CRACK IDENTIFICATION IN HOLLOW SECTION STRUCTURES THROUGH COUPLED RESPONSE MEASUREMENTS

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ABSTRACT

This paper present a feasible method for crack identification in hollow section structures based on the coupling vibration behaviour of cracked members. From the last several decades lots of techniques have been developed by many researchers to detect, locate and quantify damage by using changes to modal parameters such as natural frequency, mode shape or damping ratio. However, these approaches suffer from the fact that the structural damage has a low sensitivity to changes in these parameters.

A related option is offered through coupled response measurements. A transverse surface crack is well known to produce local flexibility due to the stress-strain singularity in the vicinity of the crack tip. The local flexibility can be represented by the way of a 6×6 matrix for a beam element that includes the crack. This matrix contains off-diagonal terms that cause coupling response along the direction corresponding to these terms. This coupling property due to the crack is evidence of the existence of the cracks.

Coupled response of cracked hollow section structures was studied. Hollow section structures demonstrate a more pronounced coupling when cracks occur. In this paper, both an analytical simulation and the early results of experimental implementation are presented. This method is fairly discriminative even for small cracks.

NOMENCLATURE

E	Young's modulus
I	Moment of inertia
A	Cross sectional Area
ρ	Density of the material
ν	Poisson's ratio
U_T	Strain energy
$J(a)$	Strain energy density function
C	Local flexibility matrix
K_{mn}	Stress intensity factor
$U_i(x,t)$	Axial vibration
$V_i(x,t)$	Lateral vibration
Q	System matrix

1. INTRODUCTION

Detection and control of damage in mechanical structures such as found in large mining machinery is an important concern to mine operators.

Among many possible damage identification methods, vibration measurements offer the potential to be an effective, inexpensive and fast tool for nondestructive testing. During the past several decades, significant amount of research has been conducted in the area of vibration-based damage identification. The main idea under this approach is that a change in a system due to damage will manifest itself as changes in the structural dynamic characteristics.

Reviews on vibration of cracked structures were reported by Dimarogonas [1], Wauer [2] and Doebling *et al* [3]. Many identification techniques have been proposed based on different selected parameters. Some authors used the change of natural frequencies [4-6] or mode shapes [7-8] as the indicator of damages while others detected structural damage directly from dynamic response in time domain or from Frequency Response Functions [9].

Despite a certain degree of success with these techniques, one common practical problem still is the sensitivity of the selected parameters to damage.

In this paper, we studied local parameters rather than those for the whole system and looked at the coupling property of the cracked member instead of just the quantitative change of parameters. It is demonstrated that this method may have a sufficiently high sensitivity to the presence of cracks.

The key idea is to model the crack section by using a local flexibility matrix, which sets up the relationship between the displacements and forces. The local flexibility matrix can be formulated from the stress intensity factors of the structure using a fracture mechanics approach. This formulation depends on the crack orientation and magnitude.

Generally, for uncracked members the local flexibility matrix is diagonal. In the presence of a crack, , some off-diagonal terms become nonzero. This means excitation along one direction (eg lateral) will cause response along other corresponding directions (eg Axial).

The simple case of local flexibility was studied by Irwin [10] for beams and by Rice and Levy [11] for plates, who related the flexibility to stress intensity factors. Papadoupolos and Dimarogonas [12] presented general picture of coupled vibration on a cracked shaft.

In this paper, a Circular Hollow Section (CHS) member is studied. After first deriving the local flexibility matrix, we then present an analytical simulation of free and forced vibration of cracked CHS beam and demonstrate crack identification by using the proposed coupling property. The results for different crack severities and locations are compared. Finally, the preliminary test results are presented that prove the feasibility of this approach. Full analysis of the experimental results was not complete at the time of writing this paper but they will be presented at the Conference.

2. LOCAL FLEXIBILITY MATRIX OF A CRACKED STRUCTURE MEMBER

A crack on a structural member introduces additional local flexibility, which is a function of the crack depth (severity) and location. This flexibility changes the dynamic behaviour of the system. In order to understand the effect of a crack upon the dynamic response of an elastic structure and furthermore to identify the cracks, one has to first establish the local stiffness or flexibility matrix of the crack member under general loading.

In general, the local flexibility for a beam can be described by the way of a local flexibility matrix, the dimension of which depends on the number of the degrees of freedom being considered, maximum 6×6.The coordinate system and the corresponding forces are shown in Figure 1. Here we use subscript 1 for the longitudinal coordinate,2 and 3 for the shear directions, 4 and 5 for bending moment and 6 for torsional degree of freedom. Using this flexibility equation, the extra displacement along any degree of freedom due to the presence of the crack is given by the following equation:

$$\vec{u} = [C]\vec{P} \tag{1}$$

where $\vec{u}$ and $\vec{P}$ are displacement and force vectors and C is the crack flexibility matrix: $\vec{u} \in R^{6\times1}$; $C \in R^{6\times6}$; $\vec{P} \in R^{6\times1}$.

The displacement u_i along the force component P_i due to the presence of the crack will be computed using Castigliano's theorem (Energy method).

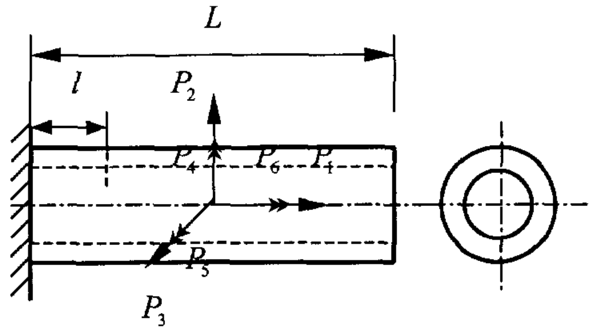

Figure 1: CHS Beam Under General Loading

If U_T is the strain energy due to a crack, the additional displacement u_i is defined as:

$$u_i = \partial U_T / \partial P_i \tag{2}$$

The strain energy has the following form:

$$U_T = \int_0^a \frac{\partial U_T}{\partial a} da = \int_0^a J(a) da \tag{3}$$

where $J(a)$ is strain energy density function, a is the crack length. Therefore:

$$u_i = \frac{\partial}{\partial P_i}\left[\int_0^a J(a) da\right] \tag{4}$$

The flexibility influence coefficient c_{ij} will be:

$$c_{ij} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j}\left[\int_0^a J(a) da\right] \tag{5}$$

From fracture mechanics the strain energy density function $J(a)$ has the general form:

$$J(a) = \frac{1}{E'}\left[\left(\sum_{l=1}^{6} K_{II}\right)^2 + \left(\sum_{l=1}^{6} K_{III}\right)^2 + \alpha\left(\sum_{l=1}^{6} K_{IIII}\right)^2\right] \tag{6}$$

Where $E^{'} = E$ for plane stress, $E^{'} = E/(1-\nu^2)$ for plane strain; $\alpha = 1+\nu$; E and ν are Young's modulus and Poisson's ratio, respectively. Then, combining the equations (5) and (6):

$$c_{ij} = \frac{1}{E^{'}}\int_0^a \left[\frac{\partial^2}{\partial P_i \partial P_j}\sum_{m=I}^{III}\left(e_m \sum_{n=1}^{6} K_{mn}\right)^2\right]da \tag{7}$$

Where $e_m = 1$ for $m = I, II$ and $e_m = \alpha$ for $m = III$; K_{mn} is the stress intensity factor of mode $m (m = I, II, III)$ due to the load $P_n (n = 1,2,\ldots 6)$.

From (7), one can judge the existence of coupling between any considered coordinates by K_{mn}. If some of the loads contribute to the same fracture mode, for example, beam under extension and bending both create tensile stress and contribute to the mode I of stress intensity factor (i.e. $K_{I1} \neq 0, K_{I5} \neq 0$), the corresponding flexibility element would be nonzero (i.e. $c_{15} \neq 0$).

Eventually, the local flexibility matrix for the beam due to the crack will have the following form:

$$C = \begin{bmatrix} c_{11} & 0 & 0 & c_{14} & c_{15} & 0 \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} \\ 0 & 0 & c_{33} & 0 & 0 & c_{36} \\ c_{41} & 0 & 0 & c_{44} & c_{45} & 0 \\ c_{51} & 0 & 0 & c_{54} & c_{55} & 0 \\ 0 & c_{62} & c_{63} & 0 & 0 & c_{66} \end{bmatrix} \tag{8}$$

This matrix relates the displacement vector $\{u\}$ to the corresponding force vector $\{P\}$ through (1).

By inversion of this local flexibility matrix we can obtain the local stiffness matrix:

$$K = C^{-1} \tag{9}$$

Due to reciprocity, the matrix C and K are symmetric for an uncracked beam. The nondiagonal terms of the matrix C show that coupling exists between longitudinal, bending and torsional vibrations because of the crack. This coupling property is a practical indicator to identify the cracks in the structure.

In the following analysis the qualitative and quantitative coupled longitudinal and bending vibration of a cracked hollow section beam will be presented. Firstly, the stress intensity factors of a hollow section beam with circumferential through-wall crack will be introduced. Then the free vibration of a cracked beam will be analysed to examine the change of natural frequencies and mode shapes. Furthermore, the coupling property of cracked beam will be shown by the frequency response functions of the system that can be derived from forced vibration analysis.

3. STRESS INTENSITY FACTORS

For the expression of the local flexibility, the crucial element is the availability of the relevant stress intensity factors. To this end, we consider the coupling between longitudinal and bending vibration. The dimension of the matrix C and K are 2×2 and can be expressed as:

$$C = \begin{bmatrix} c_{11} & c_{15} \\ c_{51} & c_{55} \end{bmatrix} \qquad K = \begin{bmatrix} k_{11} & k_{15} \\ k_{51} & k_{55} \end{bmatrix} \tag{10}$$

Using fracture mechanics[15] principles, the stress intensity factors for circumferential through-wall crack in cylinders can be expressed as follows:

Axial force P_1:

$$K_{I1} = \frac{P_1}{2\pi R t}\sqrt{\pi R \theta}F_t \tag{11}$$

Where $R = (R_o + R_i)/2$ is the mean radius; and θ is the half angle of the total through-wall crack and:

$$F_t = 1 + A_t\left[5.3303\left(\frac{\theta}{\pi}\right)^{1.5} + 18.773\left(\frac{\theta}{\pi}\right)^{4.24}\right]$$

$$A_t = \left(0.125\frac{R}{t} - 0.25\right)^{0.25} \qquad \text{For} \qquad 5 \leq \frac{R}{t} \leq 10$$

$$A_t = \left(0.4\frac{R}{t} - 3.0\right)^{0.25} \qquad \text{For} \qquad 10 \prec \frac{R}{t} \leq 20$$

Bending moment P_5:

$$K_{I5} = \frac{P_5}{\pi R^2 t}\sqrt{\pi R \theta}F_b \tag{12}$$

Where

$$F_b = 1 + A_t\left[4.5967\left(\frac{\theta}{\pi}\right)^{1.5} + 2.6422\left(\frac{\theta}{\pi}\right)^{4.24}\right]$$

A_t is same as above.

By substituting K_{mn} to equation (7) one can obtain matrix C and K.

4. FREE VIBRATION OF A CRACKED CHS BEAM

A cantilever Euler-Bemoulli CHS Beam is considered. The system is described by following differential equations:

Axial vibration: $$\frac{\partial^2 U_i}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 U_i}{\partial t^2} \quad (13)$$

Lateral vibration: $$\frac{\partial^4 V_i}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 V_i}{\partial t^2} = 0 \quad (14)$$

Where $i=1$ for the section left to the crack ($x \le l$) and $i=2$ for the section right to the crack ($l \le x \le L$).

The solutions to equations (13)----(14) can be described by formulas (15)----(18) with separate variables, where ω is the natural frequency of the system, A_u, B_u, A_v, B_v are unknown coefficients, which can be determined from the initial conditions.

$$U_1(x,t) = u_1(x)(A_u \cos\omega t + B_u \sin\omega t) \quad (15)$$
$$U_2(x,t) = u_2(x)(A_u \cos\omega t + B_u \sin\omega t) \quad (16)$$
$$V_1(x,t) = v_1(x)(A_v \cos\omega t + B_v \sin\omega t) \quad (17)$$
$$V_2(x,t) = v_2(x)(A_v \cos\omega t + B_v \sin\omega t) \quad (18)$$

By substitution of equations (15)----(18) into equations (13)----(14) and separating variables we obtain the governing equations for the spatial variable x:

$$\frac{\partial^2 u_1}{\partial x^2} + k_u^2 u_1 = 0 \quad (19)$$
$$\frac{\partial^2 u_2}{\partial x^2} + k_u^2 u_2 = 0 \quad (20)$$
$$\frac{\partial^4 v_1}{\partial x^4} - k_v^4 v_1 = 0 \quad (21)$$
$$\frac{\partial^4 v_2}{\partial x^4} - k_v^4 v_2 = 0 \quad (22)$$

Where $k_u = \sqrt{\frac{\rho}{E}}\omega$, $k_v = \left(\frac{\rho A \omega^2}{EI}\right)^{1/4}$.

Then the general solutions of equations (19)----(22) have the following form:

$$u_1(x) = A_1 \cos k_u x + A_2 \sin k_u x \quad (23)$$
$$u_2(x) = A_3 \cos k_u x + A_4 \sin k_u x \quad (24)$$
$$v_1(x) = A_5 \cosh k_v x + A_6 \sinh k_v x + A_7 \cos k_v x + A_8 \sin k_v x \quad (25)$$
$$v_2(x) = A_9 \cosh k_v x + A_{10} \sinh k_v x + A_{11} \cos k_v x + A_{12} \sin k_v x \quad (26)$$

Where $A_i, i=1,2,...12$ are unknown coefficients that will be determined by the boundary conditions. For the beam with one crack, the boundaries will include both ends and the two sides of the crack (totally 12 conditions). These conditions are homogeneous which means they don't involve functions of t. Therefore, application of the boundary conditions to the solution will yield twelve algebraic equations for $A_1, A_2, ...A_{12}$. These algebraic equations are homogeneous. The condition for the existence of nontrivial solution is that the determinant of the coefficient matrix equals to zero. This gives an equation for the determination of the system natural frequency ω.

For each value of ω_i we can get the corresponding solution of $A_1, A_2, ...A_{12}$. Then substituting $A_1, A_2, ...A_{12}$ to equations (23)----(26) we can get $u_1(x), u_2(x), v_1(x), v_2(x)$ which arethe axial and bending modes, respectively.

For a cracked cantilever beam, the boundary conditions will be:

Clamp end: $u_1(0)=0$; $v_1(0)=0$; $v_1'(0)=0$ (27)

Free end: $AEu_2'(L)=0$; $EIv_2''(L)=0$; $EIv_2'''(L)=0$ (28)

Cracked section:

$$AEu_1'(l) = AEu_2'(l) \quad (29)$$
$$EIv_1''(l) = EIv_2''(l) \quad (30)$$
$$EIv_1'''(l) = EIv_2'''(l) \quad (31)$$
$$v_1(l) = v_2(l) \quad (32)$$
$$AEu_1'(l) = k_{11}[u_2(l) - u_1(l)] + k_{15}[v_2'(l) - v_2'(l)] \quad (33)$$
$$EIv_1''(l) = k_{51}[u_2(l) - u_1(l)] + k_{55}[v_2'(l) - v_1'(l)] \quad (34)$$

By applying the solution functions (23)----(26) into the boundary conditions (27)----(34),the characteristic equation of the system can be obtained:

$$\det[Q] = 0 \quad (35)$$

This determinant is a function of the natural frequency ω and the local stiffness matrix which is dependent on the crack severity and location. The roots of the equation versus the natural frequency ω give the eigenvalues of the system. The eigenmodes of the system can also be determined as stated above.

5. FORCED VIBRATION OF A CRACKED CHS BEAM

Vibration can be excited by applying a harmonic excitation along any coordinate direction. Without a crack, the response only exists at the corresponding direction. But when even a small crack is presented, excitation in one degree of freedom gives response in all possible degrees of freedom. This coupling property can be observed by the Frequency Response Functions of the system.

Supposing a transverse harmonic excitation force $F(L,t) = f_0 \cos\omega t$ is applied at the free end of the beam, then the boundary condition of equation (28) will become:

$$EIv_2'''(L) = f_0 \tag{36}$$

The other boundary conditions will remain the same. The coefficients $A_1, A_2, \ldots A_{12}$ can now be computed by solving the linear system equation:

$$[Q]\{A\} = \{F\} \tag{37}$$

Where $\{F\} = \{0,0,\ldots f_0,\ldots 0\}^T$. Substituting the solution of $\{A\}$ back to equation (23)---(26) we can get response function $\frac{U}{F} = \frac{u(x,\omega)}{f_0}, \frac{V}{F} = \frac{v(x,\omega)}{f_0}$ for longitudinal and lateral vibration respectively. Such plots have been created for the free end of the beam ($x = L$).

6. RESULTS AND DISCUSSION

As shown in Figure 1, a Circular Hollow Section beam is examined. This is a common section used in engineering structures. In this paper, we consider a cantilever beam configuration. The following parameters are selected: beam length 1.0 m, outside diameter 48.3 mm, wall thickness 3.2 mm and crack location 0.2 m from clamp end.

Based on previous analysis, we firstly obtained the eigenfrequencies and mode shapes for uncracked beam, and then compared to cracked beam under different locations and various crack severities. The eigenfrequencies are shown in table 1 and The first four modes are presented in Figure 2. As can be seen, there is more frequency shift when the crack is located closer to the root or if the crack is more severe. From the mode shapes one can observe the discontinuity at the crack location.

An examination of the coupling effects on the Frequency Response Functions (FRFs) indicates the sensitivity of the method to the crack presence. The driving point FRFs of the free end (vertical response with vertical excitation) under different crack severity are plotted in Figure 3. It is clear that for uncracked beam only lateral response exists. However, once the crack is presented the axial response manifests itself in lateral spectra and as the crack progresses this manifestation becomes very significant. In order to judge these extra peaks are from axial response, we compared it with the FRF of the uncracked beam and also created the corresponding mode shape for further confirmation.

This coupling, which is due to the non-diagonal terms at the local flexibility matrix, is a potential indicator for crack identification.

Mode		ω_{unck}	$l = 0.2m$			Crack Severity: 10%		
Axial	Bending	(rad/s)	5%	10%	25%	$l = 0.2m$	$l = 0.5m$	$l = 0.8m$
	B1	290	2.41	9.45	42.44	9.45	2.33	0.09
	B2	1820	0.02	0.08	0.33	0.08	8.88	1.89
	B3	5098	0.72	2.87	12.01	2.87	0.01	7.20
A1		8119	1.07	4.44	15.09	4.44	2.88	0.66
	B4	9990	1.72	5.10	8.97	5.10	6.62	8.70
	B5	16514	1.49	4.42	8.77	4.42	0.00	5.66
A2		24358	0.45	1.66	0.55	1.66	7.25	2.62
	B6	24669	0.28	0.48	3.80	0.48	0.98	0.43

Table 1: Comparison of Natural Frequency Change (%) Due to Crack

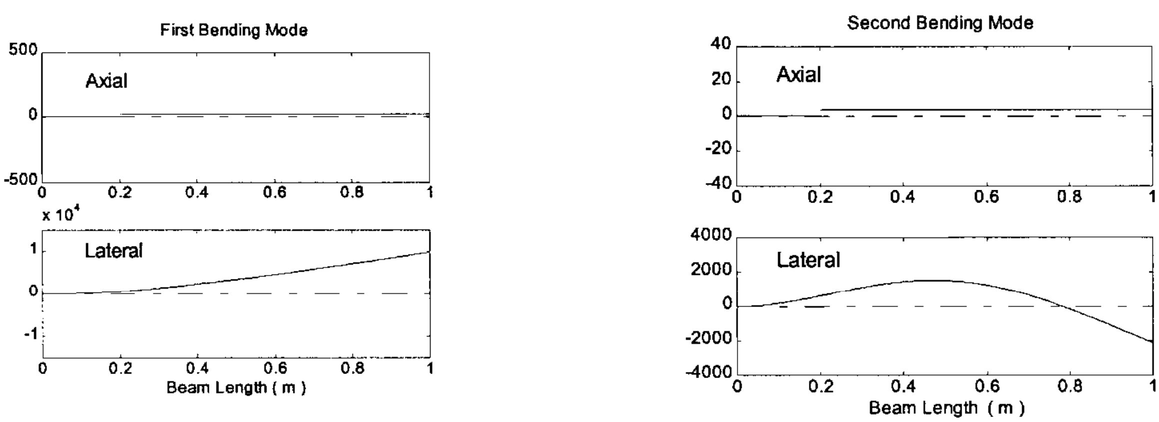

Figure 2: Mode Shapes of Cracked CHS Beam

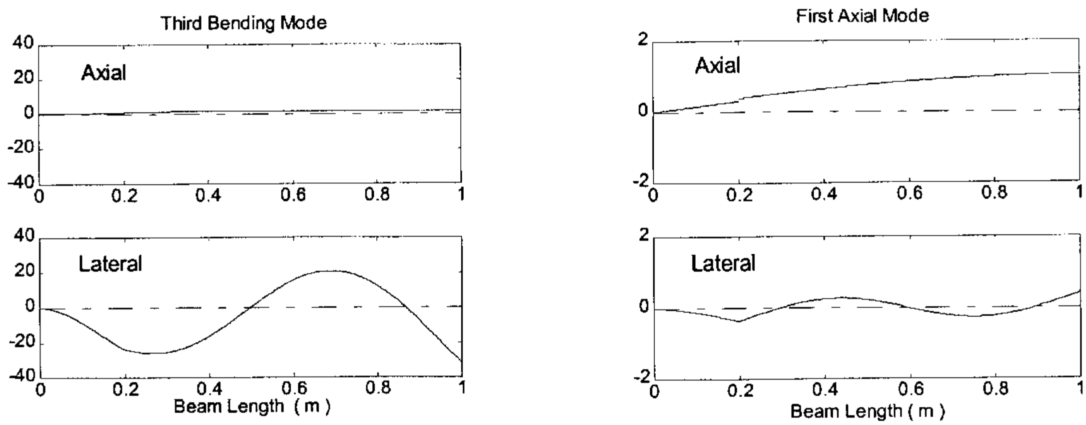

Figure 2: Mode Shapes of Cracked CHS Beam (Continued)

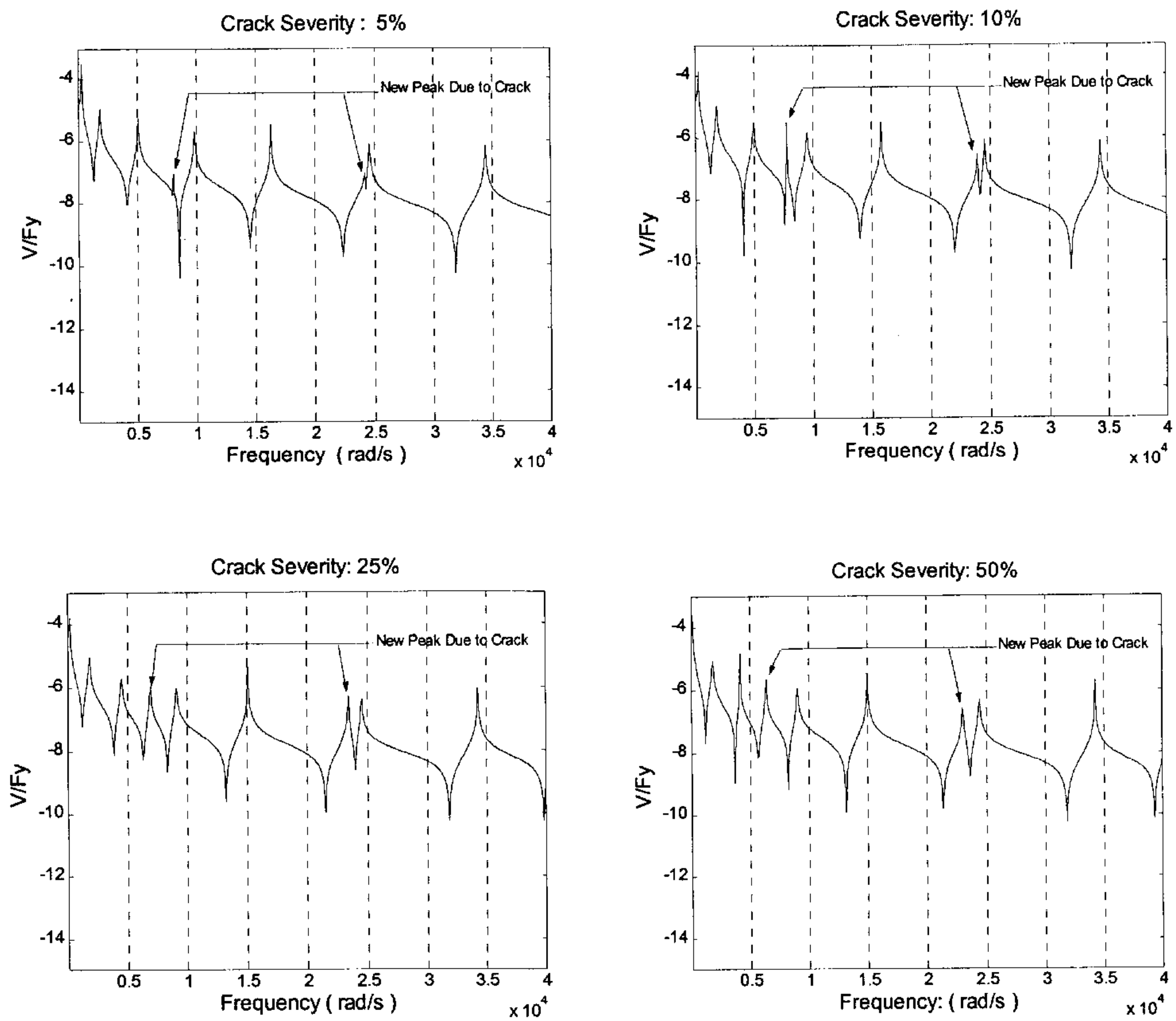

Figure 3: Comparison of FRF of CHS Beam Under different severity

7.EXPERIMENT

In order to observe the evidence of the coupling property, a CHS beam of same dimension was firmly clamped onto a strong bench. The crack was created by a very fine hacksaw to a certain degree of severity. The modal testing was conducted by impact excitation. The lateral and axial responses were measured at the free end of the beam while the excitation points were selected at various locations.

The acquisition of the excitation and response data was done by an IOTECH Wavebook data acquisition system and the signal processing was done using MATLAB. A sampling frequency of 15 kHz was used.

Both uncracked and cracked CHS beam were tested and relating Frequency Response Functions were generated. From the results we can see that extra resonance peaks appear at lateral FRFs. One of these FRFs (free end lateral driving point FRF) is plotted in Figure 4.

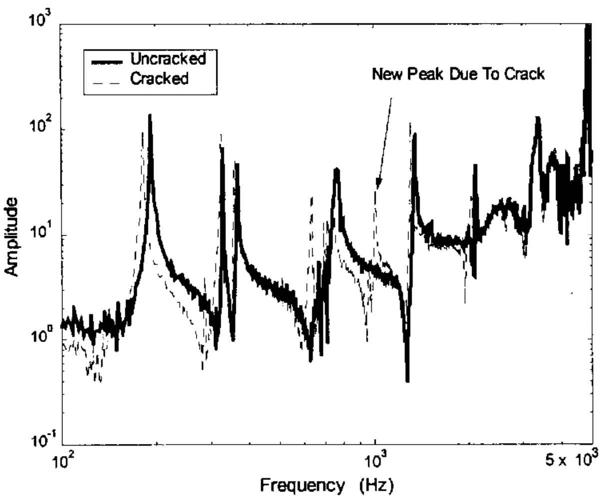

Figure 4:Experimental FRF of Cracked and Uncracked CHS Beam

8.SUMMARY AND CONCLUSIONS

The purpose of this paper is to present the feasibility of identifying cracks by using the coupling property of cracked structural members. To achieve this, we used a Circular Hollow Section (CHS) member as an example to analytically study the free and forced vibration of a cracked CHS beam and then experimentally observe the coupling behaviour of the same structure. Both methods showed that the coupling property well indicated the existence of cracks. The results of the beam under different crack severity and location were also compared.

The reason for coupling is because of the nondiagonal terms of the local flexibility matrix due to the presence of a crack. Physically, if several loadings contribute to the same mode of the stress intensity factor, the corresponding coupling will be presented.

In order to identify the cracks, one can observe the coupled response measurements through driving point FRF or extra modal modes. The side peaks on FRF plots indicate the existence of cracks.

In the case of cantilever beam, if the crack is closer to the clamp end, the coupling phenomenon will be more obvious. Similarly, the more severe of the crack, the more clear of the coupling.

This approach is based on linear fracture mechanics and open crack model. However the coupling property is still useable for crack identification. For more complex structure and the crack closure effect, further study needs to be conducted.

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