AIAA'87

AIAA-87-0549

Investigation of Weak Shock-Shock and Shock-Expansion Intersection in the Presence of a Turbulent Boundary Layer

D. J. Mee and R. J. Stalker, Univ. of
Queensland, Australia

AIAA 25th Aerospace Sciences Meeting

January 12-15, 1987/Reno, Nevada

INVESTIGATION OF WEAK SHOCK-SHOCK AND SHOCK-EXPANSION INTERSECTION IN THE PRESENCE OF A TURBULENT BOUNDARY LAYER

by

David J. Mee* and R.J. Stalker**
Department of Mechanical Engineering
University of Queensland
Australia

Abstract

An investigation of intersecting shock-shock and shock-expansion interactions with a turbulent boundary layer is reported. The form of these interactions is interpreted from experimental results at Mach 1.85. It is found that intersecting shock interactions can produce a given overall pressure rise with less likelihood of separation than an equivalent strength single shock interaction. A simple theory to predict limiting streamline deflection angles is extended to boundary layer interactions with multiple merging and intersecting shock wave and expansion fan configurations and shows reasonable agreement with experiments. The possibility that intersecting wave interactions can lead to boundary layer separation is discussed.

Nomenclature

M	Mach number
Δp	pressure rise above undisturbed pressure
Δp_{pr}	pressure rise across primary shock
S	length of vortex tube element
u	local mainstream flow velocity
v	local cross-flow at surface
β	angle of vortex tube to local mainstream flow direction
γ	ratio of specific heats
δ_{av}	average boundary layer thickness (1.9 mm)
Δ	mainstream flow deflection angle
ε	limiting streamline deflection angle relative to local mainstream flow direction
ϕ	limiting streamline deflection angle relative to undisturbed flow direction
ρ	mainstream density
ζ	angle of shock wave or centre of expansion fan to local mainstream flow direction

Subscripts

1	undisturbed upstream state
m	middle of expansion fan
n, n+1	state identification

1. Introduction

The interactions between shock waves and boundary layers have been the subject of investigations for the past 40 years (eg. see the reviews of Green[1], Adamson & Messiter[2], Settles & Dolling[3]). In recent times attention has been focused on three-dimensional interactions, of which one of the simplest and most important configurations is the fin induced, swept, normal shock interaction. A variation on this configuration is the intersection of two such swept shock waves in the presence of a turbulent boundary layer† (Figure 1). This type of interaction, found in engine intakes and between protrusions on supersonic flight vehicles, is the subject of the present paper. The important features of this interaction are discussed in the light of experimental results. A simple theory, due to McCabe[4], which has proved remarkably successful in predicting limiting streamline deflection angles in single interactions, is extended to intersecting wave configurations (including intersecting and merging shock wave and expansion fan combinations). The predicted limiting streamline patterns are then discussed and compared with experimental results.

2. Experiments

Experiments to measure surface pressures and limiting streamline patterns for shock-shock and shock-expansion interactions were performed in the University of Queensland supersonic blowdown tunnel. The tunnel has a 76 mm x 102 mm test section and a Mach number of 1.85. For the present tests the stagnation pressure was 300 kPa and the stagnation temperature, which was governed by the prevailing atmospheric conditions, was typically 295 K. The unit Reynolds number in the test section was 4.1×10^7 m^{-1}.

The shock waves and expansion fans were generated by sharp fins mounted from the tunnel side-walls. These waves interacted with a thin, turbulent boundary layer which was formed on a sharp leading edged, flat, measuring plate inserted in the tunnel test section. In the measurement region the average boundary layer thickness, δ_{av}, was calculated to be 1.9 mm.

Surface pressures were measured at an array of 0.5 mm diameter pitot tappings with a 32 channel pressure sensor module. The measuring plate could be moved relative to the wedges in order to increase the effective instrumentation density. For each experimental configuration a total of at least seven runs of the tunnel was

---

* Graduate student
** Professor, member AIAA

† The intersection of two opposing shock waves is labelled "shock-shock" and the intersection of a shock wave and an opposing expansion fan is labelled "shock-expansion".

made with the measuring plate being moved between runs. Limiting streamline patterns were recorded using the china film technique. Pressure measurements and limiting streamline patterns were recorded for the shock-shock interactions but only limiting streamline patterns were recorded for the shock-expansion tests.

3. Experimental results and discussion.

Previous investigations of swept normal shock interactions have shown that the boundary layer leads to a spreading out of the sudden changes which take place across the shock wave in the mainstream flow. After an initial inception zone close to the shock generator the interaction develops into either a cylindrically or conically symmetric form and is termed 'fully developed'. When a second swept normal shock intersects the first shock the subsequent interaction will vary depending upon whether the single shock interactions have reached a fully developed state. The interactions in the present experiments were close to a fully developed cylindrically symmetric state at the intersection point. The discussion which follows will concentrate on the intersection of two fully developed interactions.

When two shock waves intersect the pressure level downstream of intersection is higher than that behind either of the primary shocks. At the surface the pressure rises are spread either side of the shock positions because of the presence of the boundary layer. In the regions removed from the shock intersection point the pressure rises across each of the shocks can be identified clearly (see Figure 2). However, towards the shock intersection the pressure rises blend together such that the rises associated with the individual shocks cannot be separated.

The limiting streamline patterns for this form of interaction (Figure 3) show that the primary shocks cause the flow in the boundary layer close to the surface to be deflected through a greater angle than streamlines in the mainstream flow. A peak in limiting streamline deflection angle occurs at about the shock location. The deflection angles slowly decrease with further distance downstream of the shock. The limiting streamlines then start deflecting rapidly as the reflected shock wave is approached. Again essentially all deflection takes place upstream of the shock. The expansion around the tail of the wedge restricts any examination of changes in limiting streamline angles further downstream. As with the recorded surface pressures, the changes in limiting streamline deflection angles associated with the individual shocks are less distinct towards the intersection point and the maximum deflection angle there is found to be reduced.

The experimental results indicate that the limiting streamline deflection angles downstream of the reflected shocks are less than the angles downstream of the stronger of the two primary shocks. Thus there is less twisting of the boundary layer downstream of the intersection. This indicates that a given pressure rise can be achieved with intersecting shocks with less likelihood of separation than the same pressure rise achieved through a single swept shock wave, assuming that a reduction in boundary layer twist indicates a step away from separation. The limiting streamline patterns indicate that there is an accumulation of fluid near the surface towards the intersection and suggest a thickening of the boundary layer in this region.

In Mee, Stalker and Stollery[5] it is shown that reasonable predictions of surface pressures and limiting streamline patterns can be made by superposition of the results obtained for two single shock interactions.

The limiting streamline patterns of the present investigation indicate that the surface streamlines diverge slowly downstream of the shock intersection point. A convergence or divergence of limiting streamlines downstream of the intersection may be expected because of the different properties (eg. velocity, density) either side of the shear surface in the mainstream. If the limiting streamlines converge then there is the possibility that a form a separation may occur downstream of the intersection as a result of the interaction.

It is worthwhile speculating on the flowfields obtained if convergence or divergence of limiting streamlines does occur. The expected limiting streamline pattern for an interaction which produces divergence is sketched in Figure 4(a). For the weak interactions being discussed here it is expected that the deflection angles of the streamlines will vary monotonically from the surface to the free stream. Thus a diverging surface streamline pattern suggests a region of decreasing boundary layer thickness which may lead to increases in heat transfer rates and skin friction magnitudes in that region. However, with the low rates of divergence expected, these effects may be quite small.

A possible converging limiting streamline pattern is sketched in Figure 4(b). The pattern shown has a line of convergence of limiting streamlines beginning downstream of the intersection point. It is not clear that such a distinct line of convergence will necessarily appear. If the line does appear then the streamline pattern has a similar form to the 'open' separation patterns (also called 'local' and 'free vortex' separation) noted in flows over bodies of revolution at moderate to high angles of incidence[6,7]. This type of separation is characterized by the limiting streamlines that approach the separation line from either side all emanating from the same upstream line of attachment. The effects of such separation on the boundary layer and mainstream flows may be detrimental in some applications.

No converging limiting streamline patterns were obtained in the present experiments but a simple theory, due to McCabe[4], to predict limiting streamline deflection angles for single shock interactions, was extended to the intersecting configuration to determine whether the limiting streamline deflection angles in intersecting shock interactions could be predicted and if convergence or divergence is indicated downstream of the intersection. This extension is considered in the section 4.

4. Prediction of limiting streamline deflection angles.

In many practical flows it is important to know whether boundary layer separation will occur because of the effects this can have on surface heat transfer rates and on disruption to the entire flow field. Limiting streamline patterns are often used to indicate whether separation does occur for a particular configuration. While there has been considerable debate on how to interpret surface footprints of three-dimensional separation, a line of complete convergence of limiting streamlines is often taken to indicate separation. In swept, normal shock interactions a simple incipient separation criterion that has been used is that the limiting streamline deflection angle is equal to the shock wave angle. Thus it proves useful to be able to predict limiting streamline deflection angles. To this end McCabe[4] presented a simple theory for swept normal shock interactions. The success of this theory in correctly predicting deflection angles in a wide range of experiments at free stream Mach numbers up to 6.0 belies its simplicity. Indeed the theory has been criticized for its sweeping assumptions (which are discussed in Mee[8]) and it has been suggested that a more complete analysis will probably lead to poorer predictions of experimental results[9]. However, in examining intersecting wave interactions it appears worthwhile extending McCabe's theory to such configurations even if only to obtain qualitative predictions.

The theory of McCabe is based upon the examination of the passage of a vortex tube element as it is deflected by the shock, assuming that circulation around a material curve in the boundary layer is conserved. The theory assumes a cylindrically symmetric interaction and gives a deflection angle downstream of the shock wave. Experiments indicate that the limiting streamlines begin deflecting upstream of the shock wave, reach a maximum angle at about the shock location and then slowly reduce in angle. Eventually they must run parallel to the mainstream flow. The McCabe theory predicts well the maximum limiting streamline deflection angle (i.e. that at the shock location).

The initial aim of this part of the project was to extend the McCabe theory to intersecting shock interactions. However, results of superposition of limiting streamline patterns[5] suggested that the intersection of a shock wave and an expansion fan in the presence of a turbulent boundary layer would lead to a limiting streamline pattern containing a line of convergence and thus the possibility of separation. It was decided to extend McCabe's theory to this type of interaction also. The final result is a general formulation which can be applied to any number of intersecting or merging shock waves and expansion fans.

For a process (shock wave or expansion fan) which deflects the vortex tube element, AB, at state n to become CD at state n+1 (see Figure 5), it can be shown that ε_{n+1}, the deflection angle of limiting streamlines at state n+1 relative to the mainstream flow at that state, can be written as

$$\tan\varepsilon_{n+1} = \frac{\rho_{n+1}}{\rho_1}\,\frac{u_1}{u_{n+1}}\,\frac{S_{n+1}\sin\beta_{n+1}}{S_1}\,\frac{1}{\tan\beta_{n+1}}$$

where ρ and u are the freestream density and velocity, S is the length of the vortex tube element and β its angle relative to the local freestream flow direction as indicated in Figure 5. The subscript 1 refers to properties at the undisturbed upstream state.

When process n is a shock wave it is found that (see Figure 6(a))

$$\frac{\rho_{n+1}}{\rho_n} = \frac{\tan\zeta_n}{\tan(\zeta_n - \Delta_n)}$$

$$\frac{u_n}{u_{n+1}} = \frac{\cos(\zeta_n - \Delta_n)}{\cos\zeta_n}$$

$$\frac{S_{n+1}\sin\beta_{n+1}}{S_n\sin\beta_n} = \frac{\sin(\zeta_n - \Delta_n)}{\sin\zeta_n}$$

and

$$\tan\beta_{n+1} = \frac{\sin(\zeta_n - \Delta_n)}{\cos(\zeta_n - \Delta_n) - \dfrac{u_{n+1}}{u_n}\left[\cos\zeta_n - \dfrac{\sin\zeta_n}{\tan\beta_n}\right]}$$

where ζ_n is the angle of the shock relative freestream flow at state n and Δ_n is the mainstream flow deflection caused by the shock wave.

In applying the McCabe principles to the interaction of an expansion fan with a boundary layer the theory could become somewhat more complicated. Instead of mainstream flow properties changing at a plane, as is the case with a shock wave, changes occur in a spreading fan region. A simplified flow geometry is considered in order to keep the theory tractable. It is assumed that all changes in mainstream properties occur at the centre of the expansion fan. The sensitivity of the solution to this assumption was tested by comparing results with those obtained when the sudden changes were assumed to occur at the upstream and then at the downstream Mach line of the expansion fan. The inaccuracies introduced by this assumption are small compared with the expected accuracy of the theory. For process n being an expansion fan it can be shown that (see Figure 6(b))

$$\frac{\rho_{n+1}}{\rho_n} = \left[\frac{1 + (\gamma-1)/2\ M_n^2}{1 + (\gamma-1)/2\ M_{n+1}^2}\right]^{1/(\gamma-1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{M_n}{M_{n+1}}\left[\frac{1 + (\gamma-1)/2\ M_n^2}{1 + (\gamma-1)/2\ M_{n+1}^2}\right]^{-1/2}$$

$$\frac{S_{n+1}\ \sin\beta_{n+1}}{S_n\ \sin\beta_n} = \frac{\sin(\zeta_{mn} - \Delta_n)}{\sin\zeta_{mn}}$$

and

$$\tan\beta_{n+1} = \frac{\sin(\zeta_{mn} - \Delta_n)}{\cos(\zeta_{mn} - \Delta_n) - \dfrac{u_{n+1}}{u_n}\left[\cos\zeta_{mn} - \dfrac{\sin\zeta_{mn}}{\tan\beta_n}\right]}$$

where M_n and M_{n+1} are the Mach numbers at states n and n+1 and ζ_{mn} is the angle of the centre of the expansion fan to the mainstream flow at state n.

The formulation that has been presented can be applied to determine limiting streamline deflection angles downstream of any number of intersecting and merging shock waves and expansion fans. Some examples of possible wave combinations are given in Figure 7 where a maximum of two waves is indicated. Reservations should be noted in applying the theory to large numbers of wave interactions because certain assumptions made in developing the theory (eg. constant boundary layer thickness, conservation of circulation) are likely to become invalid.

5. Theoretical predictions and comparison with experiments.

The predictions of the extension of the McCabe analysis have been compared with experimental limiting streamline patterns for colliding shock-shock and a limited number of colliding shock-expansion interactions. The predictions for shock-shock intersections will be discussed first. From the limiting streamline patterns, angles were measured along a series of lines through the interaction region. A typical result, that for the interaction of shocks generated by wedges at angles of 3° and 5° to the flow, is shown in Figure 8. Also shown are the predictions of the theory which compare reasonably with the deflection angles at the shock positions. In all cases tested the theory predicts that the magnitude of the final deflection angle is less than the largest of the deflection angles behind the primary shocks.

As anticipated in section 3, the theory predicts that the final limiting streamline deflection angle is dependent upon the order in which the vortex tube element encounters the shocks. In general this results in the prediction of convergence or divergence of limiting streamlines downstream of the intersection. For a given wedge combination the theory suggests a diverging pattern at low Mach numbers and a converging one at high Mach numbers.

Theoretical predictions as a function of Mach number, for a range of wedge pairs, are shown in Figure 9. Here the rate of convergence of limiting streamlines increases with Mach number and shock strength.

A typical limiting streamline pattern for a shock-expansion intersection is given in Figure 10. The theory predicts and the experimental results indicate that the limiting streamline deflection angle is increased downstream of the intersection. The theory also predicts converging and diverging limiting streamline patterns for shock-expansion intersections. Convergence, when it is predicted, occurs at only a low rate at low Mach numbers and the rate of divergence increases with Mach number and the strength of the shock waves and expansion fans. A different form of limiting streamline pattern containing a line of convergence is suggested by the theory (and also by superposition of single interaction results[5]) when a shock wave, not quite strong enough to cause separation, intersects with an expansion fan. The limiting streamlines are then predicted to be deflected through an angle greater than that of the reflected shock. The expected limiting streamline pattern is sketched in Figure 11. This pattern also has an open separation form. The rate of convergence from upstream is much higher than that from downstream.

The deflection angles measured at the position of the reflected shock or the centre of the reflected expansion fan are compared with theoretical results in Figure 12. Again reasonable predictions of the levels are obtained. For the shock-expansion intersection experiments the observed divergence of limiting streamlines is in qualitative agreement with the theory. However, the experimental results indicate a detectable amount of divergence for the shock-shock intersections while the theory suggests a small amount of convergence of limiting streamlines downstream of the intersections. The theory does predict that there will be some divergence for this type of interaction at low Mach numbers with the change-over point occurring at Mach numbers between 1.5 and 1.7, depending on the shock generator combination. The rate of convergence or divergence increases either side of the change-over Mach number.

There is another factor, that the theory does not take into account, which explains the fact that the experimental results indicate divergence in all cases examined. That is the experimentally observed reduction in limiting streamline deflection angle with increased distance downstream of the shock in single interactions. It was noted in section 4.2 that superposition of limiting streamline patterns predicts divergence of streamlines downstream of the intersection. Such an effect will act against the tendency of convergence and possibly move higher the Mach number at which a change-over from divergence to convergence occurs from that predicted by the present theory. Experiments at higher Mach numbers are necessary to determine whether the predicted limiting streamline convergence does occur and where the change-over takes place.

Paynter[10] produced an analysis to predict boundary layer profiles downstream of multiple weak swept normal shock interactions and predicted a cross flow into the corner region downstream of the reflection of a shock wave from a wall. He suggests that this is consistent with reported experiments at a free stream Mach number of 2.4. This type of interaction is quite similar to those being considered here except for the complications that arise at shock reflection in the presence of a wall boundary layer. A cross-flow towards the wall is consistent with present predictions.

7. Conclusions.

The nature of intersecting swept normal shock wave boundary layer interactions has been discussed in the light of experimental results. The twisting of the boundary layer was found to be reduced downstream of shock intersection. Intersecting shocks will achieve a given pressure rise with less likelihood of separation than the same pressure rise achieved through a single swept shock wave. A simple theory, due to McCabe[4], has been extended to predict limiting streamline deflection angles in intersecting and merging shock wave and expansion fan interactions. The theory shows reasonable agreement with experimental results for intersecting shock-shock and shock-expansion interactions. However further experiments are required to determine the accuracy and applicability limits of the theory. Qualitative predictions for intersecting wave interactions suggest that a form of open separation may occur downstream of the intersection but no conclusive experimental evidence of this has been found.

References.

1. Green, J.E., "Interactions between shock waves and turbulent boundary layers", Prog. Aero. Sci., Vol. 11, pp. 235-340, 1970.
2. Adamson, T.C. Jr. and Messiter, A.F., "Analysis of 2D interactions between shock waves and boundary layers", Ann. Rev. Fluid Mech., Vol.12, pp. 103-138, 1980.
3. Settles, G.S. and Dolling, D.S., "Swept shock wave-boundary layer interactions", AIAA Progress in Astronautics and Aeronautics, to appear.
4. McCabe, A., " The 3-D interaction of a shock wave with a turbulent boundary layer", Aero. Quart., Aug. 1966, pp. 231-252.
5. Mee, D.J., Stalker, R.J. and Stollery, J.L., "Glancing interactions between single and intersecting oblique shock waves and a turbulent boundary layer", J. Fluid. Mech., Vol. 170, pp. 411-433, 1986.
6. Wang, K.C., "Separation patterns of boundary layer over an inclined body of revolution", AIAA J., Vol. 10, No. 8, pp. 1044- 1050, 1972.
7. Wang, K.C., "Boundary layer over a blunt body at high incidence with an open-type of separation", Proc. R. Soc. A., Vol. 340, pp. 33-55, 1974.
8. Mee, D.J., "Investigation of swept shock wave boundary layer interactions", Ph.D. Thesis, University of Queensland, (in preparation).
9. Oskam, B., "Three-dimensional flow fields generated by the interaction of a swept shock wave with a turbulent boundary layer", Princeton University, Gas Dynamics Laboratory Report No. 1313, 1976.
10. Paynter, G.C., "Analysis of multiple weak glancing shock boundary layer interactions", AIAA J., Vol. 19, No. 1, pp. 72-78, 1981.

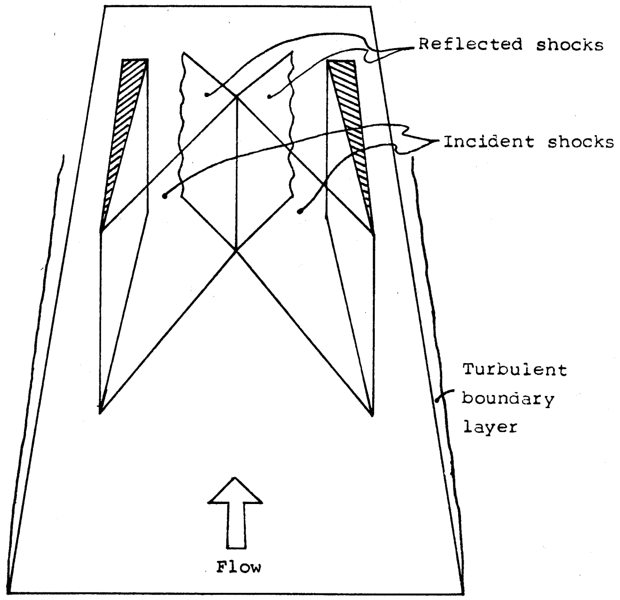


Figure 1 Intersecting shock interaction with a turbulent boundary layer.

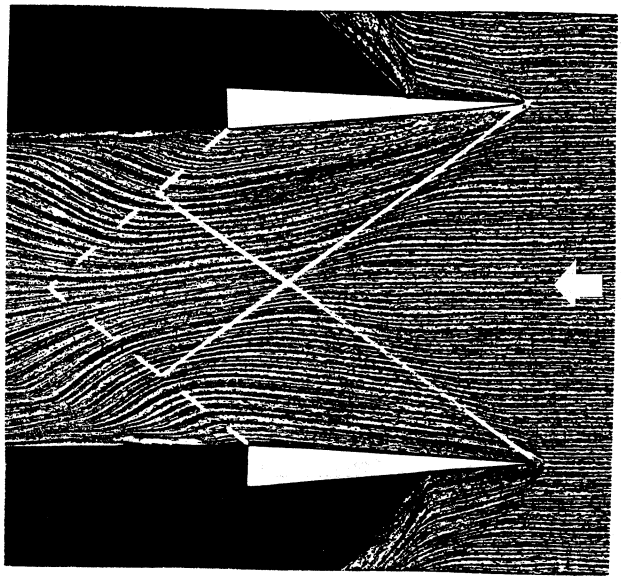


Figure 3 Limiting streamline pattern for shock intersection due to wedges at 3° and 5° to the flow. Solid lines - shocks, dashed lines - expansions.

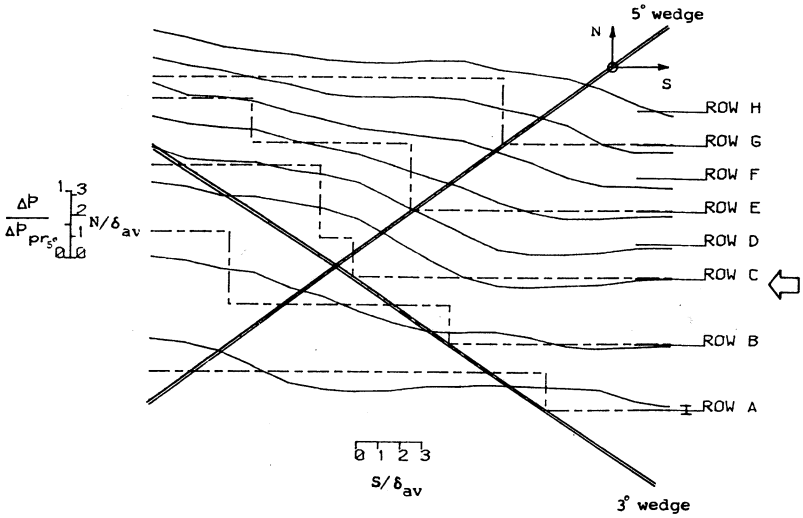


Figure 2 Surface pressures for shock intersection due to wedges at 3° and 5° to the flow. Solid lines - measured pressures, chained lines - inviscid pressures, double lines - shocks.

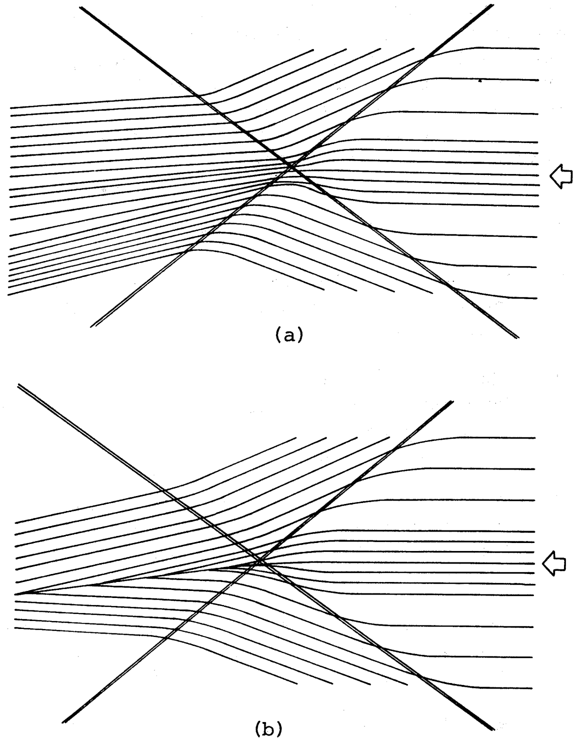


Figure 4 Proposed limiting streamline patterns for shock-shock intersections, (a) diverging, (b) converging.

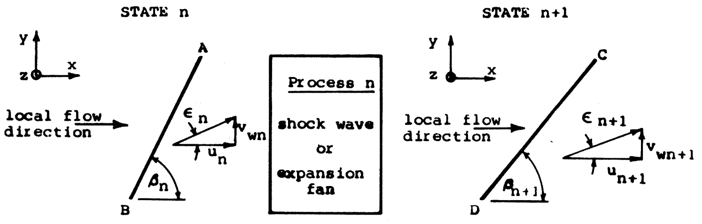
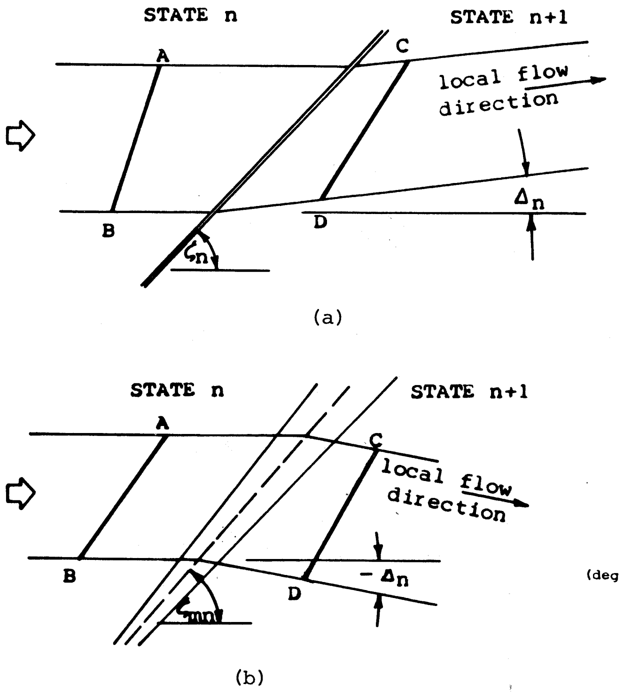


Figure 5 Notation for a process which deflects an elemental vortex tube from AB to CD.



(a)

(b)

Figure 6 Geometry and notation for process n being (a) a shock wave, and (b) an expansion fan.

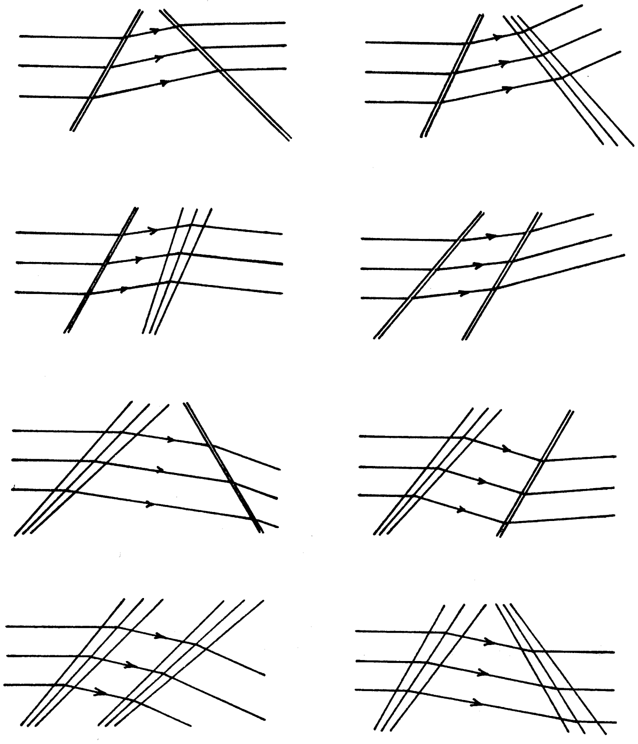


Figure 7 Possible wave combinations - two waves (flow left to right).

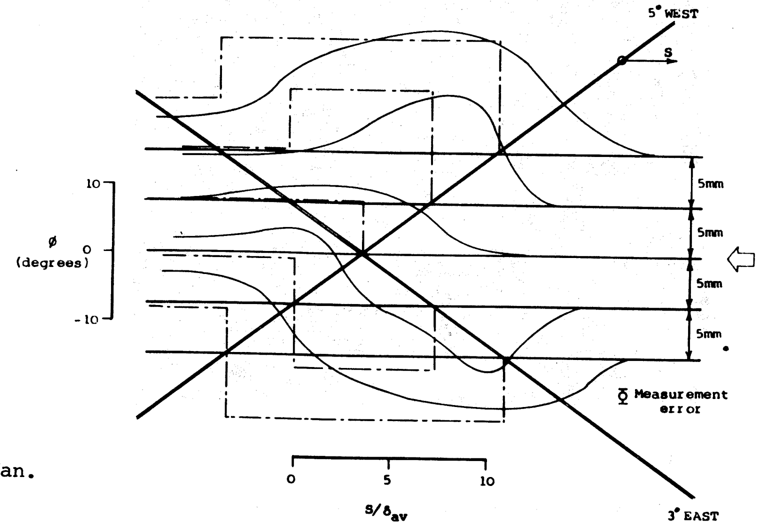


Figure 8 Limiting streamline deflection angles, ∅, for shock intersection due to wedges at 3° and 5° to the flow. Solid lines - measurements, chained lines - theoretical predictions.

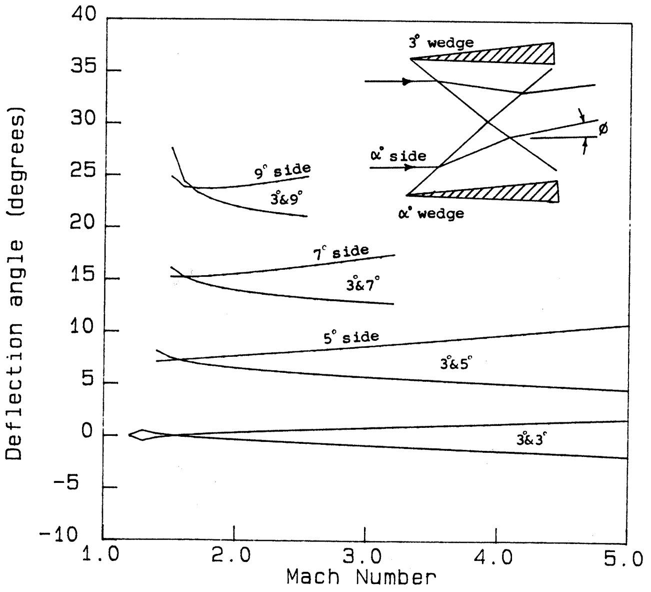


Figure 9 Limiting streamline deflection angles, ϕ, downstream of shock intersection.

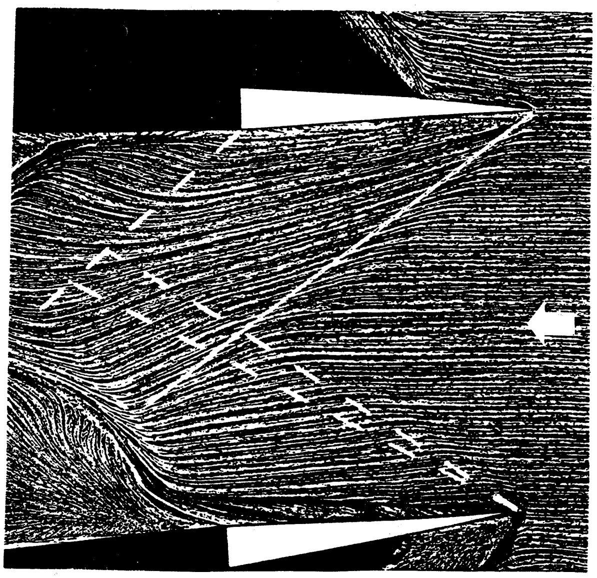


Figure 10 Limiting streamline pattern for a shock-expansion interaction due to wedges at 5° and -3° to the flow. Solid lines - shocks, dashed lines - expansions.

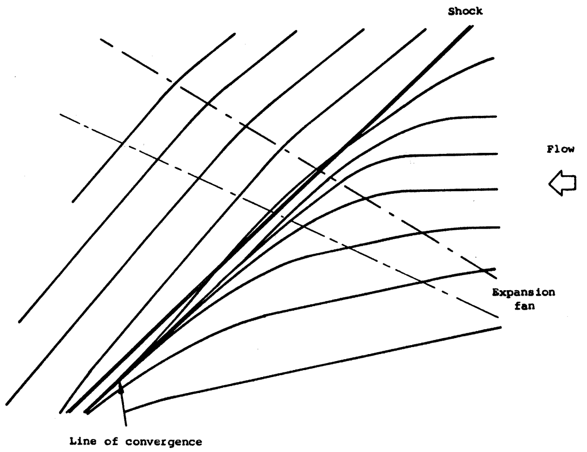


Figure 11 Proposed limiting streamline pattern for a shock-expansion interaction.[5]

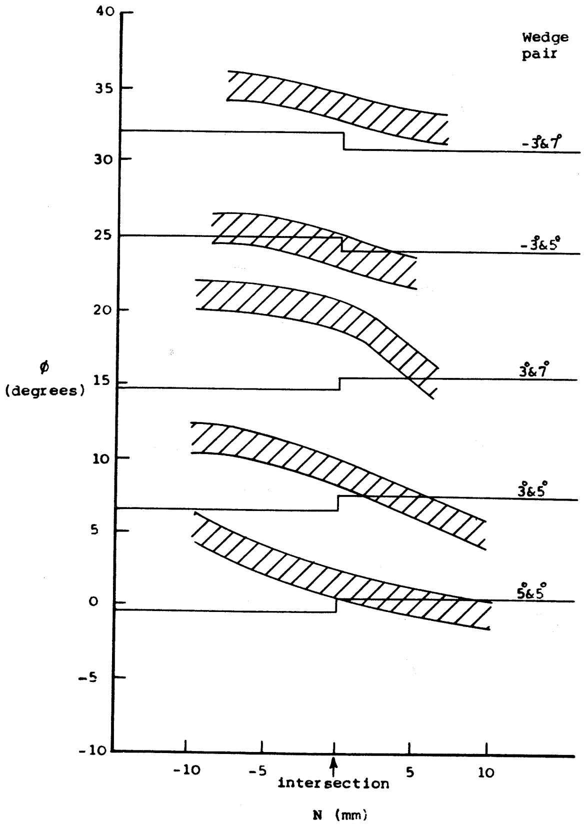


Figure 12 Limiting streamline deflection angles, ϕ, at shock or centre of expansion fan. Shaded regions - measurements, solid lines - theoretical predictions.