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# Critical Flow Constrains Flow Hydraulics in Mobile-Bed Streams : a New Hypothesis by GRANT, G.E. (1997), *Water Res. Res.*, Vol. 33, No. 2, pp. 349-358, paper 96WR03134.

**Discussion by**

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Abstract :

The writer discusses the challenging ideas proposed by GRANT (1997) on steep movable bed channel flows. First the definition of critical flow are revisited. Then, the calculation of bed shear stress in near-critical flows are discussed. New experimental data are presented, and they highlight the three-dimensional variations of boundary shear stress. It is believed further that the applicability of GRANT's (1997) hypothesis is restricted because of the confusion between critical and near-critical flows.

The writer wishes to congratulate the author for some interesting and challenging ideas on steep movable bed channel flows. The discusser would like to comment on the definition of critical flow (based on eq. (1)), the calculation of bed shear stress in near-critical flows and the applicability of the author's hypothesis. He hope that the information will assist to refine the author's developments.

## 1. Definition of critical flow conditions

Historically critical flow conditions were defined as the flow properties at the singularity of the backwater equation (BELANGER 1828, BAZIN 1865). In the general case of a channel of non constant shape and longitudinal bed slope, the one-dimensional form of the energy equation (i.e. the backwater equation) becomes :

$$\frac{\partial H}{\partial x} = \frac{\partial d}{\partial x} \cos\theta - d \sin\theta \frac{\partial \theta}{\partial x} + \frac{\partial z_o}{\partial x} - \alpha \frac{Q^2}{g A^3} \frac{\partial A}{\partial x} = - S_f \qquad (1\text{-}1)$$

where H is the mean total head [ i.e. average over the flow cross-section area], x is the longitudinal co-ordinate in the flow direction, d is the flow depth measured normal to the channel bottom, $\theta$ is the longitudinal bed slope, $z_o$ is the bed elevation, $\alpha$ is the kinetic energy correction coefficient (i.e.

Coriolis coefficient), Q is the water discharge, A is the cross-section area, B is the free-surface width, $S_f$ is the friction slope :

$$S_f = f \frac{1}{D_H} \frac{V^2}{2\,g} \qquad (1\text{-}2)$$

f is the Darcy friction factor, $D_H$ is the hydraulic diameter and V is the mean flow velocity (V = Q /A).

For a constant channel shape, $\partial A = B\,\partial d$ and equation (1-1) becomes :

$$\frac{\partial d}{\partial x} = \frac{\sin\theta - S_f + d\sin\theta \frac{\partial \theta}{\partial x}}{\cos\theta - \alpha \frac{Q^2 B}{g\,A^3}} \qquad (1\text{-}3)$$

More recently (BAKHMETEFF 1912,1932, HENDERSON 1966) critical flow conditions were defined as the flow properties (depth and velocity) for which the mean specific energy is minimum for a given flow rate and channel cross-section shape. The mean specific energy is defined as :

$$E = d\cos\theta + \alpha \frac{V^2}{2\,g} \qquad (1\text{-}4)$$

assuming a hydrostatic pressure distribution. E is similar to the energy per unit mass, measured with the channel bottom as the datum. In the general case of a non-rectangular channel (e.g. HENDERSON 1966, p. 51), the mean specific energy is minimum for :

$$\frac{\partial E}{\partial d} = \cos\theta - \alpha \frac{Q^2 B}{g\,A^3} = 0 \qquad (1\text{-}5)$$

Today this second definition of critical flow (i.e. Eq. (1-3)) is commonly used. Equation (1-5) and Equation (1-3) for θ constant yield the criterion for critical flow conditions :

$$\alpha \frac{Q^2 B}{g\cos\theta\,A^3} = 1 \qquad (1\text{-}6)$$

Equation (1-6) is more general than the author's equation (1) which did not take into account the bed slope effect nor the cross-section shape. α is larger than unity although rarely exceeds 1.15, and the ratio $\alpha/\cos\theta$ typically ranges between 1 and 1.3 in natural streams.

## 2. Near-critical flows and bed-shear stress

Near-critical flows may be defined as flow situations characterised by the occurrence of critical or nearly-critical flow conditions over a "reasonably-long" distance and time period. The specific energy/flow depth diagram shows that, near the critical flow conditions, a very-small change of energy (e.g. caused by a bottom or sidewall irregularity) can induce a very-large change of flow depth. Near-critical flows are indeed characterised by the development of large free-surface undulations (e.g. IMAI and NAKAGAWA 1992, CHANSON and MONTES 1995, CHANSON 1995,1996, MONTES and CHANSON 1998). Experimental observations showed further that "undular flows" may take place for $0.3 \le Fr \le 3$, but this range of flow conditions could be broader depending upon the boundary conditions.

The discusser performed new experiments in a 20-m long fixed-bed channel of rectangular cross-section (W = 0.25 m) to investigate the boundary shear stress under an undular hydraulic jump. The bed shear stress was measured with a Prandtl-Pitot tube (∅=3.3 mm) used as a Preston tube (see appendix).

Bed-shear stress measurements under an undular jump (in a fixed-bed channel) are presented in figure 1-1, in which the bed shear stress at various positions across the channel (z/W = 0 at the sidewall, z/W = 0.5 on the centreline) is plotted as a function of the dimensionless distance $x/d_c$ where x is the distance from the channel intake and $d_c$ is the critical flow depth. On the same graph the dimensionless flow depth $d/d_c$ is shown also.

The discusser's data show large fluctuations of bed shear stress in the cross-wise and longitudinal directions. Typically the bed shear is minimum below the wave crests and maximum underneath the wave troughs. The results are similar to the findings of IMAI and NAKAGAWA (1992).

The results have direct implications to natural mobile-bed channels. Considering a flat movable-bed stream, an undular flow might take place during a flood event or in an estuary during a period of the tide. Below the free-surface undulations, the movable bed becomes subjected to an non-uniform boundary shear stress distribution (fig. 1-1) and, as a result, erosion may take place underneath the wave troughs while accretion occurs below the wave crests. Altogether the conditions are favourable for the development of standing-wave bed forms (in phase with the free-surface standing waves).

Note further that the boundary shear stress is consistently smaller near the wall (black square in fig. 1-1) than on the channel centreline (white square, fig. 1-1). Hence sediment motion is likely to be more intense near the channel centreline than next to the banks.

## 3. Discussion

The author's hypothesis and the corollary that "assumption of critical flow would essentially replace the standard flow resistance equations" might not be an ultimate achievement. Even if the limiting state of mobile-bed flows is critical flow, can the flow conditions be predicted accurately ?

The discusser does not think so. Indeed the present discussion has highlighted some aspects of the complexity of undular flows. Figure 1-1 highlights that the boundary shear stress distribution along undular flow (in a fixed-bed channel) is affected by large changes in both the lateral and longitudinal directions. In figure 1-1, the bed shear stresses fluctuate by one order of magnitude in the longitudinal and lateral directions. The author's use of empirical flow resistance correlations (author's equations (3), (8) and (13)) imply an "uniformly distributed" boundary shear stress. Such an assumption is refuted by the discusser's results (fig. 1-1).

Further the author's developments deriving from his "hypothesis" are based on empirical correlations which do not reflect the physical nature of the energy losses nor the interactions between the flow and the movable boundaries. In an alluvial channel, the boundary friction is related to the skin friction (or grain-related friction) and to the form losses caused by the bed forms. Although the skin friction shear stress (or effective shear stress) may be calculated accurately, the estimate of the bed-form shear stress is difficult. Fundamental experiments in irregular open channels (KAZEMIPOUR and APELT 1983) showed that the form losses could account for up to 92% of the total loss. For undular flows, there is little experimental data to predict accurately the form losses associated with the standing wave bed forms, but this situation does not justify the use of "overall bed shear stress" correlations.

In summary the author's hypothesis of critical flow cannot be considered as an ultimate simplification. The present discussion and the works of IMAI and NAKAGAWA (1992), CHANSON (1995), CHANSON and MONTES (1995), and MONTES and CHANSON (1998) highlighted the complexity of undular flows.

## APPENDIX : CALIBRATION OF PITOT TUBE

The Pitot tube was calibrated in uniform equilibrium flows. The calibration curve (Pitot tube velocity versus boundary shear stress) was best fitted by :

$$\tau_o = 3.1821 \, V_b^{2.0275} \qquad (2\text{-}1)$$

was similar to calibration curves obtained by PRESTON (1954) and PATEL (1965), and MACINTOSH (1990) who used the same experimental channel. $\tau_o$ is the boundary shear stress and $V_b$ is the velocity measured by the Pitot tube lying on the bed.

**REFERENCES**

BAKHMETEFF, B.A. (1912). "O Neravnomernom Dwijenii Jidkosti v Otkrytom Rusle." ('Varied Flow in Open Channel.') St Petersburg, Russia (in Russian).

BAKHMETEFF, B.A. (1932). "Hydraulics of Open Channels." *McGraw-Hill*, New York, USA, 1st ed., 329 pages.

BAZIN, H. (1865). "Recherches Expérimentales sur l'Ecoulement de l'Eau dans les Canaux Découverts." ('Experimental Research on Water Flow in Open Channels.') *Mémoires présentés par divers savants à l'Académie des Sciences*, Paris, France, Vol. 19, pp. 1-494 (in French).

BELANGER, J.B. (1828). "Essai sur la Solution Numérique de quelques Problèmes Relatifs au Mouvement Permanent des Eaux Courantes." ('Essay on the Numerical Solution of Some Problems relative to Steady Flow of Water.') *Carilian-Goeury*, Paris, France (in French).

CHANSON, H. (1996). "Free-Surface Flows with Near-Critical Flow Conditions." *Can Jl of Civ. Eng.*, Vol. 23, No. 6, pp. 1272-1284.

CHANSON, H. (1995). "Flow Characteristics of Undular Hydraulic Jumps. Comparison with Near-Critical Flows." *Report CH45/95*, Dept. of Civil Engineering, University of Queensland, Australia, June, 202 pages.

CHANSON, H., and MONTES, J.S. (1995). "Characteristics of Undular Hydraulic Jumps. Experimental Apparatus and Flow Patterns." *Jl of Hyd. Engrg.*, ASCE, Vol. 121, No. 2, pp. 129-144. Discussion : Vol. 123, No. 2, pp. 161-164.

HENDERSON, F.M. (1966). "Open Channel Flow." *MacMillan Company*, New York, USA.

IMAI, S., and NAKAGAWA, T. (1992). "On Transverse Variation of Velocity and Bed Shear Stress in Hydraulic Jumps in a Rectangular Open Channel." *Acta Mechanica*, Vol. 93, pp. 191-203.

KAZEMIPOUR, A.K., and APELT, C.J. (1983). "Effects of Irregularity of Form on Energy Losses in Open Channel Flow." *Aust. Civil Engrg Trans.*, I.E.Aust., Vol. CE25, pp. 294-299.

MACINTOSH, J.C. (1990). "Hydraulic Characteristics in Channels of Complex Cross-Section." *Ph.D. thesis*, Univ. of Queensland, Dept. of Civil Eng., Australia, Nov., 487 pages.

MONTES, J.S., and CHANSON, H. (1998). "Characteristics of Undular Hydraulic Jumps. Results and Calculations." *Jl of Hyd. Engrg.*, ASCE, Vol. 124, No. 2, pp. 192-205.

CHANSON, H. (1999). "Critical Flow Constrains Flow Hydraulics in Mobile-Bed Streams : a New Hypothesis - Comment." *Water Res. Res.*, Vol. 35, No. 3, p. 903-907 (ISSN 0043-1397).

PATEL, V.C. (1965). "Calibration of the Preston Tube and Limitations on its use in Pressure Gradients." *Jl of Fluid Mech.*, Vol. 23, No. 1, pp. 185-208.

PRESTON, J.H. (1954). "The Determination of Turbulent Skin Friction by Means of Pitot Tubes." *Jl Roy. Aeronaut. Soc., London*, Vol. 58, February.

**NOTATION**

| | |
|---|---|
| A | cross-section area ($m^2$); |
| B | free-surface width (m); |
| $D_H$ | hydraulic diameter (m); |
| d | flow depth (m); |
| $d_c$ | critical flow depth (m); |
| E | mean specific energy (m); |
| f | Darcy friction factor; |
| g | gravity constant ($m/s^2$); |
| H | mean total head (m); |
| Q | discharge ($m^3/s$); |
| $S_f$ | friction slope; |
| V | mean flow velocity (m/s) : V = Q/A; |
| $V_b$ | velocity (m/s) measured close to the channel bed; |
| W | channel width (m); |
| x | longitudinal co-ordinate (m); |
| z | cross-wave co-ordinate (m) : z = 0 at the wall; |
| $z_o$ | bed elevation (m); |
| $\theta$ | mean channel slope; |
| $\tau_o$ | bed shear stress (Pa). |

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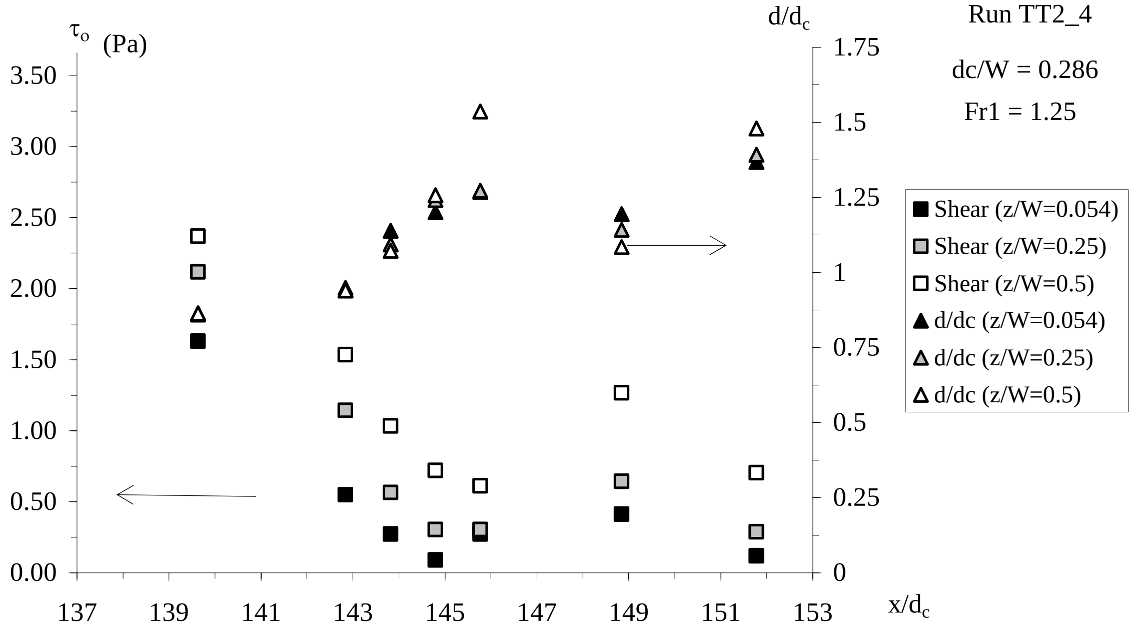
Fig. 1-1 - Bed shear stress along an undular hydraulic jump

(A) Flow conditions : Fr = 1.25, $d_c/W$ = 0.286, W = 0.25 m (Run TT2_4)




(B) Flow conditions : Fr = 1.35, $d_c/W$ = 0.286, W = 0.25 m (Run TT2_1)