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Flow resistance in skimming flow: A critical review

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ABSTRACT: During very large rainfall events, dams and weirs must be equipped with adequate flood release facilities for a safe dissipation of the kinetic energy of the flow. With stepped spillway design, it is essential to predict accurately the flow resistance associated with the steps. The authors investigate the flow resistance of skimming flows and associated form losses. New experiments were performed with channel slopes ranging from 5.7° up to 55°. The results provide a better understanding of the basic flow patterns and flow resistance mechanisms. They emphasise that form loss is dominant. Altogether more than 35 model studies and 4 prototype investigations (totalising more than 650 data) are re-analysed.

1. INTRODUCTION

During large rainfall events, the flood waters make way through, above and beside man-made dams. Significant damage may occur if the kinetic energy of the flow is not dissipated safely. One type of flood release facility is the stepped spillway design. It is characterised by significant flow resistance and associated energy dissipation taking place on the steps. The world's oldest stepped spillway is presumably the overflow stepped weir in Akarnania, Greece, built around B.C. 1,300. The weir, 10.5-m high with a 25-m long crest, is equipped with a stepped overflow (14 steps, $\theta \sim 45°$, h = 0.6 to 0.9-m). Although modern RCC stepped spillways are designed for a skimming flow regime (Fig. 1), there is some controversy on an accurate estimate of the flow resistance. Model and prototype data exhibit little correlation, being scattered over three orders of magnitude (e.g. CHANSON 1995a).

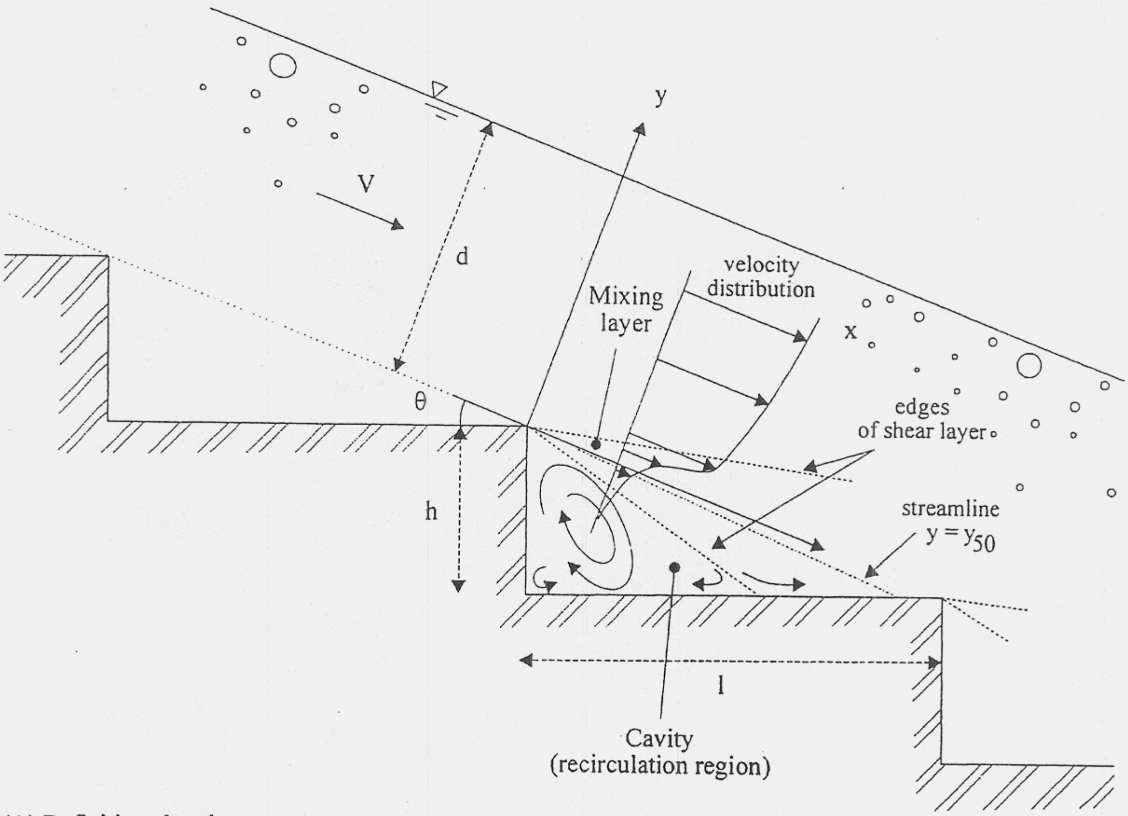
In the present paper, the authors investigate the flow resistance in skimming flows. New experiments were performed with channel slopes ranging from 5.7 up to 55°. Basic flow patterns are presented and the experimental results are compared with existing data: i.e., over 35 model and 4 prototype studies are re-analysed including more than 650 data (Table 3). It is the purpose of this paper to assess critically the overall state of this field and to present new compelling conclusions valid for both flat and steep spillways with skimming flows.

2. BASIC EQUATIONS

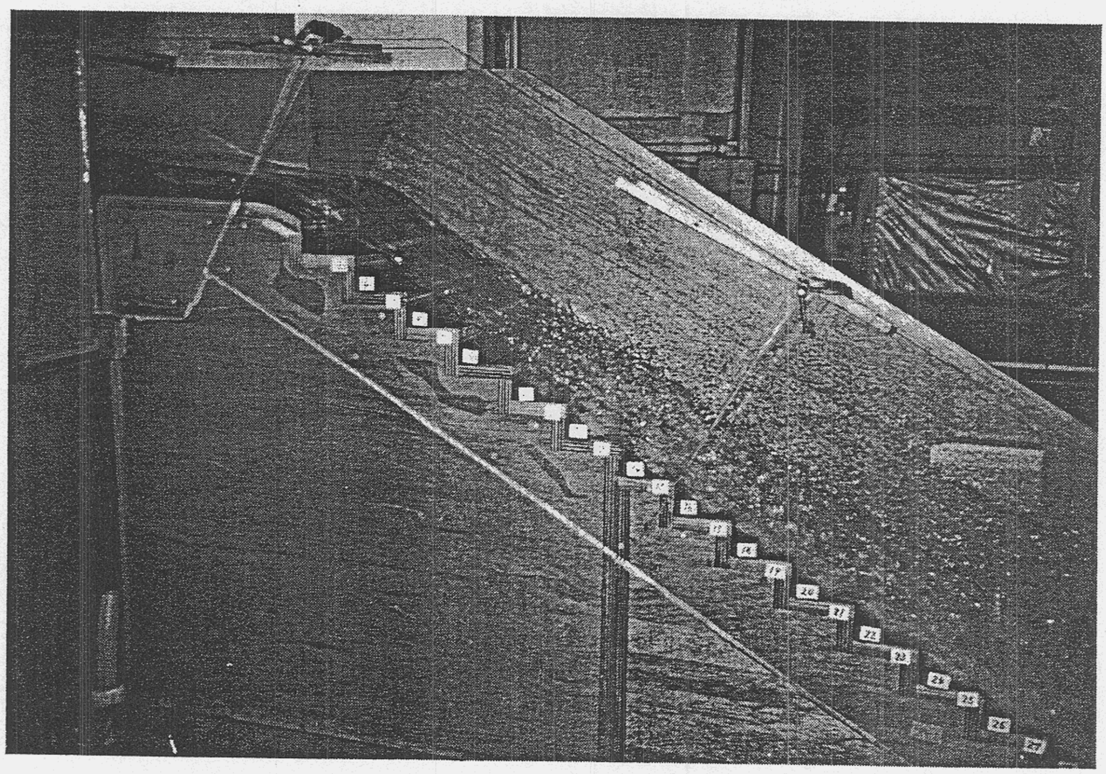
While the flow resistance on smooth-invert chutes is primarily skin friction, skimming flows over stepped chute are characterised by significant form losses. The water skims over the step edges with formation of recirculating vortices between the main stream and the step corners (Fig. 1). In *uniform equilibrium flows*, the momentum principle states that the boundary friction force equals EXACTLY the gravity force component in the flow direction : i.e., the WEIGHT-OF-WATER component acting parallel to the pseudo-bottom formed by the step edges. It yields:

$$\tau_o * P_w = \rho_w * g * A_w * \sin\theta \qquad \text{Uniform equilibrium} \quad (1)$$

where τ_o is the average shear stress between the skimming flow and the recirculating fluid underneath, P_w is



(A) Definition sketch



(B) Experimental channel at Nihon University (θ = 30 deg., W = 0.4 m, h = 0.05 m, d_c/h = 0.99)

Fig. 1 - Skimming flow

the wetted perimeter, ρ_w is the water density, g is the gravity constant, A_w is the water flow cross-section area and θ is the mean bed inclination angle. For a wide channel, Equation (1) becomes :

$$\tau_o = \left(\int_{y=0}^{y=Y_{90}} \rho_w * (1 - C) * dy \right) * g * \sin\theta \qquad \text{Uniform equilibrium (wide channel)} \quad (2)$$

where C is the void fraction and Y_{90} is the distance normal to the pseudo-bottom where C = 90%. KAZEMIPOUR and APELT (1983) stressed that to try to account for the form losses with a Gauckler-Manning or Darcy-Weisbach formula is unsatisfactory. Nevertheless it is still common practice, including for skimming flows and this study is no exception. By analogy with clear-water open channel flows, the average boundary shear stress may be expressed in terms of the Darcy friction factor f as:

$$\tau_o = \frac{f}{8} * \rho_w * V^2 \qquad (3)$$

where V is the mean flow velocity (or equivalent clear-water flow velocity) defined as:

$$V = q_w * \left(\int_{y=0}^{y=Y_{90}} (1 - C) * dy \right)^{-1} \qquad (4)$$

In *gradually-varied flows*, the friction factor *MUST* be deduced from the friction slope S_f. [The friction slope is the slope of the total head line (HENDERSON 1966, CHANSON 1999).]. It yields :

$$V^2 = \frac{8 * g}{f} * \left(\int_{y=0}^{y=Y_{90}} (1 - C) * dy \right) * S_f \qquad \text{Gradually-varied flow (wide channel)} \quad (5)$$

Free-surface aeration is always substantial in prototype skimming flows and its effects should not be neglected. Downstream of the air entrainment inception point, the distribution of air concentration may be described by a diffusion model :

$$C = 1 - \tanh^2\left(K' - \frac{y}{2*D'*Y_{90}} \right) \qquad (6)$$

where tanh is the hyperbolic tangent function and y is the distance normal to the pseudo-bottom formed by the step edges, D' is a dimensionless turbulent diffusivity and K' is an integration constant (CHANSON 1995b,1997). D' and K' are functions of the mean air content C_{mean} only and they may be estimated as:

$$D' = (0.848*C_{mean} - 0.00302)/(1 + 1.1375*C_{mean} - 2.2925*C_{mean}^2) \qquad C_{mean} < 0.7 \quad (7)$$

$$K' = 0.32745015 + 0.5/D' \qquad (8)$$

Equation (6) was tested successfully with stepped chute data obtained in models and prototype (BAKER 1994, RUFF and FRIZELL 1994, TOZZI et al. 1998, CHAMANI and RAJARATNAM 1999, MATOS et al. 1999, Present study).

3. FLOW RESISTANCE IN SKIMMING FLOWS

Recent laboratory studies of stepped channels provided significant contributions to the understanding of stepped channel flows (e.g. OHTSU and YASUDA 1997). Dynamic similarity of skimming flows is however complex because of the role of the steps in enhancing turbulent dissipation and free-surface aeration. For uniform equilibrium flow down a prismatic rectangular channel with horizontal steps, a dominant flow feature is the momentum exchange between the free-stream and the cavity flow within the steps (Fig. 1). A complete dimensional analysis yields :

$$F\left(\frac{V}{\sqrt{g * d}} ; \rho_w \frac{V * d}{\mu_w} ; \frac{g * \mu_w^4}{\rho_w * \sigma^3} ; C_{mean} ; \frac{d}{h} ; \frac{W}{h} ; \theta ; \frac{k_s}{h} \right) = 0 \qquad (9)$$

where V and d are the mean flow velocity and flow depth at uniform equilibrium flow conditions, W the channel width, k_s the skin roughness height, g the gravity acceleration, μ_w and ρ_w the dynamic viscosity and density of water respectively and σ the surface tension. From left to right, the dimensionless terms are Froude, Reynolds and Morton numbers, the amount of entrained air and the last four characterise the cavity shape and the skin friction effects on the cavity walls.

Equation (9) illustrates that a Froude similitude does not describe the complexity of stepped spillway flows. BaCaRa (1997) described a systematic laboratory investigation of the M'Bali dam spillway with model scales of 1/10, 1/21.3, 1/25 and 1/42.7. For the scales 1/25 and 1/42.7, the flow resistance was improperly reproduced with a Froude similitude. Further the Proceedings of the International Symposium on Scale Effects in Modelling Hydraulic Structures (IAHR, Esslingen, Germany, 1984, H. KOBUS editor) suggested that air entrainment is poorly reproduced on small-size scale models (also CHANSON 1997).

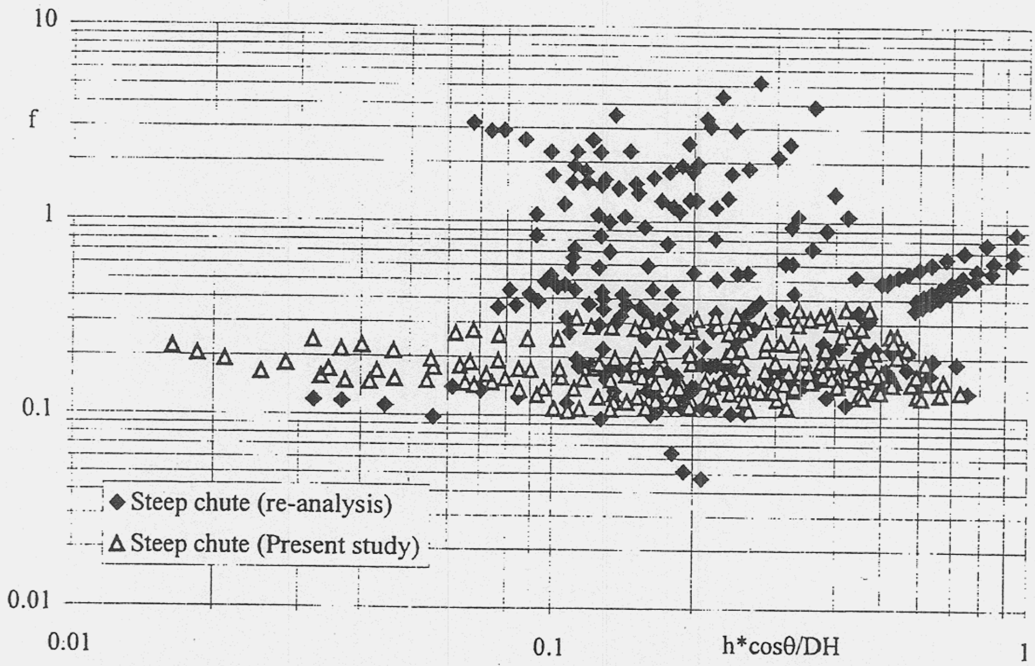
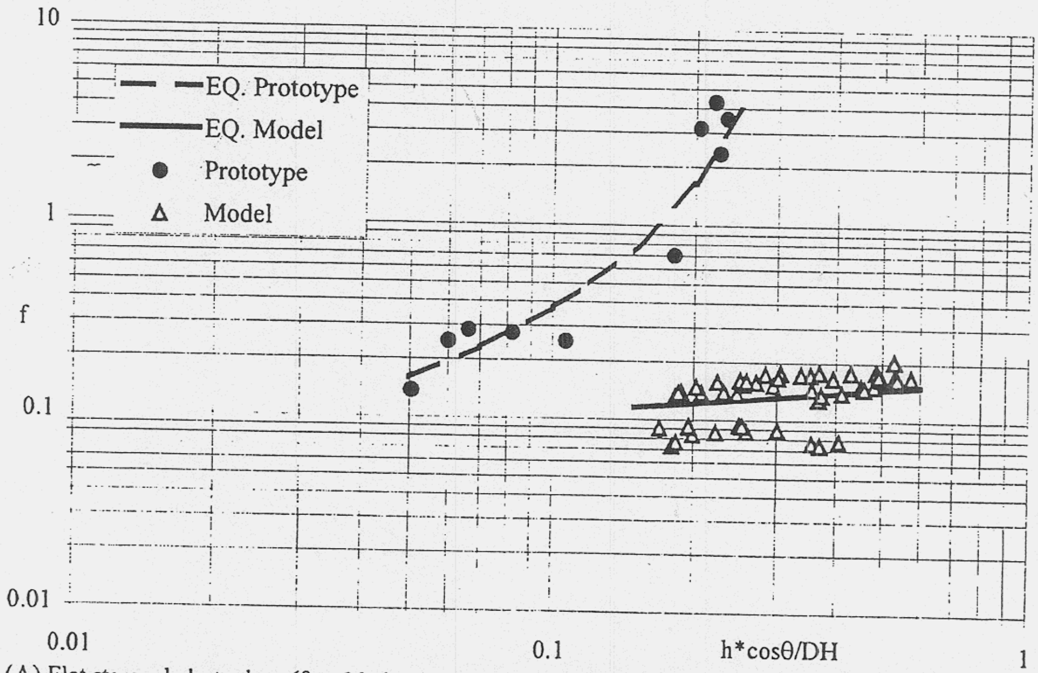
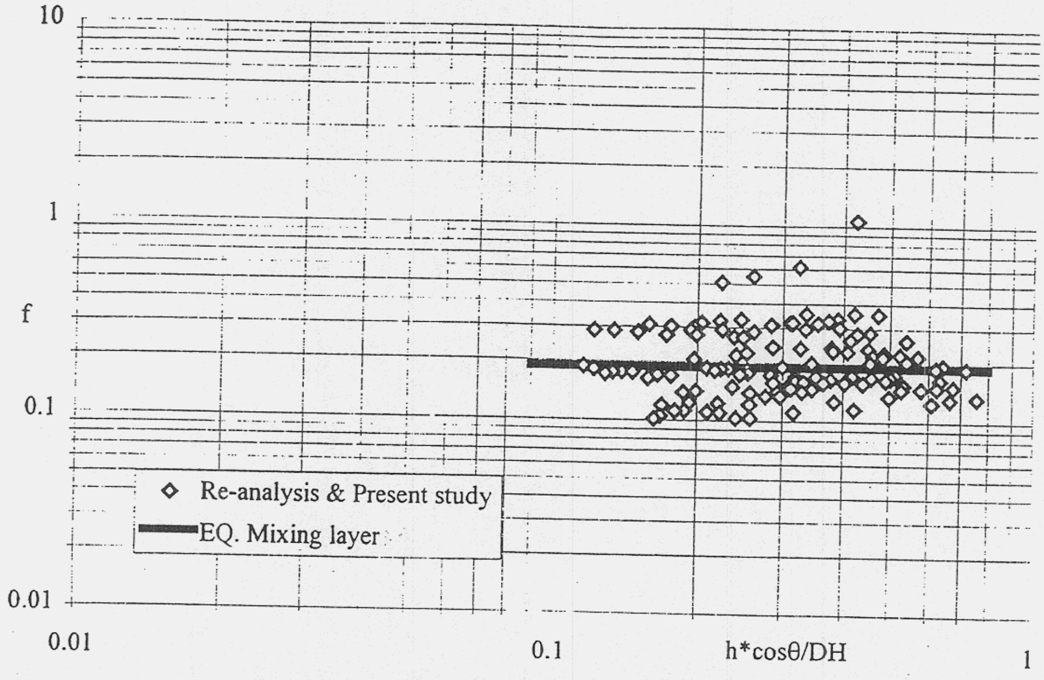


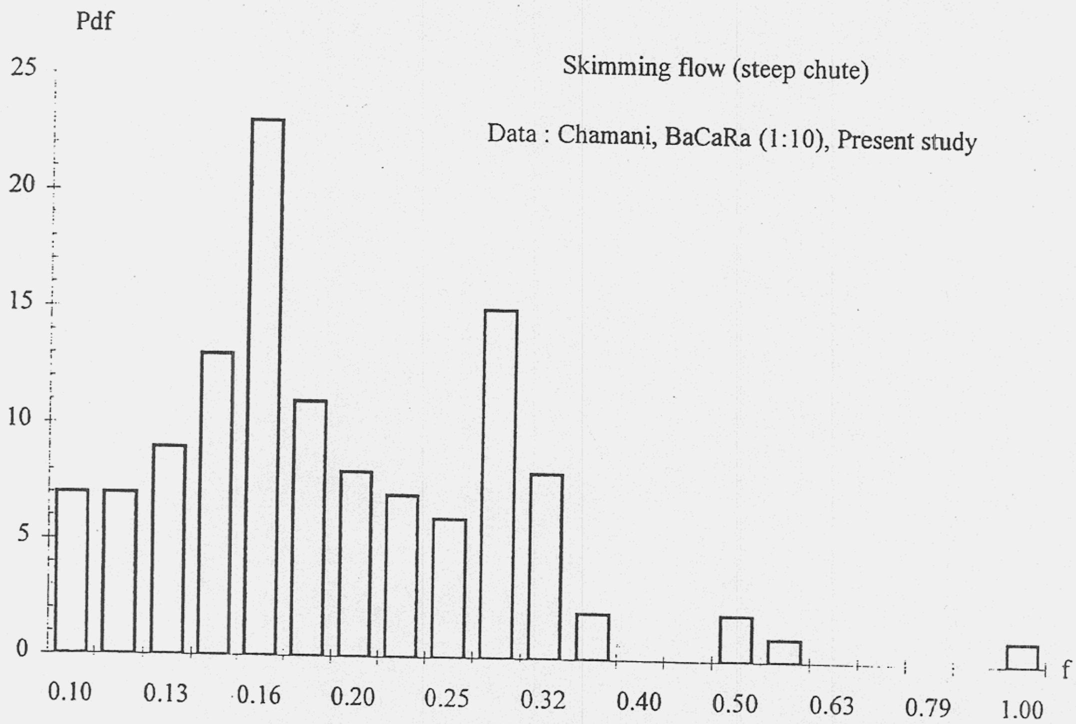
Fig. 2 - Darcy friction factor of skimming flows on steep stepped chute (θ > 20 deg.) {414 data}



(A) Flat stepped chute data (θ < 20 deg.) {62 data} - Comparison with Equations (10a) and (10b)



(B) Steep stepped chute data (θ > 20 deg.) {125 data} - Comparison with Equation (12)



(C) Probability distribution function of steep chute friction factor (θ > 20 deg.) {125 data}

Fig. 3 - Flow resistance in skimming flow: conditional analysis

New experimental study

A systematic study of stepped spillway flows was undertaken at Nihon University (Fig. 1B, YASUDA and OHTSU 1999). Experiments were performed in four 0.4-m wide stepped channels with pseudo-bottom slopes of 5.7°, 11.3°, 19°, 30° and 55°, step heights ranging from 0.002 to 0.08 m and flow rates between 0.008 and 0.08 m^2/s. Each channel was followed by a horizontal stilling basin. Clear-water depths were recorded with a pointer gauge. Pressure and velocity in clear-water flows were measured with a Pitot tube.

Void fractions were recorded with an optical fibre probe (single-tip). The skimming flow depth was measured by two methods : the clear-water flow depth deduced from void fraction profiles and an indirect method based upon the water depth measurement in the stilling basin. Comparisons showed agreement between the two methods within 5%.

Data analysis

The experimental data were analysed using Equations (1) and (5) for uniform equilibrium and gradually-varied flows respectively. They are compared with the re-analysis of over 35 model and prototype studies using the same equations. The combined analysis (over 650 data) shows that the flow resistance is larger on stepped chutes than on smooth channels, but the steep stepped chute data show very little correlation (Fig. 2). The study highlights further the lack of uniformity in the experimental procedure and data processing. [Too many studies gave incomplete experimental information with uncertainties of up to 200%.]
A conditional analysis was applied. Only the following data were retained: prototype data, large-size model data ($h > 0.020$ m, $Re > 1E+5$), and model studies published between Jan. 1997 and June 1999. This analysis gives more 'statistical weight' to prototype data and recent works. Basic results of the conditional analysis indicate different trends for flat and steep chutes (Fig. 3A & 3B). For flat chutes, prototype data (obtained with downward-inclined steps) exhibit higher flow resistance than laboratory data:

$$\frac{1}{\sqrt{f}} = -1.224 - 1.245 * Ln\left(\frac{h*\cos\theta}{D_H}\right) \qquad \text{Flat chute } (\theta < 20°) \text{ - Prototypes} \quad (10a)$$

$$\frac{1}{\sqrt{f}} = 2.430 - 0.2676 * Ln\left(\frac{h*\cos\theta}{D_H}\right) \qquad \text{Flat chute } (\theta < 20°) \text{ - Models} \quad (10b)$$

with normalised correlation coefficients of 0.9525 (10 data) and 0.4872 (188 data) respectively. By comparison, the Colebrook-White formula for flat rough walls is :

$$\frac{1}{\sqrt{f}} = 1.14 - 4.605 * Ln\left(\frac{k_s}{D_H}\right) \qquad \text{Fully-rough turbulent} \quad (11)$$

For steep chutes, the friction factor data present no obvious correlation but they are distributed around two dominant values : $f \approx 0.17$ and 0.30 (125 data) (Fig. 3C).

4. DISCUSSION

Skimming flows are characterised by unsteady momentum exchanges between the main stream and cavity flows. The recirculating fluid will, at irregular time intervals, flow outward into the main flow and be replaced by fresh fluid. The ejection mechanism appears sequential. Once one cavity outflow occurs, it induces a sequence of outflows at the downstream cavities. The process was observed visually for all investigated slopes (Present study) but the sequential mechanism was possibly most effective on the steep chute geometries. This similar pattern is observed with skimming flows past strip roughness (DJENEDI et al. 1994, ELAVARASAN et al. 1995) while the sequential fluid ejection process was observed on the M'Bali stepped spillway model by Professor LEJEUNE.
For flat slopes ($\theta < 20°$), the flow resistance includes the form drag of the recirculating cavity plus skin friction on the downstream end of each step. The free-surface is not parallel to the pseudo-bottom but it exhibits an 'undular' pattern in phase with the step geometry. Between adjacent steps, the cavity of recirculating fluid is affected by the interference of the mixing layer onto the downstream step.
For steep slopes ($\theta > 20°$), there is no skin friction between the main flow and the step faces, and the form drag associated with the recirculation predominates. The cavity flow is three-dimensional as visually observed during the present study and by Professor LEJEUNE on the M'Bali model. The flow resistance is a function of the recirculation process and energy dissipation is dominated by the transfer of momentum between the cavity flow and the main stream. A simplified analytical model (App. I) suggests that the pseudo-boundary shear stress may be expressed, in dimensionless form, as :

$$f = \frac{2}{\sqrt{\pi}} * \frac{1}{K} \qquad (12)$$

where $1/K$ is the dimensionless rate of expansion of the shear layer (Fig. 1). In air-water mixing layers, BRATTBERG and CHANSON (1998) observed $K \sim 6$ for $V = 2$ to 8 m/s. Equation (12) predicts f about 0.2

for a steep chute skimming flow: i.e., close to the observed friction factor in skimming flow (Fig. 3B).
It is believed that the channel width affects the development and number of recirculating cells at each step. For steep chutes ($\theta > 20°$), data from CHAMANI and RAJARATNAM (1999) and the present study suggest higher friction factors for low aspect ratios (i.e. $W/h \leq 10$) with identical flow conditions and geometry. Note that both studies used a constant channel breadth, 0.3-m and 0.4-m respectively, and the effect of the channel width was not specifically investigated.

5. CONCLUSION

In skimming flows down stepped chutes, the external edges of the steps form a pseudo-bottom over which the flow passes. Beneath this, recirculating vortices develop and are maintained through the transmission of shear stress from the waters flowing past the step edges. Skimming flows are characterised by a large flow resistance which is caused by *FORM LOSSES*. The flow resistance is consistently larger than on smooth-invert channels. Unsteady interactions between the recirculating cavities and the main stream are important. Flow visualisations suggest irregular fluid ejections from the cavity into the main stream, the process being sequential from upstream to downstream. On flat chutes, the flow resistance is a combination of skin friction on the horizontal faces of the steps and form drag associated with the recirculating cavity. The flow resistance data differ from steep chute data and they may be correlated with the relative step roughness (Eq. (10)). On steep chutes, the flow resistance must be analysed as a form drag. It may be estimated from the maximum shear stress in the shear layers: Equation (12) presents a simple model of form loss that agrees well with steep chute data. Although they exhibit some scatter, the data are distributed around two dominant values ($f \sim 0.17$ and 0.30).
The study emphasises the complexity of skimming flow on stepped chutes. The flow resistance and energy dissipation processes are dominated by the form losses and cavity recirculation.

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Source of data: BaCaRa (1991,1997), BAKER (1994), BAYAT (1991), BINDO et al. (1993), CHAMANI and RAJARATNAM (1999), CHRISTODOULOU (1993), DIEZ-CASCON et al. (1991), FRIZELL (1992), GORDIENKO (1967), GRINCHUK et al. (1994), NOORI (1984), PEYRAS et al. (1991,1992), SORENSEN (1985), TOZZI (1992), TOZZI et al. (1998), and Present Study. {Full bibliographic details in CHANSON 1995a, pp. 210-229}

APPENDIX I - ESTIMATING BOUNDARY SHEAR STRESS ALONG A RECIRCULATING CAVITY

At each step, the cavity flow is driven by the developing shear layer and the associated transfer of momentum (Fig. 1). The mixing layer is basically a free shear layer. The equivalent boundary shear stress of the cavity flow equals the maximum shear stress in the shear layer that may be modelled by a mixing length model:

$$\tau_o = \tau_{max} = \rho * \nu_T * \left(\frac{\partial V}{\partial y}\right)_{y=y_{50}} \qquad \text{(I-1)}$$

where ν_T is the momentum exchange coefficient, y_{50} is the location of the streamline $V = V_o/2$, and V_o is the free-stream velocity (e.g. Goertler model). The equivalent friction factor equals :

$$f = \frac{8 * \tau_{max}}{\rho * V_o^2} = \frac{2}{\sqrt{\pi} * K} \qquad \text{(I-2)}$$

where $1/K$ is the dimensionless rate of expansion of the shear layer. In air-water mixing layers of plane plunging jets, BRATTBERG and CHANSON (1998) observed $K \sim 6$ for velocities ranging from 2 to 8 m/s. For monophase flows, $K \sim 12$. Equation (I-2) predicts a form drag : $f \approx 0.2$ for skimming flow, a result close to friction factor data in steep stepped flows. BRATTBERG and CHANSON (1998) showed further that $y_{50}/d_o = -0.094*(x/d_o+5.3)$, implying the presence of a stagnation point associated with maximum pressures on the horizontal face of the step. This is consistent with the pressure measurements of FRIZELL (1992) and LEJEUNE and LEJEUNE (1994).

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