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GENERAL SOLUTIONS FOR THE INITIAL RUN-UP OF A BREAKING TSUNAMI FRONT

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ABSTRACT: An assessment of potential damage on coastlines due to tsunami requires prediction of the likely flow depths and impact forces on structures. Since tsunami have a long wavelength relative to the water depth, models that solve the non-linear shallow water equations (NLSWE) are appropriate. Carrier et al (2003) presented solutions for tsunami that do not break. However, where the leading front of the tsunami breaks, the wave front is more similar to a bore approaching the shoreline. An analytical treatment for the run-up driven by an incident bore was developed by Shen & Meyer (1963). However, the Shen & Meyer result is only unique close to the shoreline, and the flow depths and velocities depend on the seaward boundary condition. Here, a new numerical model is presented that solves the NLSWE for the case of a broken tsunami front arriving at the shoreline. It is a simple model that allows specification of either the flow depths or velocities at the seaward boundary, and the solution method allows the effects of friction to be incorporated. The model is able to reproduce the Shen & Meyer analytical solution for the initial motion of the shoreline. The potential damage to coastal structures and other infrastructure may be assessed by calculation of impact and drag forces due to the predicted flow depths and velocities.
Keywords: Breaking tsunami, run-up

1. INTRODUCTION

Investigation of the potential impact of tsunami on coastlines requires an understanding of the hydrodynamics, and for this purpose models that solve the non-linear shallow water equations (NLSWE) have been widely used. For the case of waves that do not break before reaching the shoreline, Carrier & Greenspan [1] developed analytic solutions which describe the smooth water motion. Further solutions for non-breaking tsunami were developed by Carrier et al. [2]. However, the leading front of tsunami events often break before arriving at the shoreline [3], as seen in photographs of the December 2004 Indian Ocean tsunami arriving on the coast of Thailand (Fig. 1).

Fig. 1: Photographic evidence of breaking tsunami, Thailand, December 2004 (Associated Press)

Where the wave breaks, an analytic solution for a single bore arriving at a plane beach was derived by Shen & Meyer [4]. This solution describes the motion of the shoreline as a parabola, with a single free parameter, the initial shoreline velocity. The Shen & Meyer solution has been used by Peregrine & Williams [5] and Pritchard & Hogg [6] to represent not only the shoreline motion but also the depth and flow velocities throughout a run-up event. However, the Shen & Meyer result is only unique close to the shoreline, and different flow depths and velocities within the run-up may be obtained by varying the seaward boundary conditions. This paper presents numerical solutions of the NLSWE that predict flow depths and velocities for various alternative seaward boundary conditions.

2. NUMERICAL MODEL DEVELOPMENT

The non-linear shallow water equations are derived by writing the mass and momentum conservation equations with the assumption of hydrostatic pressure. In dimensional form, adopting the definitions illustrated in Fig. 2, they are written:

$$\begin{gathered}\frac{\partial h^*}{\partial t^*}+\frac{\partial\left(u^*h^*\right)}{\partial x^*}=0\\ \frac{\partial u^*}{\partial t^*}+u\frac{\partial u^*}{\partial x^*}+g\cos\gamma\frac{\partial h^*}{\partial x^*}=-g\sin\gamma\end{gathered}\qquad(2.1)$$

where x^* is distance along the slope, t^* is time, h^* (x^*,t^*) is the water depth perpendicular to the slope and u^* (x^*, t^*) is the flow velocity parallel to the slope.

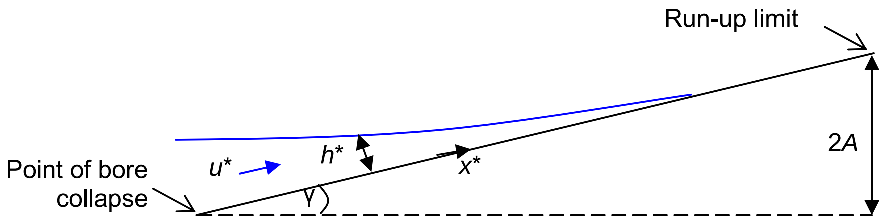


Fig. 2: Definition sketch

Following the approach of Peregrine & Williams [5], the parameters are made dimensionless and scaled on the vertical run-up height $2A$:

$$x=\frac{x^*\sin\gamma}{A}\qquad t=t^*\sin\gamma\sqrt{\frac{g}{A}}\qquad h=\frac{h^*\cos\gamma}{A}\qquad u=\frac{u^*}{\sqrt{gA}}\qquad(2.2)$$

The equations (2.1) may then be written:

$$\begin{gathered}\frac{\partial h}{\partial t}+\frac{\partial(uh)}{\partial x}=0\\ \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\partial h}{\partial x}+1=0\end{gathered}\qquad(2.3)$$

Direct numerical solution of these equations is possible, but requires special procedures for dealing with the moving boundary at the shoreline [7]. However, by rewriting equations (2.3) in characteristic form a numerical scheme may be developed without the need for special consideration of the shoreline. Making use of the local long wave celerity $c=\sqrt{h}$, equations (2.3) may be written:

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t}+(u+c)\frac{\partial}{\partial x}\right)(u+2c+t)=0\\
&\left(\frac{\partial}{\partial t}+(u-c)\frac{\partial}{\partial x}\right)(u-2c+t)=0
\end{aligned}
\tag{2.4}
$$

Hence the characteristic relations of equations (2.3) are:

$$\frac{d}{dt}\left(u+2c+t\right)=0 \quad \text{along} \quad \frac{dx}{dt}=u+c \tag{2.5}$$

$$\frac{d}{dt}\left(u-2c+t\right)=0 \quad \text{along} \quad \frac{dx}{dt}=u-c \tag{2.6}$$

Equations (2.5) describe the forward (or advancing) characteristics and equations (2.6) the backward (or receding) characteristics. These characteristic curves correspond physically to the paths of infinitesimal wave disturbances. The path of the shoreline following bore collapse represents both a forward and backward characteristic, since $c=\sqrt{h}=0$ at the shoreline.

The numerical scheme solves for the flow depths and velocities along the paths defined by (2.5) and (2.6). The initial conditions for the solution are specified at the origin, which is the point where the incoming bore reaches the dry beach. The seaward boundary is defined by the x = 0 axis (the original still water shoreline), along which either flow depths or velocities may be specified. Details of the solution technique are provided in the Appendix.

The Shen & Meyer analytical solution for the shoreline motion is

$$x_s(t)=2t-\tfrac{1}{2}t^2 \tag{2.7}$$

and the water depths and flow velocities close to the shoreline are given by:

$$h(x,t)=\frac{\left(x_s(t)-x\right)^2}{9t^2}=\frac{(4t-t^2-2x)^2}{36t^2} \tag{2.8}$$

$$u(x,t)=\frac{2(t-t^2+x)}{3t} \tag{2.9}$$

Peregrine & Williams [5] and Pritchard & Hogg [6] applied (2.8) and (2.9) throughout the run-up, in the absence of other solutions. However, Shen & Meyer noted that the results (2.8) and (2.9) are only valid in the vicinity of the shoreline. It has been found that (2.8) significantly underestimates the depth away from the shoreline of real events [8]. Accurate prediction of the depth is critical for correct assessment of potential impacts on coastal environments.

The numerical model is able to reproduce the Shen & Meyer solution by specifying a value of $u+2c+t=2$ on the incoming forward characteristics at the seaward boundary (a comparison is shown in Figure 3). Alternative solutions for the depths and velocities may be obtained by specifying either depth or velocity time series along the seaward boundary. It can be seen that the Shen & Meyer solution is only one of many possible distributions of h and u within the run-up, depending on the chosen seaward boundary condition.

The force on coastal structures due to a tsunami run-up event may be estimated by calculating the momentum flux. In non-dimensional terms the momentum flux per unit breadth can be written:

$$F=hu^2 \tag{2.11}$$

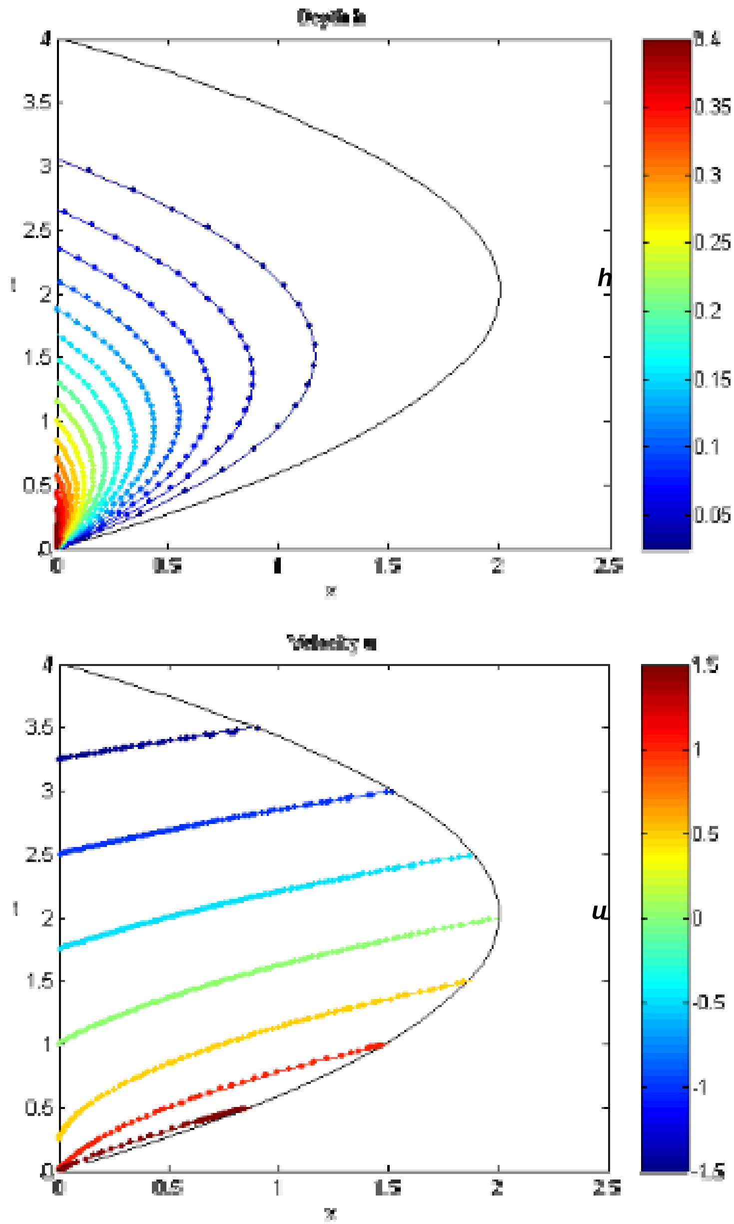


Fig 3: Contours of *h* and *u* from the numerical model (dots) showing close agreement with the Shen & Meyer (1963) analytical solution (lines)

3. MODEL RESULTS

Figures 4, 5 and 6 present model results for 3 different seaward boundary conditions:

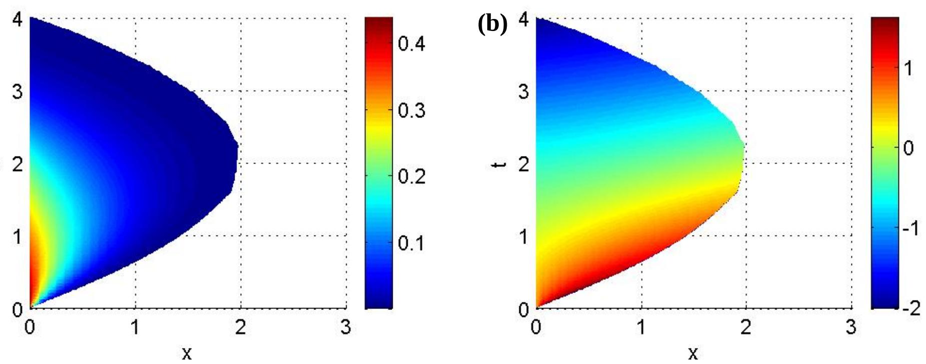
- Case I: Water depth decreasing along the $x = 0$ axis as $h = \frac{4}{9} - \frac{t}{8}$
- Case II: Water depth constant along the $x = 0$ axis $h = \frac{4}{9}$
- Case III: Water depth increasing along the $x = 0$ axis as $h = \frac{4}{9} + \frac{t}{8}$

It can be seen that the momentum flux (and hence the force on coastal structures) has a maximum close to the shoreline singularity at the origin. This peak momentum flux in the uprush is similar in magnitude for each of the cases considered. In Case I, the run-up lens becomes thin and the peak momentum flux in the backwash is not as large as the uprush. However, in Case II a secondary bore forms in the backwash and the peak momentum flux almost as large as that of the uprush. In Case III a secondary backwash bore forms earlier and further up the slope from the initial shoreline position. Here the numerical solution begins to break down and the momentum flux reaches another maximum, larger than that in the uprush. Note that for real tsunami, the front may consist of several sequential incoming bores; in this case the model results are only valid for the initial uprush since additional incoming bores would override the initial shoreline motion and drive the flow further inland. Further scenarios may be considered by specifying the flow velocity at x = 0, rather than the depth.

Comparison may be made with the results of Carrier et al. [2]. Their results indicated peak momentum flux in the backwash for the case of a leading elevation wave, and a peak momentum flux in the uprush for a leading depression wave. Our results indicate that for breaking tsunami the momentum flux in the backwash is strongly dependent upon the specified seaward boundary condition. For all cases considered the peak momentum flux in the uprush is significant due to the collapse of the bore as it arrives on the dry beach. Note that our results are valid for both leading elevation and leading depression waves, but that the point of arrival on the dry beach would be further offshore for a leading depression wave.

The model allows specification of arbitrary depths or velocities along the x = 0 axis. The three cases presented here are simply examples of possible scenarios, and further investigation is required to establish a physically realistic seaward boundary condition. The effects of bed shear stress may also be included in the model by adding a friction term to the momentum equation.

(a)



(b)

(c)

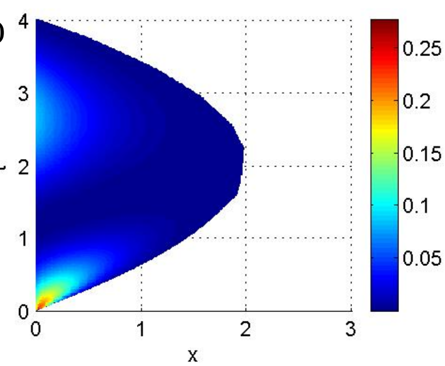
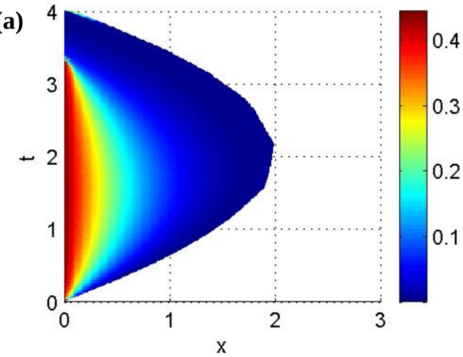


Fig. 4: Case I

Model results (non-dimensional units)

$h = \frac{4}{9} - \frac{t}{8}$ **on the seaward boundary**

(a) Depth
(b) Velocity
(c) Momentum Flux

(a)


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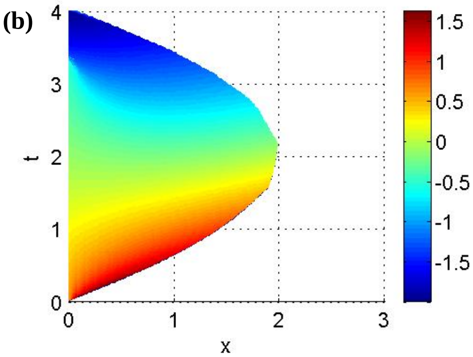
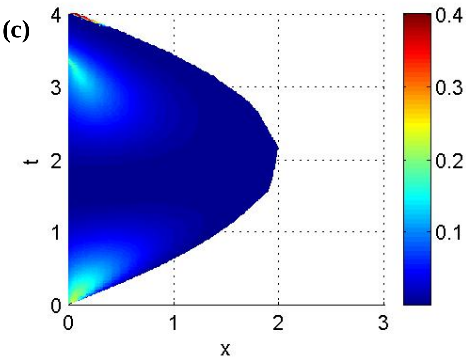


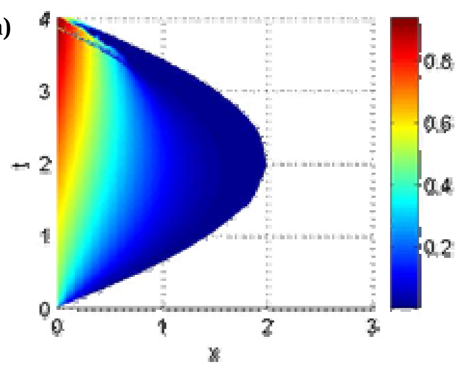
Fig. 5: Case II

Model results (non-dimensional units)

$h = \frac{4}{9}$ **on the seaward boundary**

(a) Depth
(b) Velocity
(c) Momentum Flux

(c)


(a)


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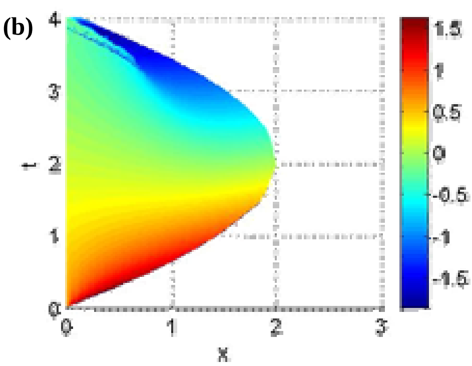
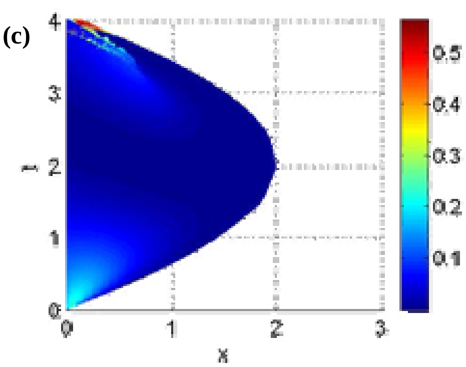


Fig. 6: Case III

Model results (non-dimensional units)

$h = \frac{4}{9} + \frac{t}{8}$ **on the seaward boundary**

(a) Depth
(b) Velocity
(c) Momentum Flux

(c)


4. CONCLUSION

The hydrodynamics during the initial run-up of a breaking tsunami front have been investigated by solving the non-linear shallow water equations using the method of characteristics. The model is able to reproduce the Shen & Meyer analytical solution for run-up due to broken waves. The seaward boundary condition may be specified in terms of water depths or velocities, and the potential impact of tsunami on coastal environments may be assessed by calculation of the momentum flux. The model is easy to adapt to location-specific conditions and may be extended by including a friction term in the momentum equation. Modeling results for three different seaward boundary conditions indicate that a local peak of the momentum flux is always found close to the shoreline singularity corresponding to bore collapse. A secondary peak of momentum flux occurs in the backwash, but the location and magnitude of this peak depends on the chosen seaward boundary condition. Further assessment of available data will enable specification of a physically realistic seaward boundary condition and more accurate prediction of the potential damage to coastal environments due to breaking tsunami.

5. APPENDIX: METHOD OF CHARACTERISTICS SOLUTION

The initiation of the swash event is the initial condition for the solution at $x = 0$, $t = 0$. Backward characteristics radiate from the shoreline singularity at $x = 0$, $t = 0$, each characteristic corresponding to a constant value of $\beta = u - 2c + t$. At the seaward boundary (the $x = 0$ axis) the depth c^2 is specified, and the value of β is given by the outgoing backward characteristic. Hence the value of u on the boundary is determined by solving $\beta = u - 2c + t$. Figure 7 illustrates the solution technique in the vicinity of the origin. The solution begins at the seaward boundary and proceeds along each forward characteristic in turn. The location of the intersection points of the characteristics and the corresponding values of h and u are determined by simultaneous solution of equations (2.5) and (2.6). For example, the location of point **b2** in Figure 7 is determined by solving $x_{b2} = x_{a2} + (u_{a2} + c_{a2})(t_{b2} - t_{a2})$ and $x_{b2} = x_{b1} + (u_{b1} - c_{b1})(t_{b2} - t_{b1})$ simultaneously, and the values of u_{b2} and c_{b2} are found by solving $u_{a2} + c_{a2} + t_{a2} = u_{b2} + c_{b2} + t_{b2}$ and $u_{b1} - c_{b1} + t_{b1} = u_{b2} - c_{b2} + t_{b2}$ simultaneously. The timestep Δt was set to 5×10^{-6} close to the shoreline, and 5×10^{-4} elsewhere.

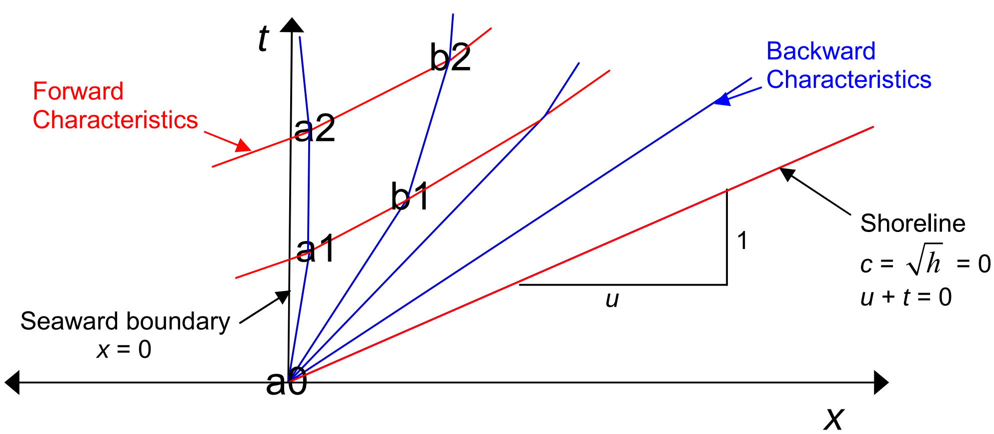


Fig. 7: Illustration of numerical solution technique

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