which leads to a low run-time complexity and low-latency. One potential disadvantage of the zero-forcing approach is that it may lead to severe noise-enhancement if the crosstalk channel matrix is poorly conditioned.

Thankfully, in DSL channels CWDD has been shown to ensure a well conditioned channel matrix, leading to near-optimal performance of the zero-forcing canceller[4]. This theoretical result is confirmed through our simulations in Section IV where we see that the zero-forcing canceller achieves near-optimal performance.

IIIC. Decision Feedback Canceller

Decision feedback equalization is traditionally used for canceling inter-symbol interference, however this approach has also been proposed for crosstalk cancellation in DSL[2].

The decision feedback canceller consists of a feed-forward and a feedback filter. The feed-forward filter converts the crosstalk channel matrix into one that is upper triangular, and hence the crosstalk obeys a form of causality, in the sense that each user only experiences crosstalk from previous users. This allows decision feedback to be used to detect each of the users in turn, before subtracting the crosstalk they cause to the remaining undetected users[2].

In practice this is implemented through a QR decomposition of the crosstalk channel matrix

$$\mathbf{H}_k = \mathbf{Q}_k\mathbf{R}_k.$$

Here $\mathbf{Q}_k$ is a unitary matrix, whilst $\mathbf{R}_k$ is upper triangular.

The matrix $\mathbf{Q}_k^H$ forms the feed-forward filter which transforms the received vector of (1) to

$$\mathbf{w}_k = \mathbf{Q}_k^H\mathbf{y}_k = \mathbf{R}_k\mathbf{x}_k + \mathbf{Q}_k^H\mathbf{z}_k.$$

Since $\mathbf{Q}_k$ is unitary, the feed-forward filter does not alter the statistics of the noise $\mathbf{z}_k$, which we assume to be spatially white. If the noise is spatially coloured then a pre-whitening operation needs to be integrated into the feed-forward filter. This is relatively straight-forward to implement in practice, and has no effect on complexity, so we continue under the assumption of spatially white noise.

Now that the channel has been converted into an upper-triangular matrix $\mathbf{R}_k$, decision feedback can be applied to cancel the remaining crosstalk. The estimate for user n is formed by subtracting the crosstalk components of the previously detected users

$$\hat{x}_k^n = \text{dec}\left[\frac{w_k^n}{r_k^{n,n}} - \sum_{m=n+1}^{N} \frac{r_k^{n,m}}{r_k^{n,n}} \hat{x}_k^m\right],$$

where $w_k^n = [\mathbf{w}_k]_n$ and $r_k^{n,m} = [\mathbf{R}_k]_{n,m}$.

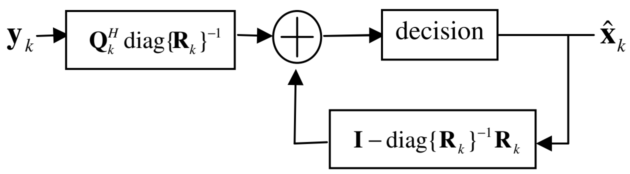

Figure 1: Decision feedback equalizer

The decision feedback canceller will perform very close to the theoretical channel capacity provided that the previously detected users have been detected error-free[2]. In practice this is not the case, and each user will experience errors due to the noise within the channel. When a user is erroneously detected, the decision feedback operation will actually create more crosstalk, leading to error-propagation and a significant reduction in performance.

An important contribution of this work is to examine the effect of decision errors in the decision feedback canceller. This is something that has been ignored in prior work that is based on the assumption of error-free detection, an assumption that is of course invalid in practice[2].

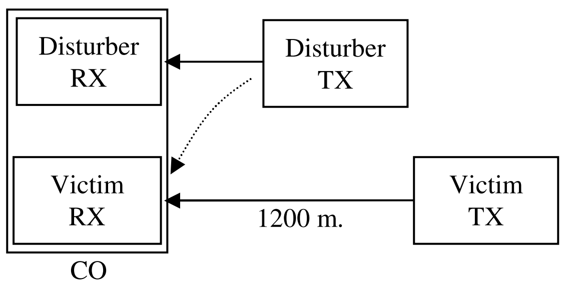

Figure 2: Upstream VDSL simulation environment

IV. Performance

The performance of single user detection, zero forcing cancellation and decision feedback cancellation will be compared in this section. The simulation scenario is depicted in Fig. 2. The victim line was fixed at a length of 1200 m. whilst the length of the disturbing line was allowed to vary from 100 m. to 1200 m.

The symbol error rate was calculated on tone 1205, which corresponds to the highest tone in upstream band 1 under the 998 FDD band plan[3]. In simulation we assume a line diameter of 0.5 mm (24 AWG). Empirical models were used to generate the direct and crosstalk channels[3]. The AWGN was assumed to have a PSD of −133 dBm/Hz for all tones and lines. Tone spacing was set to 4.3125 kHz and the DMT symbol-rate was set to 4 kHz as per VDSL standards[3].

The symbol error rate (SER) on the 1200 m. victim line is shown in Fig. 3 as we vary the length of the disturber's line. In the case of single-user detection the SER is highest for short disturber line lengths. This is to be expect since for short disturber line lengths the near-far effect will be most severe, as the crosstalk signal travels only a short distance into the victim, completely dominating the signal of interest, which has already attenuated quite significantly over the 1200 m. line.

Clearly the application of crosstalk cancellation, either zero-forcing or decision feedback, leads to a significant reduction in SER, and brings the unencoded SER down to an acceptable level of $2x10^{-4}$, which corresponds to a coded symbol error rate of 10^{-7}. In practice this will allow many more tones to be used for transmission, increasing the overall data-rate. This corresponds quite nicely with the results seen in previous work, which predicted large data-rate gains from an analytical perspective[2][4]. In this paper we have confirmed these benefits through more accurate Monte-Carlo simulation.

It is interesting to note that both the zero-forcing and decision feedback cancellers operate very close to the crosstalk free bound, so both of these techniques exhibit near-optimal performance.

This confirms prior analytical work, which showed that the zero-forcing canceller causes negligible noise enhancement due to the CWDD nature of DSL channels.

It also reveals a previously unsuspected insight, that the performance of the decision feedback canceller is relatively unaffected by decision error propagation.

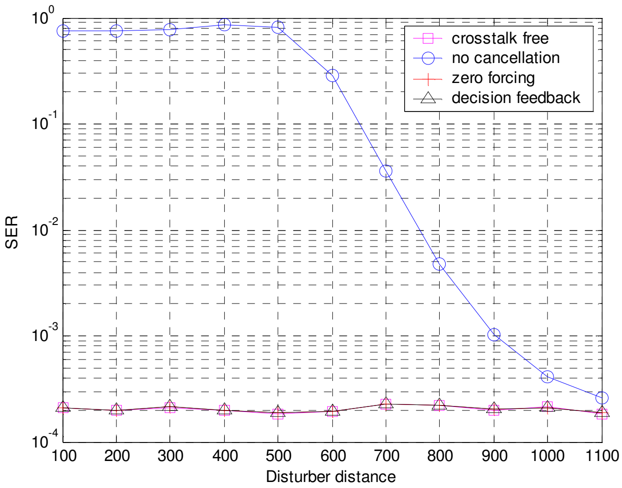

Figure 3: Crosstalk cancellation performance

Hence we have confirmed that the zero-forcing and decision feedback cancellers are simple, low complexity designs with near-optimal performance. This is an observation alluded to through analysis in previous work, and in this paper we see the same result, this time reinforced through numerical simulation.

V. Conclusions

This paper compared the performance of the zero forcing and decision feedback cancellers. Unlike previous work, a comparison was made on the basis of numerical simulation rather than through analysis alone. This allowed the effects of noise enhancement and decision error propagation to be evaluated directly, issues that were ignored in the analysis of previous work.

It was seen that both the zero-forcing and decision feedback cancellers achieve near-optimal performance. The zero-forcing canceller has a lower run-time complexity and is preferable when the noise is spatially white. When strong alien crosstalk sources exist near the central office, the noise will be correlated between lines. In this case a noise pre-whitening operation must be performed. This noise pre-whitening often destroys the CWDD property of the crosstalk channel matrix, and the zero-forcing canceller then loses its near-optimal performance. In this case the decision feedback canceller is preferable. An important area for future work is a more detailed study into the effects of noise correlation on crosstalk canceller performance.

References

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