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Technical Note
Apparatus to Determine Static and Dynamic Elastic Moduli

By

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Introduction

The earliest recognition of the discrepancy between statically and dynamically determined elastic moduli of rock was reported by Zisman (1933) followed by Ide (1936). Zisman tested a granite and found that the dynamic Young's moduli were 20 per cent greater than the static value. The dynamic moduli quoted by Zisman and Ide were measured by a resonant frequency method detailed in Ide (1936), at atmospheric pressure. It is now recognised that dynamic values of Young's modulus also vary as the stress is varied [King (1966), Baronet al. (1964), Wyllie et al. (1956), Walsh (1966)].

In general the dynamic values of Young's modulus have been found to be significantly greater than the static values. It was also noted that the discrepancy was far greater for "soft" rocks, such as sandstone, than for "hard" rocks, such as granite [Rinehart et al. (1961), Sutherland (1962), Cannaday (1964), Simmons and Brace (1965) and King (1966)].

The discrepancies between E_D and E_s have been widely attributed to microcracks and pores, though the situation is complicated by the fact that results are dependent upon both rock microstructure and level of confinement during tests and that both E_n and F_s are effected differently [Ide (1936), Brace (1965 a) and Walsh (1965 a)].

For rocks such as sandstone where cracks are responsible for a relatively small proportion of the total porosity, the compressibility of the specimen decreases (under increasing confining pressure) to a nearly constant value. However, the final compressibility is not that of a "solid" sandstone but that of a porous sandstone, since porosity, other than that caused by narrow cracks, is virtually impossible to remove even under very high con-fining pressures Walsh (1965 b).

According to Brace (1965b) above confining stresses of 2 - 3 kPa, most cracks are closed and their effect is largely eliminated; behaviour of rocks above these pressures is referred to as intrinsic.

The increase in E_D with increasing stress can be attributed to the closing under stress of cracks within the rock, since the elastic waves increase in velocity as the material becomes compacted [Volarovich (1967), Walsh (1966)]. This explanation is also consistent with the observation that the effect is more pronounced at low stresses and is more marked in the direction of the major principal stresses [Anderson et al. (1974)]. Sliding of the surfaces of closed cracks past one another has the effect of decreasing the value of Young's modulus [Wyllie et al. (1956)].

Zisman (1933) provided a qualitative explanation of the consistently higher magnitude of E_D as compared to E_s . He suggested that a wave pulse or packet of energy passing through the rock on entering a cavity (crack, pore) suffers a loss of energy (due to reflection and refraction) at the air/rock, rock/air interfaces. A small amount of wave energy is transmitted across the boundary and a large amount scattered around the cavities. However, as the cavities form a random array in the rock the scattered energy is of little importance and the pulse passes through the rock largely unaffected.

Simmons and Brace (1965) summarised this qualitative explanation by saying that at frequencies of a few mega-cycles a pulse of elastic energy may be expected to bypass cracks; a major part of the energy passes through the specimen at about the speed it would if no cracks were present. Low frequency and static measurements are therefore affected to a greater degree than the dynamic measurements.

Several authors, for example King (1966) and Hardy and Kim (1970), stress the importance of testing under triaxial stress conditions as this is the physical model closest to the in-situ condition. It is also suggested that under triaxially loaded conditions the behaviour of the rock fabric is more readily understood and predictable than in the uniaxially loaded condition.

In uniaxial compression it may be expected that cracks and fractures close rapidly normal to the direction of applied stress and values of E_D and E_s increase Thill and Peng (1969). However, as considerable expansion takes place in the plane normal to the direction of applied stress there is the possibility of the formation of stress induced cracks roughly parallel with the loading direction. This possibility in addition to the problems that occur approaching failure stress, further complicates the measurement of E_D and E_s , and significantly affects the results obtained.

Equipment Development

To allow one to investigate possible differences between statically and dynamically determined elastic rock moduli, such as those on Scarborough sandstone reported within this Note, it is necessary to measure both static and dynamic modulus for each specimen of rock. In any such study aimed at quantifying the effects of rock texture such as microcracks, microfractures and porosity on moduli, testing must be conducted under both triaxial and uniaxial conditions. A further essential requirement, which has not been adequately catered for in earlier investigations, is that moduli measurements be taken "simultaneously", to eliminate problems and inaccuracies arising from hysteresis effects.

It is probable that load cycling would have minimal effect on rock texture under the testing conditions outlined in this note. However, since the primary objective of the tests was to determine the effects of texture on moduli and since load cycling could produce irreversible changes in rock texture, it was decided that sequential measurement of E_D and E_s would make a comparison of the moduli tenuous and unreliable.

For static modulus determinations under both uniaxial and triaxial conditions, conventional resistance strain gauge techniques were used.

However, equipment is not commercially available to allow one to simultaneously measure static and dynamic moduli under triaxial conditions, features regarded as essential in the current study. Development, construction and proving of such an apparatus therefore constitute a necessary pre-requisite to any study such as that planned. Such an apparatus should have the following features.

1. Rapid and adequately accurate measurement of P and S wave velocities under uniaxial and triaxial loading conditions.
2. Readout of resistance strain gauges attached to the sample under test simultaneous with wave velocity measurements.
3. Based upon some widely accepted and applied triaxial cell.
4. Appropriate axial and confining loading pressures can be applied.

Measurement of dynamic modulus is based upon a pulse propagation technique employing piezoelectric transducers. The pulse propagation method involves

measurement of the velocities of the compressional wave (P) and the shear wave (S) associated with a physical pulse established within the test rock.

Piezoelectric ceramics were chosen to produce compressional and shear waves as they could be manufactured to a suitably small size for incorporation into the equipment. The P-wave and S-wave transducers consist of suitably shaped piezoelectric elements, sintered to produce a ceramic and composed of barium titanate, lead zirconate and lead titanate. Proportions of these components can be varied- in production to yield desired ranges of physical and electromechanical properties. The particular ceramic selected is a proprietary mixture of lead titanate and lead zirconate manufactured by Plessy Ducon Pty. Ltd., Villawood, Sydney, New South Wales, and has a frequency constant of 1900 kHzmm. The measured frequencies of the transmitting and receiving transducers were 344 (P), 260 (S) and 343 (P), 253 (S) kHz respectively. The frequency constant is used in conjunction with optimum operating pulse wavelength to determine transducer thickness Howarth (1979).

The ceramic transducers were cut from cylinders of diameter 25 mm. The P-wave elements (Fig. 1) were simply discs of 25 mm diameter and 6.33 mm thickness; the S-wave elements (Fig. 1) producing plane trans-verse shear waves were in sets of three blocks of dimensions 6.33 mm thick

P = COMPRESSIONAL WAVE ELEMENT S = SHEAR WAVE ELEMENTS

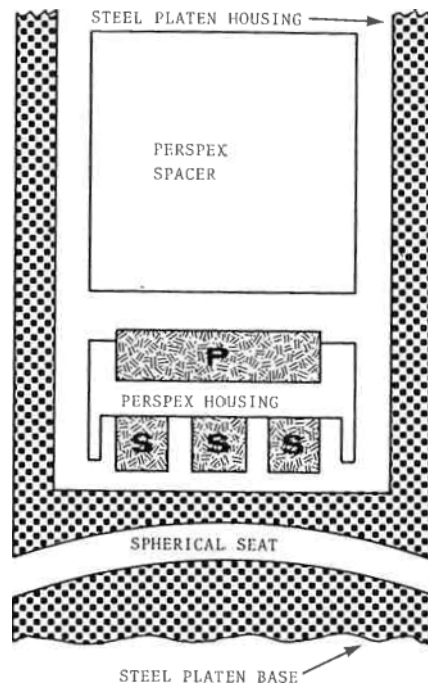


Fig. 1. Exploded diagram showing internal arrangement of components within the steel platens

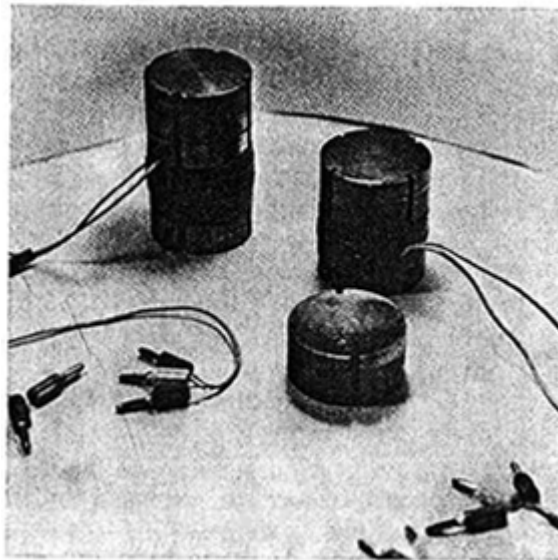


Fig. 2. Protective steel platens suitable to fit an NX Hoek triaxial cell

by 6.10 mm wide by 18.20 mm long, this composite form being suggested by King (1966) to eliminate cross-coupling of unwanted resonant nodes. Details of the ceramic polarizing procedure undertaken by the author and the transducer platen assembly can be found in Howarth (1976).

The advantage of the apparatus lies in the concurrent determination of E_D (dynamic Young's modulus) and E_s (static Young's modulus). Values of E_s were measured using conventional bonded resistance strain gauges. Such techniques are well documented and need no further discussion here. The determination of E_D was facilitated by the incorporation of both P and S wave transducer elements in the final assembly as shown in Fig. 1.

The transducers were housed in two protective steel platens shown in Fig. 2, suitable to fit an NX Hoek triaxial cell.

Proving of Apparatus

The proving tests conducted are listed below.

1. P-Wave and S-wave velocities in two standard materials, mild steel and brass, were determined and compared with accepted standard values. - Table 1 and Table 2.
2. E_s and E_D for a standard isotropic, homogeneous, structureless and linearly elastic material, mild steel, were determined and compared. - Table 3.

Table 1. Comparison of Measured P-Wave and S-Wave Velocities in Mild Steel Cylinders with Accepted Standard Values. Weast (1975)

Specimen	Density tonne m ⁻³	P-wave velocity ms ⁻¹	S-Wave velocity ms ⁻¹	Variation from standard	
				P-wave	S- w ave
1	7.835	5510	3440	-7.5	+
2	7.835	5650	3330	-5.2	+
Standard	7.850	5960	3235		

Table 2. Comparison of Measured P-Wave and S-Wave Velocities in a Brass Cylinder with Accepted Standard Values. Weast (1975)

Specimen	Density tonne m ⁻³	P-wave velocity ms ⁻¹	S-Wave velocity ms ⁻¹	Variation from standard velocity	
				P-wave	S-
Test	8.416	4630	2030	-1.5	-
Standard	8.600	4700	2110	-	-

For isotropic, homogeneous, linearly elastic materials such as steels, lacking the textural discontinuities characteristic of rocks, E_s and E_D are equal. Since one of the important areas of application of the test apparatus is in study of rock texture by measuring differences in E_s and E_D , it follows that to be acceptable the apparatus must yield equal moduli values where in fact no difference occurs in the test specimen.

Table 3. Comparison of Measured Values of E_s and E_D for Two Mild Steel Cylinders

Specimen	E_D	E_s	Variation of E_s from E_D
	GPa	GPa	
1	218±8	204±1	6
2	215±4	202±2	6
Standard*	207	207	0

Dieter (1961)

Experiments on Scarborough Sandstone

Scarborough sandstone in a coal measures sandstone that outcrops at Coal Cliff which is 30 km south of Sydney, Australia. The rock may be described as medium grained, lithic, with angular quartz grains in an argillaceous matrix showing evidence of bedding planes. The average grain size was determined to be approximately 1 mm.

Table 4. Properties of Scarborough Sandstone

Property	Normal to bedding planes	Parallel to bedding planes
Grain size	1 mm	1 mm
Uniaxial compressive strength	97 MPa	127 MPa
Dry density	2196 kg/m ³	2196 kg/m ³
Cohesive strength	11 MPa	15 MPa
Angle of internal friction (range)	47-69°	47-67°
E_s (secant 50 MPa vert* 10 MPa horiz.)	14.6 GPa	16.7 GPa
E_D (50 MPa vert" 10 MPa horiz)	26.5 GPa	25.9 GPa

*Approximate

Note. Elastic moduli were determined on specimens in the vertical (long axis) orientation at a stress level of 50 MPa. During these tests the horizontal confining pressure was kept constant at 10 MPa.

Mechanical property tests were undertaken, in accordance with standard procedures described in Brown (1981). All tests including the static and dynamic elastic moduli experiments were conducted on specimens with a moisture content of zero. Scarborough sandstone has the measured properties detailed in Table 4. These results demonstrate that the material is slightly anisotropic.

Cylindrical rock cores were prepared suitable to fit an NX Hoek triaxial cell. They were instrumented with bonded resistance strain gauges prior to insertion within the cell. Tests were undertaken on two sets of cores, one taken normal to the direction of bedding and the other parallel to the direction of bedding. The two sets of cores were

prepared as the material was clearly anisotropic as demonstrated by the mechanical property tests.

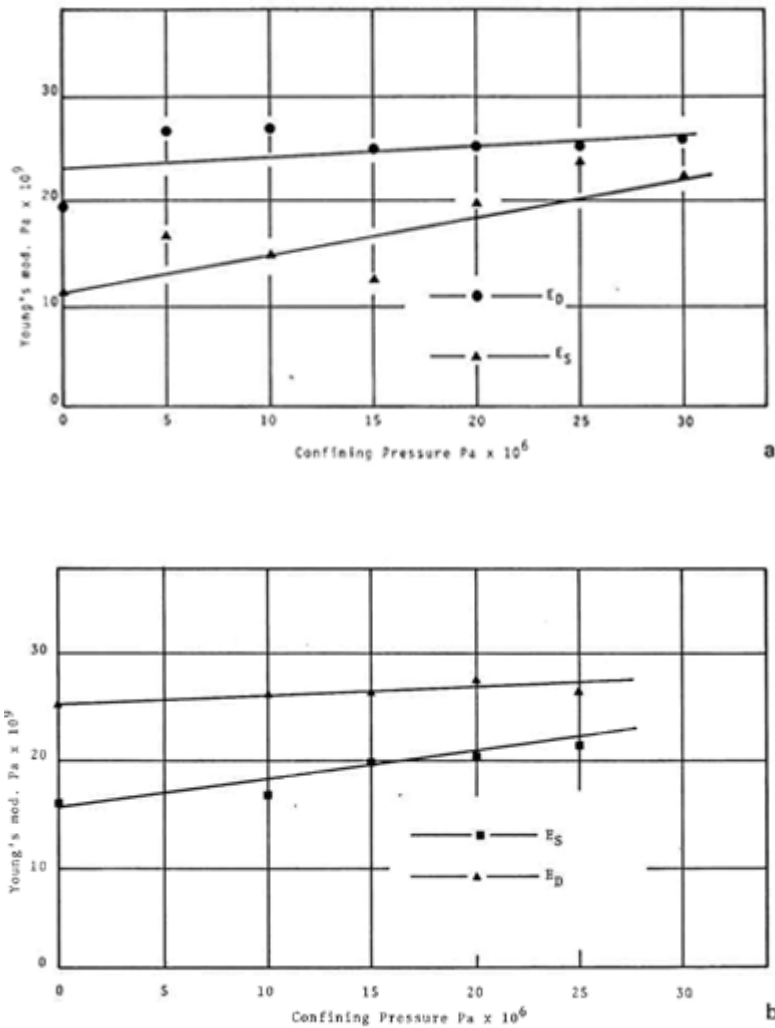


Fig. 3. a) Cores taken normal to the direction of bedding (moisture content equals zero per cent). b) Cores taken parallel to the direction of bedding (moisture content equals zero per cent)
 Plot of E_D and E_S against horizontal confining stress in Scarborough sandstone (vertical stress level = 50 MPa approx. see text)

A series of experiments were undertaken where the samples were loaded axially while maintaining a constant horizontal confining pressure. Measurements of vertical and horizontal strain and P and S wave velocities were taken at regular intervals during the loading and unloading cycles. These experiments were repeated for various levels of horizontal confinement and the results are shown in Figs. 3 a and 3 b. The number of experiments undertaken to prepare Figs. 3 a and 3 b were twenty seven and twenty four respectively. Each point was extracted at an appropriate axial (vertical) stress level of 50 MPa. However, details of the computational procedures in obtaining these values can be found in (Howarth, 1976).

Discussion

The results of this preliminary experimental work show that as confinement increases E_D and E_S tend toward each other.

It is reasonable to assume that as the stress is increased the rock becomes more compact and hence the values of E_n and E_s increase as the rock becomes "stiffer". At low confining pressures, microcracks and micro-fissures are open and because the dynamic method of measuring moduli is far less sensitive to the presence and closure of cracks and fissures than the static method, the difference between E_D and E_s is large. However, as the stress is increased and the microcracks and fissures tend to close, E_s rises far more rapidly than the corresponding value of E_D , due to this differential sensitivity. Several authors, [Boozer et al. (1963) and Kuster and Toksoz (1974)] have partly attributed differences in E_D and E_s to the degree of water saturation, however, since in these tests the moisture content was zero this explanation is inapplicable.

The anisotropy of the material is also clearly demonstrated in Figs. 3 a and 3b. However, on superposition of the two graphs it can be seen that at high confining pressures the effect of anisotropy has decreased. This may be due to the more homogeneous texture of the rock that will result from the high stresses imposed on it.

Conclusions

1. The apparatus makes it possible to concurrently measure E_s and E_D of cylindrical rock specimens under triaxial conditions in the NX Hoek cell.
2. Accuracy of the apparatus is comparable to usual dynamic modulus testing.
3. Preliminary test results for one particular rock type, show that under triaxial conditions the differences between E_D and E_s are smaller at high confining pressures than at low confining pressures. Circumstantial evidence suggests that microcracks and microfissures are responsible for this phenomenon. Further work needs to be undertaken in other rock types to substantiate these preliminary findings.

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References

- Anderson, I., et al. (1974): The Effect of Oriented Cracks on Seismic Velocities. *J. Geophys. Res.* 26, 4011-4015.
- Baron, G., et al. (1964): The Effect of Stresses and Deformations on the Velocity of Sound in Rock. Fourth International Conference on Strata Control and Rock Mechanics: Columbia University, New York. 474-483.
- Boozer, G. E., Hiller, K. H., Serdengecti, S. (1963): Effects of Pore Fluids on the Deformation Behaviour of Rocks Subjected to Triaxial Compression. *Proc. 5th U. S. Rock Mech. Symp. Minneapolis, U. S. A.* 579-625.
- Brace, W. F. (1965 a): Relation of Elastic Properties of Rocks to Fabric. *J. Geophys. Res.* 70 (No. 22), 5657-5667.
- Brace, W. F. (1965b): Some New Measurements of Linear Compressibility of Rocks. *J. Geophys. Res.* 70 (No.2), 391-398.
- Brown, E. T. (Ed.) (1981) : *Rock Characterization Testing and Monitoring. ISRM Suggested Methods.* Pergamon (U. K.), pp. 211.
- Cannaday, F. X. (1964): Modulus of Elasticity of a Rock Determined by Four Different Methods, *U. S. B. M., R. I.* 6533.
- Dieter, G. E. (1961): *Mechanical Metallurgy.* New York: McGraw Hill.
- Hardy, H. R., Kim, Y. S. (1970): Detection of a Low Level Critical Stress in Geologic Materials Using Ultrasonic Techniques, *Proceedings of the Twelfth Symposium on Rock Mechanics. Missouri, U. S. A. Soc. Min. Engrs. - Amer. Inst. Min. Metall. Petrol Engrs.* 301-327.
- Howarth, D. F. (1976): A Comparison of Statically and Dynamically Determined Elastic Moduli of Scarborough Sandstone. M. E. Thesis, University of New South Wales, Australia, 136.
- Howarth, D. F. (1979): Development of an Apparatus for Concurrent Determination of

- Static and Dynamic Young's Moduli of Rock Cores. *Aust. Geomech. J.* 9, 14-21.
- Ide, J. M. (1936): Comparison of Statically and Dynamically Determined Young's Modulus of Rocks, *Proc. Nat. Acad. Sciences, U. S. A.*, 81-92.
- King, M. S. (1966): Wave Velocities in Rocks as a Function of Changes in Overburden Pressure and Pore Fluid Saturants, *Geophysics* 31 (No. 1), 50-73.
- Kuster, G. T., Toksoz, M. N. (1974): Velocity and Attenuation of Seismic Waves in Two Phase Media. Theoretical Calculations. Pt. 1. *Geophysics* 39, 587-606.
- Rinehart, J. S., Fortin, J.-P., Baugin, P. (1961): Propagation Velocity of Longitudinal Waves in Rock. Effect of State of Stress, Stress Level of the Wave, Water Content, Porosity, Temperature Stratification and Texture. *Proc. Fourth Symp. Rock Mech.* 119-135, Penn. State University.
- Simmons, G., Brace, W. I. (1965) : Comparison of Static and Dynamic Measurements of Compressibility of Rocks: *J. Geophysical Res.* 70 (No. 22), 5649-5656.
- Sutherland, R. B. (1962): Some Dynamic and Static Properties of Rock. *Rock Mechanics. Proc. Fifth Symp. Rock Mech.*, University Minnesota, 473-491.
- Thill, R. E., Peng, S. S. (1969): Statistical Comparison of the Pulse and Resonance Methods for Determining Elastic Moduli. *U. S. B. M., R. I.* 7831.
- Volarovich, M. P. (1967): An Experimental Investigation of Rupture and Elastic Wave Propagation and Absorption in Rocks Under High Confining Pressure. *Geophys. J. R. Astr. Soc.* 14, 73-79.
- Walsh, J. B. (1966): Seismic Wave Attenuation in Rock Due to Friction. *J. Geophysical Res.* 71 (No. 10), 2591-2599.
- Walsh, J. B. (1965 a): The Effect of Cracks on the Uniaxial Elastic Compression of Rocks. *J. Geophysical Res.* 70 (No. 2), 399-411.
- Walsh, J. B. (1965b): The Effect of Cracks on the Compressibility of Rock. *J. Geophysical Res.* 70 (No. 2), 381-389.
- Weast, R. C. (Ed.) (1975): *Handbook of Chemistry & Physics.* CRC Press.
- Wyllie, M. R. J., Gregory, A. R., Gardner, G. H. F. (1956): Elastic Wave Velocities in Heterogeneous and Porous Media. *Geophys.* 21 (No. 1), 41-70.
- Zisman, W. A. (1933): Comparison of the Statically and Seismologically Determined Elastic Constants of Rocks. *Proc. Nat. Acad. Sci.* 19, 680-686.