Simple Thermal Model for Vertical-Cavity, Surface-Emitting Lasers

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***Abstract*— A simple, multi-mode, rate equation based thermal model has been developed for vertical-cavity, surface-emitting lasers. The misalignment between cavity supported modes and the material gain peak is considered to be the primary cause of the higher order transverse modes and power rolloff observed in these devices. Experiments are performed on a *Mode 8085-2008* VCSEL to determine its modal composition over a range of currents and temperatures. The rate equation model was fitted to these modally resolved light current curves with good success.**

I. Introduction

Vertical-cavity, surface emitting lasers (VCSELs) have grown enormously in popularity and performance since their inception. Manufactured with a very short resonant cavity, VCSELs lase in a single longitudinal mode, and emit light perpendicularly to the plane of crystal growth - allowing on-wafer testing, and the construction of two dimensional transmitter arrays. These properties, in addition to their low threshold current, high efficiency, and small volume, make them ideal for short range optical communications applications.

Single mode operation of VCSELs is frequently modelled by a set of rate equations describing the photon density, carrier density, and phase [1], [2]. This paper proposes a system-level, rate equation model that describes the thermal behaviour of multi-mode VCSELs in a simple and accurate manner.

The thermal power rolloff and higher order mode behaviour are primarily attributed to the misalignment between the material gain and cavity resonance peaks, rather than the change of threshold current [3]. This behaviour can be represented by a modified set of rate equations; maintaining the simplicity of previous rate equation models [4], but providing greater physical insight into VCSEL operation. Subsequently, the laser can be described in fewer, more meaningful, parameters with higher accuracy outside the range of measurement.

Modally resolved, light current characteristics have been extracted from measurements on a *Mode 8085-2008* VCSEL, over a range of ambient temperatures from 10 – 45°C. The calculated modal behaviour shows excellent agreement with the measured values over all temperatures.

II. Model Development

The rate equations, Eqs. (1) – (3), are the foundation of the thermal VCSEL model presented in this paper. The equations are simple and flexible, and are applicable to both directly modulated and continuous wave laser operation.

$$\frac{dS(t)}{dt} = \Gamma g \frac{N(t) - N_0}{1 + \epsilon S(t)} S(t) - \frac{S(t)}{\tau_\mathrm{p}} + \frac{\Gamma \beta N(t)}{\tau_\mathrm{n}} \tag{1}$$

$$\frac{dN(t)}{dt} = \frac{I(t)}{qV_\mathrm{a}} - g \frac{N(t) - N_0}{1 + \epsilon S(t)} S(t) - \frac{N(t)}{\tau_\mathrm{n}} \tag{2}$$

$$P(t) = \frac{V_\mathrm{a} \eta h \nu}{2 \Gamma \tau_\mathrm{p}} S(t) \tag{3}$$

where,

$N(t)$ is the carrier density,
$S(t)$ is the photon density,
$I(t)$ is the drive current,
q is the electron charge,
V_a is the volume of the active region,
τ_n is the electron lifetime,
g is the gain slope constant,
N_t is the carrier density at threshold,
ϵ is the nonlinear gain coefficient,
Γ is the optical confinement factor,
τ_p is the photon lifetime,
η is the quantum efficiency of the laser,
ν is the lasing wavelength,
and β is the percentage of spontaneous emission that contributes to the lasing mode.

Through several approximations, and the replacement $\Phi = 2q/(h\nu\eta)$, Eqs. (1) – (3) can be reduced to Eq. (4) under steady state conditions [5], [6].

$$(\Phi P)^2 - (I - I_\mathrm{th} - I_\mathrm{s})\Phi P - I_\mathrm{s} I = 0 \tag{4}$$

Equation (4) describes the ideal laser light-current curve with a threshold current of I_th, a differential efficiency of $1/\Phi$, and a spontaneous emission term of I_s.

Unfortunately practical VCSEL performance does not match the linear, single mode behaviour indicated by the rate equations. Despite VCSELs' single *longitudinal* lasing mode, they typically operate under several *transverse* modes. Additionally, a significant power rolloff can be observed at currents above threshold. While spatial hole burning [7] and carrier diffusion [8] contribute to this behaviour, thermal effects are generally accepted as the primary cause [4], [9]–[12].

High operating current densities and resistive DBR mirrors make VCSEL performance extremely dependant on thermal influences [13]. This dependence was modelled by Hasnain *et al* [12]:

$$T(I) = T_{\mathrm{s}} + \alpha I + \beta I^2 \tag{5}$$

where, T_{s} is the ambient temperature, and α and β are extracted parameters that depend on the thermal properties of the laser.

In response to an increase in drive current and VCSEL temperature, the cavity supported lasing wavelength, λ_{c}, and the peak material gain, λ_{p}, will shift relative to each other. This process is referred to as *cavity detuning* [14]. The maximum gain is achieved when the Fabry Perot resonance occurs at the same wavelength as the peak material gain. With increasing current, the fundamental mode shifts across the gain profile, resulting in the observed power rolloff. Additionally, the lower wavelength, higher order transverse modes will lase more efficiently as they shift towards the gain peak. The relative misalignment between the transverse modes and material gain is used here to model the power and modal characteristics of VCSELs.

The shift of material gain and lasing wavelength are both approximately linear with temperature [15], which allows the wavelength dependent gain profile to be mapped to temperature. By determining the current to temperature relationship, the effect of cavity detuning can be represented in a simple current dependent equation:

$$\rho_i(I) = \exp\left[-\frac{(I - I_{\mathrm{off}_{\mathrm{i}}}(T))^2}{2\sigma^2\gamma_i}\right] \tag{6}$$

where, $I_{\mathrm{off}_{\mathrm{i}}}$ represents the initial, temperature dependent, misalignment between the material and cavity gain peaks, and σ is the standard deviation of the material gain. The material gain profile has been approximated by a Gaussian distribution, similar to the quadratic shape assumed by [12] and [16]. While Eq. (5) gives a quadratic relationship between temperature and drive current, experimental data, Fig. 1, indicates that it is close to linear. To simplify Eq. (6), the curve is considered to be linear over the lasing range of each mode, with a gradient represented by γ_i.

In this manner, the effect of drive current and ambient temperature can be considered simultaneously. The modified multi-mode rate equations are presented below:

$$\frac{dS_1}{dt} = \Gamma g_1(N(t) - N_{0_1})\frac{S_1}{\rho_1}(t) - \frac{S_1(t)}{\rho_1\tau_{\mathrm{p}}} + \frac{\Gamma\beta N(t)}{\tau_{\mathrm{n}}} \tag{7a}$$

$$\frac{dS_2}{dt} = \Gamma g_2(N(t) - N_{0_2})\frac{S_2(t)}{\rho_2} - \frac{S_2(t)}{\rho_2\tau_{\mathrm{p}}} + \frac{\Gamma\beta N(t)}{\tau_{\mathrm{n}}} \tag{7b}$$

$$\vdots$$

$$\frac{dS_N}{dt} = \Gamma g_N(N(t) - N_{0_{\mathrm{N}}})\frac{S_N(t)}{\rho_N} - \frac{S_N(t)}{\rho_N\tau_{\mathrm{p}}} + \frac{\Gamma\beta N(t)}{\tau_{\mathrm{n}}} \tag{7c}$$

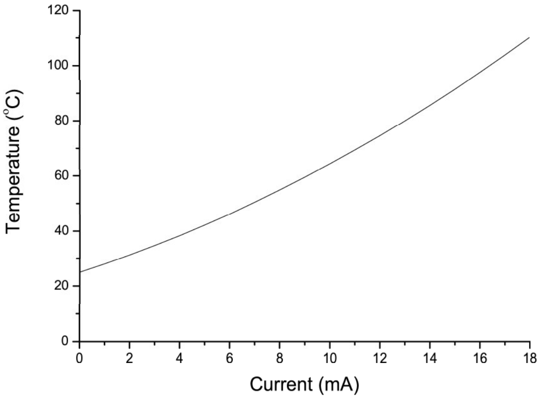


Fig. 1. Extracted Temperature from a *Mode 8085-2008* Oxide Confined VCSEL. $T(I) = T_{\mathrm{s}} + 2.94I + 0.01I^2$

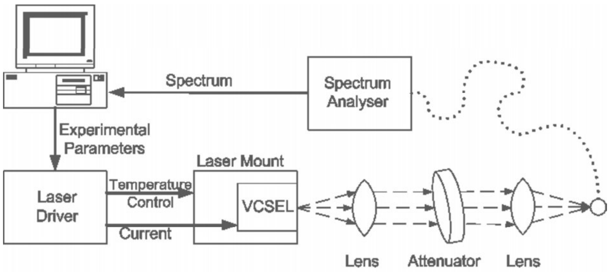


Fig. 2. Experimental Setup.

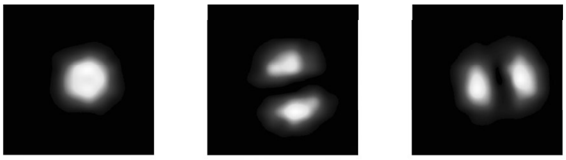
$$\frac{dN(t)}{dt} = \frac{I(t)}{qV_{\mathrm{a}}} - \sum_{i=1}^{N}\left(g_i(N(t) - N_{0_{\mathrm{i}}})\frac{S_i(t)}{\rho_i}\right) - \frac{N(t)}{\tau_{\mathrm{n}}} \tag{8}$$

III. Experiment and Fitting

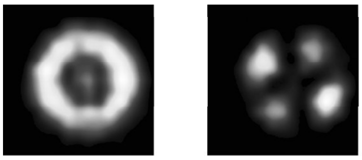
Measurements were performed on a *Mode 8085-2008* VCSEL to determine its spectral and power characteristics. The optical spectrum was recorded as the drive current was varied from 0-18mA, and the operating temperature was increased from $10 - 45$°C, Fig. 2. From these spectra, the mode peaks were isolated and a modally resolved light current curve could be constructed for each temperature. The near field laser beam was scanned by a fibre probe. Figure 3 shows the three dominant, spectrally separated, transverse modes that were identified: the fundamental Gaussian (HG_{00}), a doughnut ($HG_{01}+HG_{10}$), and a leopard combination ($HG_{02}+HG_{20}+HG_{11}$).

Equations (7) and(8) were solved for the steady state case, and fitted to the experimental data, Fig. 4. I_{th}, I_{s}, Φ, and γ were extracted for each mode and kept constant over the entire temperature range, Tab. I; σ was 2.5mA.

Changing the ambient temperature has the same detuning effect as increasing the drive current: shifting the lasing modes across the gain profile. Figure 5 shows that with increasing temperature, the current offset decreases. $I_{\mathrm{off}_{\mathrm{i}}}$ is the drive current at which mode i is perfectly aligned with the peak



(a) HG_{00} (b) HG_{01} (c) HG_{10}



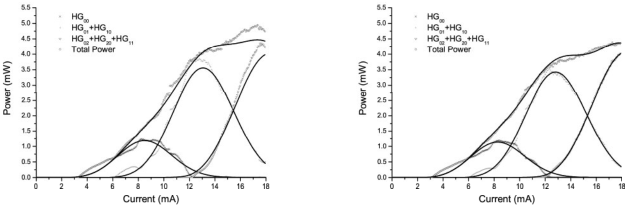
(d) HG_{02} and HG_{20} (e) HG_{11}

Fig. 3. Dominant transverse modes.

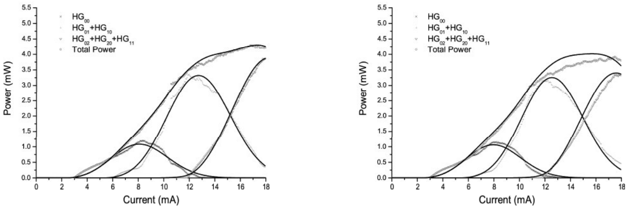
TABLE I

EXTRACTED FITTING PARAMETERS

	HG_{00}	HG_{10+01}	$HG_{20+02+11}$
$I_{\rm th}$ (A)	0.0026	0.0056	0.0119
Φ (A/W)	4.66	1.99	1.45
$I_{\rm s}$ (A)	9.30×10^{-7}	6.02×10^{-6}	1.62×10^{-5}
γ	0.9	1	1.2



(a) 10°C (b) 20°C



(c) 30°C (d) 40°C

Fig. 4. Modally resolved light current curves. Fitted (-), Experimental (+).

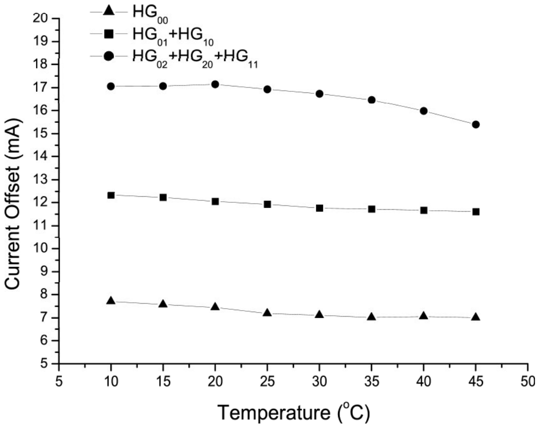


Fig. 5. The shift of $I_{\rm off}$ with temperature.

material gain.

IV. CONCLUSION

Modifications have been made to the single mode rate equations to simply and accurately model the multi-mode characteristics of vertical-cavity, surface-emitting lasers. The model was successfully fitted to experimental data over a range of ambient temperatures. The excellent agreement between calculation and experiment, in addition to the progression of $I_{\rm off}$ with temperature, suggests that cavity detuning plays a major role in the thermal behaviour of VCSELs.

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