Non-Newtonian effects in simple models of mantle convection

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Abstract

One of the difficulties with self consistent plate-mantle models capturing multiple physical features, such as elasticity, non-Newtonian flow properties, and temperature dependence, is that the individual behaviours cannot be considered in isolation. For instance, if a viscous mantle convection model is generalized idealistically to include hypo-elasticity, then problems based on Earth-like Rayleigh numbers exhibit almost insurmountable numerical stability issues due to spurious softening associated with the co-rotational stress terms. These difficulties can be avoided if a stress limiter is introduced in the form of a power law rheology or yield criterion. A general Eulerian model is discussed and it is shown that the basic convection modes of a cooling planet are reproduced.

Keywords: Mantle convection; Non-Newtonian flow; Stress rate; Episodicity; Visco-elastic-plastic deformation; Numerical stability

1. Introduction

We apply a general Eulerian model [1] applicable to a wide range of existing fluid dynamics problems. The approach considers combined Newtonian and power law creep as well as elasticity and temperature dependence of the creep parameters. As the deformations involved in geological deformation are large, the constitutive relationships must contain geometric terms to ensure that the tensor properties of the model are conserved. A model with such properties is described as being 'objective'.

A wide range of objective, incompressible, visco-elastic-plastic models exist that differ in the choice of the objective stress rate, such as the Jaumann, Oldroyd, or Truesdell rates (refer to Kolymbas et al. [2] for a recent discussion). The salient features of these objective stress rates can be studied with homogeneous simple and pure shear flows. For simple shear flows, a comparison can be made between the shear stress–shear strain curves for a constant applied shear strain rate, assuming infinitesimal theory (i.e. no co-rotational stress terms) and adopting Jaumann and Naghdi models respectively.

Viscous deformation is described by a combined Newtonian and power law creep model. The power law viscosity includes a contribution from dislocation glide, a typical power law exponent ($n = 3$), and a contribution from plastic deformations with temperature independent coefficients and a large exponent ($n = 15$). The effective viscosity is given by:

$$\eta_{eff} = \left(\frac{1}{\eta_N} + \frac{1}{\eta_N \left(\frac{\tau}{\tau_0}\right)^{1-n}} + \frac{1}{\eta_Y \left(\frac{\tau}{\tau_Y}\right)^{1-n_{ol}}}\right)^{-1} \tag{1}$$

where η_N is the temperature dependent Newtonian viscosity, η_Y is a reference viscosity for the plastic deformation, τ_0 is the transition stress, τ_Y is the yield stress, and τ is the second deviatoric stress invariant.

We illustrate the various non-Newtonian effects in Figs 1 and 2 by means of results for the relatively extreme case $DE = (\eta_N/\mu)\dot{\gamma} = 1$ (e.g. $(10^{25}$ Pas$/10^{11}$ Pa$)10^{-14}$ s^{-1}). The stress response for $\dot{\gamma} =$ const with and without Jaumann terms is compared. Also shown for comparison is the response for a Maxwell model based on Naghdi's definition of the co-rotational rate (see [2]).

The responses of the infinitesimal model (no co-rotational terms) and the Naghdi definition (with spin $\omega = -\dot{\gamma}/2(1 + (\gamma/2)^2)$ are qualitatively similar, whereas the Jaumann model (with spin $\omega = -\dot{\gamma}/2$) exhibits a spurious softening effect. As there is no experimental

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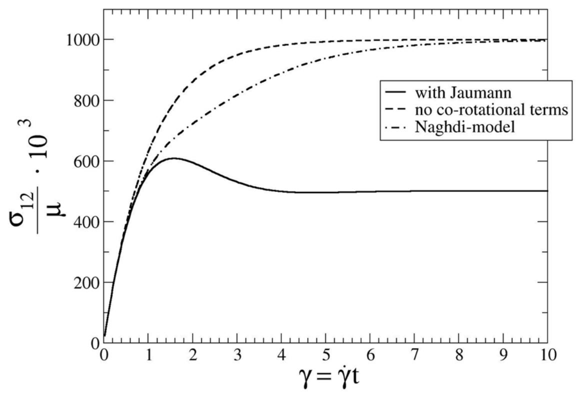

Fig. 1. The simple shear of a Maxwell model with Newtonian (linear) rheology and Deborah number De = (η_N/ μ)\gamma= 1.

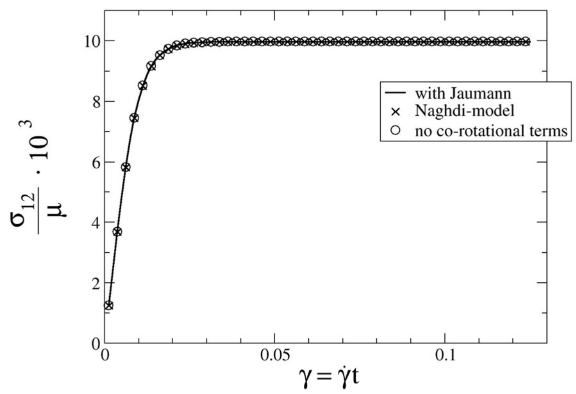

Fig. 2. Simple shear of a Maxwell model with a combined Newtonian and power law (n = 3) rheology. The dimensionless strain rate is De = 1 as in the previous case. The transition stress is $\tau_0 = 10^{-3}\,\mu$.

evidence for this kind of purely geometric softening in rocks and metals, the effect is considered to be an unwanted side effect. However, does this mean we have to abandon the Jaumann model, which is computationally much more efficient and simpler to implement than the Naghdi model for instance?

Figure 2 shows the results for a combined Newtonian and power law rheology with $n = 3$ assuming the same dimensionless strain rate as in the examples displayed in Fig. 1. In this case the spurious softening behaviour disappears as do most of the differences between the three models. We conclude (see [1]) that the spurious softening in simple shear disappears if stress limiters in the form of power law creep or a yield criterion (e.g. [2]) are taken into account.

While the co-rotational terms are insignificant in simple shear and pure shear they are of crucial importance in so called geometric instability problems such as folding (buckling) and necking (boudinage). See Biot [3] for discussions and analytical solutions of a wide variety of geometric instability problems.

2. Episodicity

The basic modes of convection applicable to a cooling planet, such as stagnant lid, episodic resurfacing and mobile lid convection have been reproduced with the non-linear viscoelastic approach and are shown in Figs 3(a), 3(b), and 3(c) respectively. The vertical spikes on top of the velocity streak-line plot in each of the subfigures are representations of the cold boundary velocities: larger spikes represent lower velocities, and

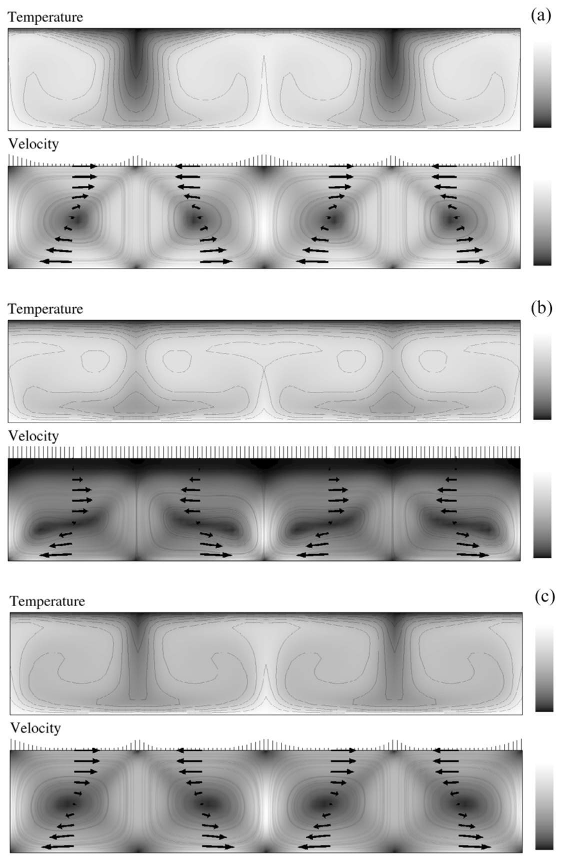

Fig. 3. (a) Typical temperature and velocity distributions for episodic convection at a maximum of the Nusselt number (refer to Fig. 4). (b) Typical temperature and velocity distributions for episodic convection at a minimum of the Nusselt number (refer to Fig. 4). (c) Typical temperature and velocity distributions at steady state for mobile lid convection. For mobile lid convection, significant parts of the top layer move like rigid bodies.

smaller spikes represent higher velocities. A comparison between the Nusselt numbers for the stagnant, episodic, and mobile lid cases is shown in Fig. 4.

A slight but noticeable shift in parameter values and validity fields for cases including elasticity has been recorded. In addition, the buffering action of elasticity permits solutions to extreme viscosity variations and introduces long-range interactions. This results in an ordering and stabilization of patterns of convection at high Rayleigh numbers, replacing smaller-scale turbulence by larger planetary-scale re-mobilization.

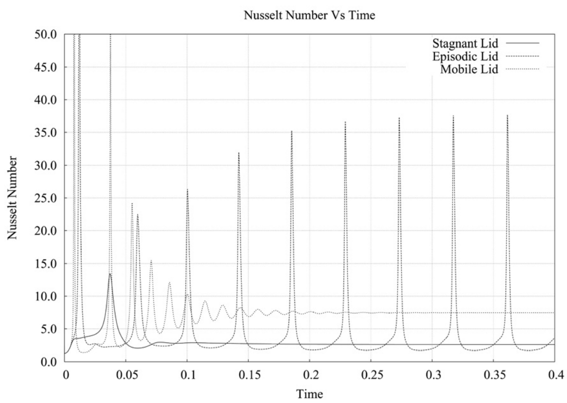

Fig. 4. A comparison of Nusselt numbers for stagnant-lid (lowest with steady state) episodic and mobile lid convection. The yield stress, τ_Y, is three times the transition stress $\tau_0 = 0.866 \times 10^{2.5}$ (i.e. the transition from Newtonian power law creep) and dimensionless shear modulus 10^4. An Arrhenius relation describes the temperature dependence of creep with a viscosity contrast across the layer of 10^5. The power law exponents are $n = 3$ and $n = 15$ (i.e. dislocation glide and plastic deformation).

3. Conclusions

Due to increasing computational power, there has been a significant break-through in large-scale geodynamical modeling over recent years [4,5,6,7]. Large-scale convection models can now reproduce basic modes of planetary tectonics as self-consistent features of the same physical planetary heat transfer problem. Yet, while such unified models have demonstrated significant detail, a fundamental rheological ingredient has been left unexplored. To obtain models with Earth-like tectonics, a further fine-tuning of rheology is required. Thus, the original motivation for this study was to integrate elasticity into a non-linear convection model to deal with the important problem of coupling a solid-like lithosphere to a fluid-like mantle in a self-consistent manner. Preliminary results are discussed in the present study.

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References

[1] Mühlhaus H-B, Regenauer-Lieb K. A self-consistent plate mantle model that includes elasticity: computational aspects and application to basic modes of convection. Geophys J Int 2004 (submitted). (Preprint available at: http://www.esscc.uq.edu.au/~muhlhaus/PDF/viscoel-revise-26–08–04-Hans.doc)

[2] Kolymbas D, Herle I. Shear and objective stress rates in hypoplasticity. Int J Numer Analytical Methods Geomech 2003;27(9):733–744.

[3] Biot M. The Mechanics of Incremental Deformations. New York: John Wiley, 1965.

[4] Moresi L, Dufour F, Mühlhaus H-B. Mantle convection models with viscoelastic/brittle lithosphere: numerical methodology and plate tectonic modeling. Pure Appl Geophys 2002;159(10):2335–2356.

[5] Moresi L, Solomatov V. Mantle convection with a brittle lithosphere: thoughts on the global tectonic styles of the earth and venus. Geophys J Int 1998;133(3):669–682.

[6] Tackley P. Self-consistent generation of tectonic plates in time-dependent, three-dimensional mantle convection simulations, 1: pseudoplastic yielding. Geochem Geophys Geosyst 2000;1(8):doi: 10.1029/2000GC000036.

[7] Tackley P. Self-consistent generation of tectonic plates in time-dependent, three-dimensional mantle convection simulations, 2: strain weakening and asthenosphere. Geochem Geophys Geosyst 2000;1(8):doi: 10.1029/2000GC000043.