Applying the Extended Mathews stability graph to stress relaxation, site-specific effects and narrow-vein stoping

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Abstract

The original Mathews method for predicting stope stability has been extended and now contains 483 open stoping and caving case histories over a wide range of geotechnical conditions and stope dimensions. The mathematical framework upon which the Extended Mathews stability graph is based and the large database has facilitated examination of a number of outstanding issues surrounding the application of empirical stability graphs. This paper summarises how the framework of the Extended Mathews stability graph framework has been applied to quantify the effect of stress relaxation upon excavation stability, examine site-specific effects and highlight the poor correlation between stability graph parameters for narrow stope stability.

Back-analysis of case studies where stope surfaces were relaxed has enabled the effect of stress relaxation upon excavation stability to be quantified and bounded. Detailed statistical analyses have demonstrated that a reliable stable-failure boundary requires at least 150 case histories, of which a minimum of 10% should be unstable stope surfaces. Marginal site-specific effects were observed for the operating conditions captured within the database. Apparent site-specific effects noted in previous literature were found to be attributable to operating conditions inadequately represented in the database. Statistical analysis of overbreak from 115 narrow-vein case studies has demonstrated that operating conditions in narrow-vein mines differ sufficiently to warrant changes in the model framework to account for undercutting of stope walls and drill and blast parameters. Backfill abutments were found to behave the same as solid rock abutments.

INTRODUCTION

The Extended Mathews stability graph (Mawdesley et al. 2001; Trueman & Mawdesley 2003) is a variant of the original Mathews stability graph method (Mathews et al. 1981). In addition to the prediction of open stope stability, the Extended Mathews stability graph method can be used to predict rock mass cavability (Trueman & Mawdesley 2003). For a description of the use and development of the Extended Mathews method, refer to the paper by Mawdesley and Trueman in these proceedings.

As part of the second Australian Minerals Industry Research Association (AMIRA) Blasting and Reinforcement Technology (BARTII) Project, the Extended Mathews method was assessed for its suitability with regards to narrow-vein stope stability. As an outcome of this investigation, a number of outstanding issues pertaining to the application of stability graphs were addressed. Concerned about the applicability of existing stability graphs to narrow-vein stoping, several narrow-vein operations expressed interest in developing their own site-specific stability graphs. Therefore, the requirements of site-specific stability graphs were addressed by determining the optimal number and type of case studies required.

Due to the large length to width aspect ratios of narrow stopes, they are prone to stress relaxation in both the hangingwall and footwall, especially if the principal stress direction is perpendicular to strike. For this reason the effect of stress relaxation on stope stability has been thoroughly investigated and a new set of guidelines for the treatment of different types of stress relaxation proposed. The suitability of existing stability graphs to narrow stoping was also assessed through the back-analysis of 115 new narrow-vein case studies. This study confirmed that existing stability graph parameters have the potential to be very poorly correlated to the stability of narrow stopes. Additional narrow stoping case studies are currently being analysed to examine the effect of: undercutting of stope walls, drill and blast parameters, stress damage, and moving backfill abutments. The aim is to develop a stope stability prediction tool specifically suited to narrow stoping operating conditions.

STRESS RELAXATION

One of the outstanding issues regarding the application of stability graphs to both open stope stability and cavability is the effect of stress relaxation. While several authors refer to the adverse effect of stress relaxation on excavation stability (Bawden 1993; Diederichs & Kaiser 1999; Kaiser et al. 2001; Kaiser et al. 1997; Suorineni et al. 2001), some empirical evidence suggests stress relaxation may not have a significant effect on excavation stability (Potvin 1988; Tyler & Trueman 1993; Wang et al. 2002). Based upon back analysis of relaxed case studies, both Mathews et al. (1981) and Potvin (1988) recommend that the stress factor A should be set equal to one, implying that stress relaxation does not significantly affect stope stability. Since Potvin's and Mathews' original recommendation, Diederichs and Kaiser (1999) have proposed an adjustment to stress factor A for cases where the minimum principal stress is less than zero. Therefore, the literature appears to give conflicting recommendations for the treatment of stress relaxation in terms of stability graphs. An investigation was undertaken to evaluate and review both approaches to determining stress factor A (Stewart & Trueman 2003A).

Three-dimensional analysis of stress relaxation

Stewart and Trueman (2003A) investigated whether the apparent discrepancies in the literature could be attributed to the influence of the intermediate principal stress and/or minimum principal stress direction. Therefore, a three-dimensional method of analysis was required that could consider whether misclassification of failed stopes could be due to a particular type of stress relaxation. Linear elastic stresses were evaluated for 55 case studies using both two and three dimensional elastic stress models. The relaxation case studies analysed were collated from the literature (Mathews et al. 1981; Potvin 1988; Pakalnis 1986; Pine et al. 1992; Pakalnis et al. 1991; Dunne & Pakalnis 1996).

Stewart and Trueman (2003A) analysed the following three types of stress relaxation: partial relaxation, full relaxation and tangential relaxation. Partial relaxation was defined as excavation surfaces where linear elastic three-dimensional modelling of $\sigma3$ is less than 0.2 MPa, while $\sigma2$ and $\sigma1$ both exceed 0.2 MPa. Suorineni (1998) determined that 0.2 MPa was the critical upper range of stress that would clamp rock wedges. Full relaxation was defined as excavation surfaces where linear elastic three-dimensional model estimates of $\sigma3$ and $\sigma2$ are both less than 0.2 MPa. Tangential relaxation was defined as excavation surfaces where at least one of the modelled principal stresses is less than 0.2 MPa and the associated stress direction diverges less than 20 degrees from parallel to the excavation wall in a three-dimensional analysis. It is important to note that in the case of three-dimensional analysis, the angle between the associated stress direction and the stope surface must be determined both with respect to the stope surface dip and the stope surface strike. If the angle subtended by the stress direction and the stope surface dip or strike is less than 20 degrees, then this stress direction was considered tangential.

Effect of different types of stress relaxation

Misclassification statistics (sensitivity and specificity) were used within the framework of the Extended Mathews stability graph (Mawdesley 2002) to investigate the effect of partial relaxation, full relaxation and tangential relaxation. Sensitivity is defined as the probability that a true case will be correctly classified (Parker & Davis 1999). Therefore, with respect to the Extended Mathews stable-failure boundary, sensitivity refers to the proportion of stable case studies that correctly plot above the stable-failure boundary. Conversely, specificity is the probability that an unstable case study will correctly plot below the stable-failure boundary. Parker and Davis (1999) define the sum of sensitivity and specificity as the accuracy of the test classification.

The misclassification statistics contained in Table 1 were obtained with stress factor A equal to one. The misclassification statistics for partial relaxation were found to be very similar to those obtained for the non-relaxed Extended Mathews case studies (Stewart & Trueman 2003A). This suggests partial relaxation does not affect nor cause excavation instability. By contrast, a low specificity was obtained for the fully relaxed cases, which indicates that full relaxation has an adverse impact upon excavation stability. The practical consequence of this result is that in cases of full relaxation, where at least two principal stresses are less than 0.2 MPa, existing stability graphs will frequently incorrectly predict a stable condition. Similarly, poor specificity was obtained for cases of tangential relaxation.

Table 1 Misclassification statistics for different types of stress relaxation

Type of relaxation	*Sensitivity*	*Specificity*	*Accuracy*
Not relaxed (Extended Mathews database)	81.3%	83.6%	1.65
Partial stress relaxation	83.3%	79.1%	1.62
Full stress relaxation	90.9%	44.4%	1.35
Tangential stress relaxation	85.7%	45.5%	1.31

Review of existing methods of quantifying stress relaxation

Diederichs and Kaiser (1999) used voussoir beam theory to develop a theoretical mechanistic model to examine the effect of stress relaxation on excavation stability and proposed the following adjustment for stress factor in cases of stress relaxation:

$$A = 0.9e^{11(\sigma_T/UCS)} \quad \sigma T<0 \qquad (1)$$

Stewart and Trueman (2003A) back-analysed 55 relaxed case studies to compare Diederichs' and Kaiser's relationship with the original Mathews method of assessing the impact of stress relaxation. Misclassification statistics were used to evaluate each method for the three types of relaxation. The results of the evaluation are contained in Table 2. If a two-dimensional stress analysis is used to estimate the induced stress as implied by Diederichs and Kaiser, the original Mathews method was found to be more accurate than Diederichs' and Kaiser's relationship.

Table 2 Misclassification statistics for existing methods of quantifying the effect of stress relaxation

Method to quantify effect of stress relaxation	**Partial relaxation**		**Full relaxation**	**Tangential relaxation**
A=1				
Sensitivity	90%		90.9%	85.7%
Specificity	92.3%		44.4%	45.5%
Accuracy	1.82		1.35	1.31
	Map3D	Phase2	Map3D	Phase2
$A= 0.9e^{11(\sigma_T/UCS)}$				
Sensitivity	88.9%	0 %	90%	90%
Specificity	85.7%	94.1 %	45.5%	44.4%
Accuracy	1.74	0.94	1.36	1.34

New adjustment to stress factor A

The poor specificity obtained for cases of full and tangential relaxation has led to the development of new adjustments for these types of stress relaxation. Three separate approaches were examined. The first approach was to empirically develop an adjustment based upon the hypothesis that the adjustment magnitude would be related to the normalised tensile stress, where the tensile stress is normalised with respect to uniaxial compressive strength (Stewart & Trueman 2003A). The second approach was to evaluate the Hoek-Brown tensile failure criterion as a predictor of instability (Stewart & Trueman 2003A). The final approach was to empirically determine an adjustment to the stress adjustment factor A by optimising the accuracy of the stable-failure boundary for cases of tangential and full relaxation. The latter approach was found to be the most successful.

In the case of both full relaxation and tangential relaxation, experimentation with a series of adjustments resulted in an optimal stability prediction (highest accuracy) being achieved when A is assigned a value of 0.7. Table 3 compares the misclassification statistics obtained for each type of stress relaxation when A is set to 0.7 and when A is set to one. From the results of those analyses Stewart and Trueman (2003A) recommended that in cases of full and tangential relaxation, the stress factor A should be set to 0.7 to account for the destabilising effect of these types of stress relaxation. In the case of partial relaxation, no adjustment was required; i.e. a stress reduction factor, A, equal to one gives the best accuracy. These recommendations are based upon three-dimensional linear elastic modelling of induced stresses where the induced stress was taken at the mid-point of the excavation wall. The reason for using three-dimensional stress modelling is that in cases where the stope length to width aspect ratio is less than five, a two-dimensional stress model may predict that the stope surface is relaxed, but in actuality it is not. While these adjustments were developed within the Extended Mathews stability graph framework, there is no apparent reason why these adjustments would not be applicable to the Modified stability graph (Potvin 1988) and the ELOS dilution graph (Clark & Pakalnis 1997). These new guidelines are also applicable to stress relaxation in block or panel caves when predicting cavability using the Extended Mathews method.

Table 3 Factor A and misclassification statistics

	Misclassification statistics			
Factor A		Partial relaxation	Full relaxation	Tangential relaxation
1	Sensitivity	90.0%	90.9%	85.7%
	Specificity	92.3%	44.4%	45.5%
	Accuracy	1.82	1.35	1.31
0.7	Sensitivity	80.0%	81.8%	78.6%
	Specificity	92.3%	88.9%	81.8%
	Accuracy	1.72	1.71	1.60

SITE-SPECIFIC GRAPHS

Some concern has been expressed by a number of authors about the general applicability of the Mathews method (Stewart & Forsyth 1995; Bawden 1993; Trueman et al. 2000). Stewart and Forsyth (1995), whilst acknowledging that there are some indications that the method may be generally applied, emphasised the potential bias in the limited database and recommended users concentrate on collecting sufficient stoping case histories to define their own stability zones. Bawden (1993) also suggested that the analysis of stable versus failed stopes can be used to derive a stability boundary for a particular operation. From their experience of back-analysing a large database from the Mount Charlotte gold mine in Western Australia, Trueman et al. (2000) concluded that the model gave reasonable predictions of stope surface stability, at least for steeply dipping deposits in moderately good to good quality rock. Nevertheless, Trueman et al. (2000) followed the guidelines of Stewart and Forsyth (1995) and developed a site-specific graph for Mount Charlotte. There has been significant interest in the development of site-specific stability graphs within the Australian metalliferous mining industry in general and particularly for narrow-vein operations. The question of the data requirements for a reliable site-specific stability graph has been addressed by Stewart and Trueman (2001A).

Data requirements and site-specific effects

Stewart and Trueman (2001A) developed a new analytical mathematical technique to address the issue of how many and what type of case studies are required to develop a reliable site-specific stability graph. A new technique was required because in the case of logistic regression there is no agreed upon technique to quantify the quality of the logistic regression boundary (Whiten 2001). Logistic regression does not have a technique equivalent to the analysis of variance approach that would be used for a least squares regression model (Devore 1991). For this reason, the authors have developed a new approach that analyses the effect of changing database parameters, such as the number of case histories, on the variance of model parameters.

The analysis of variance in logit model parameters indicated that approximately 150 case histories are required to minimise variance in model parameters of which at least 10% need to be of a different stability classification. These requirements can be used as a guide for developing site-specific stable-failure boundaries and as a check that sufficient case histories have been collected for a generic model.

An assessment was also made of how site specific the Extended Mathews method is (Stewart & Trueman 2001A). Only marginal site-specific effects were observed for the operating conditions captured within the database. Stewart and Trueman concluded that the apparent site-specific effects noted in previous literature are attributable to operating conditions inadequately represented in the database. Such operating conditions could induce erroneous stability predictions at any site, and are therefore not truly site-specific.

NARROW STOPE STABILITY MODEL

There are a number of operating conditions that make the use of existing empirical models problematic for narrow open stope design. A number of operating conditions have been hypothesised to have a significantly larger influence upon narrow stope stability than on large open stope stability. These operating conditions include the effect of undercut stope walls, drill and blast parameters, backfill abutments, stress relaxation, stress damage to stope walls associated with the retreating brow as well as irregular stope geometry (Stewart 2003).

In an effort to address the issue of narrow stope operating conditions, some sites have attempted or considered developing their own stability graphs. However, the time required to collate 150 case studies for a reliable site-specific stability graph would result in a substantial delay. Furthermore, at the feasibility stage, estimates of dilution are frequently required prior to the possibility of developing a site-specific stability graph. To overcome these issues, Stewart (2003) is currently developing a narrow stope stability model to take into account operating conditions that are considered to be specifically relevant to narrow stope stability. A number of operating conditions specific to narrow stoping have been investigated through the back analysis of 115 narrow-vein case studies.

Narrow-vein case studies

A back-analysis was carried out of 115 narrow-vein (74 stable and 41 failures) stope surfaces from the Barkers ore body at the Kundana Gold Mine, firstly in terms of stability graph parameters, stability number and shape factor, and then secondly, using comparative statistics to evaluate the significance of drill and blast patterns, the number of backfilled abutments, undercutting of stope walls and indirectly the effect of drill-hole accuracy on overbreak (Stewart & Trueman 2001B; Stewart & Trueman 2003B).

The analysis confirmed the hypothesis that the stability of narrow stopes is poorly predicted by existing stability chart parameters. A poor correlation was found between stope stability and both the Mathews stability number, N, and hydraulic radius, HR (see Figure 1a and Figure 1b). Given that both the stability number, N and the shape factor S correlate well with stability in the vast majority of Mathews method case histories, this suggests there is an overriding influence on stability at Barkers not accounted for in the Mathews method.

Figure 1a Barkers case studies plotted on Extended Mathews stability graph with failure defined as overbreak exceeding 1 m

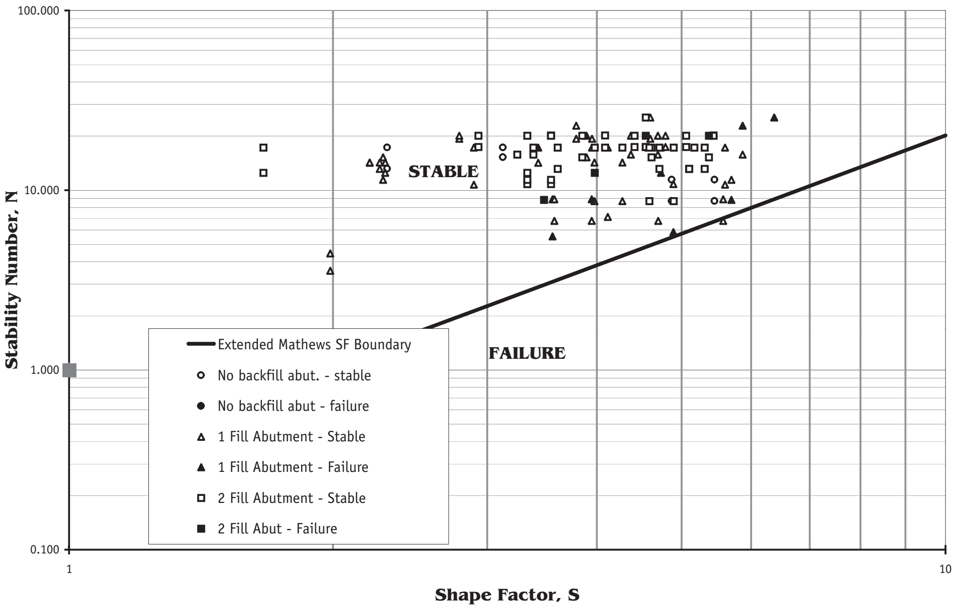
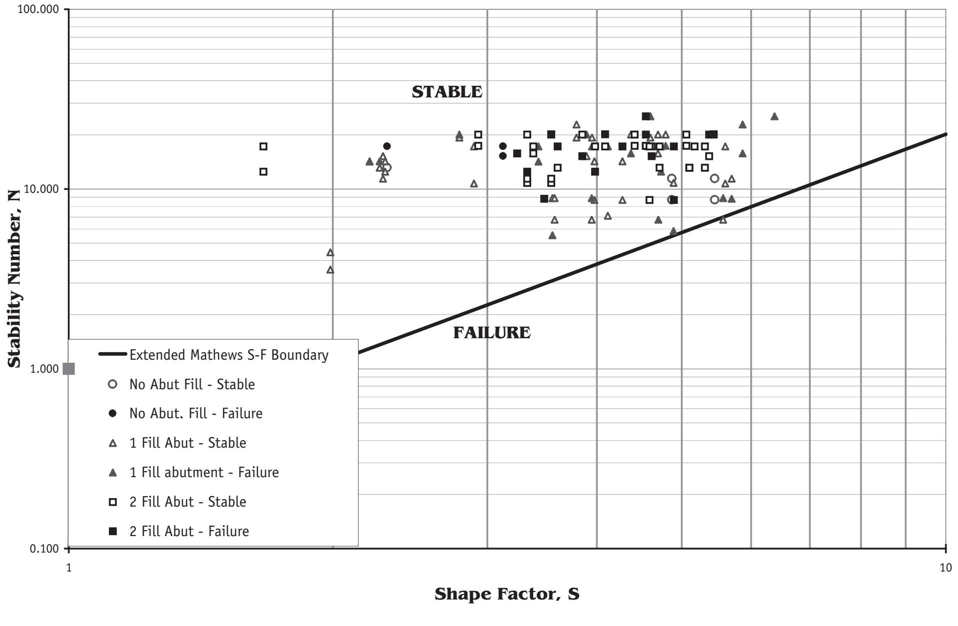


Figure 1b Barkers case studies plotted on Extended Mathews stability graph with failure defined as overbreak exceeding 0.5 m



Blast pattern was found to have a statistically significant effect on overbreak. In terms of the drill and blast patterns used at the mine, the 'in-line 3' pattern performed significantly better than both the 'staggered' and 'dice 5' patterns for the vein geometries at the time. Undercut footwalls were found to behave in a similar manner to non-undercut hangingwalls. There was no evidence that backfill abutments behave differently from solid rock abutments in terms of determination of stable stope dimensions. Drill-hole accuracy was indirectly examined by considering the effect of stope heights within the limits of 13 to 20 metres. Within these limits, stope height did not affect the magnitude of overbreak.

In summary, the Barkers case study highlighted the need for additional parameters to be taken into account when attempting to predict narrow stope stability. In particular, the potential for drill and blast parameters to significantly affect dilution was clearly evident.

CONCLUSIONS

New data has been added to the Mathews database to extend it to 483 case histories. Logistic regression was used to optimise the placement of stability zones.

Three-dimensional linear elastic modelling was used to identify and define three types of stress relaxation: partial, full and tangential relaxation. Empirical back-analysis of 55 case studies indicated that in the case of full and tangential relaxation (as determined from three-dimensional linear elastic modelling), applying a stress factor equal to 0.7 significantly improved the predictive ability of the stability graph approach.

It has been demonstrated that 150 case studies, of which at least 10% lie on the other side of the boundary, are required to define a reliable stable-failure boundary. Only marginal site-specific effects were observed for the operating conditions captured within the database.

Concerns about the applicability of the existing stability graphs to narrow stopes led to the current development of an operating condition specific stability model for the design of narrow stopes. This was deemed a practical solution to the delays associated with collating sufficient data for site-specific stability graphs. Back-analysis of stope stability at a narrow-vein gold mine showed poor correlation to the Extended Mathews stability graph parameters (stability number and hydraulic radius), thereby supporting the hypothesis that existing stability graphs may be unreliable for the prediction of narrow stope stability.

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