Discussion

Hydraulic condition for undular-jump formations

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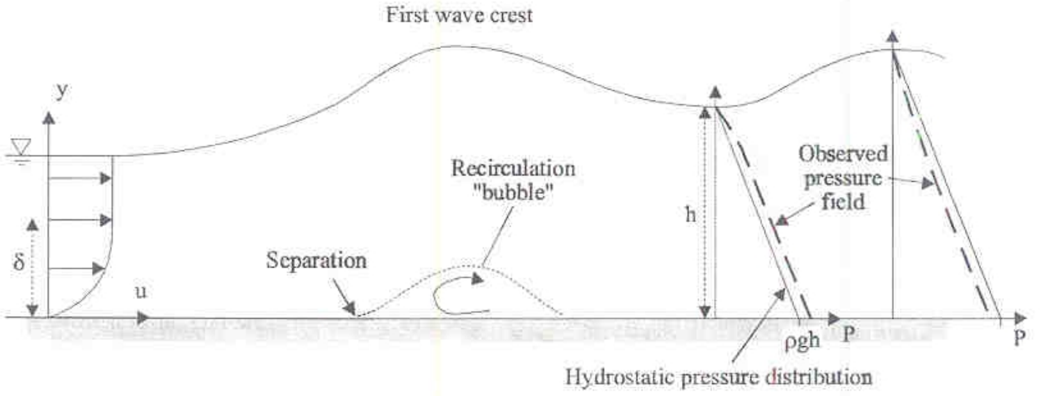
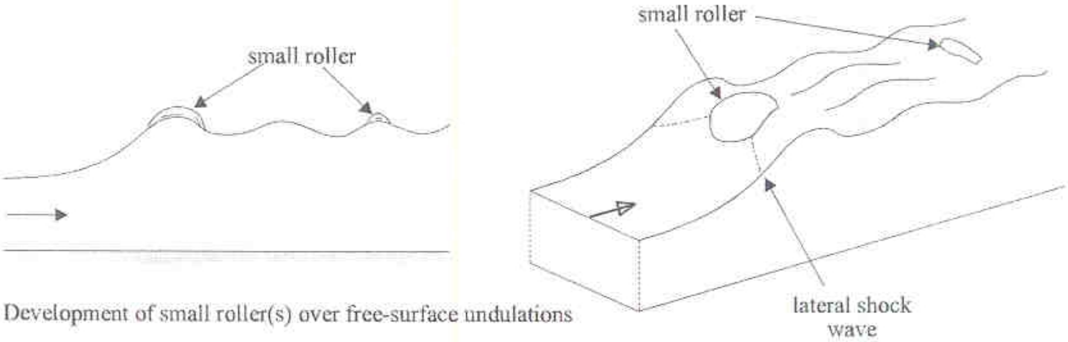
An undular hydraulic jump is characterised a series of steady, stationary free-surface undulations developing downstream of the jump toe. During their classical experiments in the Canal de Bourgogne, Pont-aqueduc de Roquefavour and Pont-aqueduc de Crau, DARCY and BAZIN (1865) observed undular jumps with marked undulations extending over lengths of up to 150 m. While there might be recreational applications, design engineers tend to avoid undular hydraulic jump formation in natural channels because of higher required embankment heights to prevent overtopping and because of the risks of bed scour and large bed form formation (CHANSON 1995,2000). The propagation of free-surface undulations and waves may impose additional impact loads, perturbations and vibrations on downstream canal structures : e.g., gates, locks, weirs. In one case, the occurrence of an undular jump immediately upstream of the intake perturbed the pump operation. Similarly undular tidal bores are well-known navigation disturbances. It is therefore essential to predict accurately the limiting conditions for free-surface undulations, and the authors' contribution is important.

During the 20th century, a number of undular jump studies were based upon an analogy with undular surge advancing in still water. MONTES (1979,1986) showed that the analogy is improper and does not account for the internal flow characteristics. The writers did well to emphasise the effects of inflow conditions on the undular hydraulic jump flow. An important flow feature of undular jumps with developing inflow is the presence of a recirculation region beneath the first wave crest (Fig. 1). Separation occurs upstream of the wave crest and recirculation was visualised near the channel centreline by FAWER (1937), MONTES (1986), YASUDA et al. (1993) and the discusser. This recirculation 'bubble' does not exist with fully-developed inflow conditions (CHANSON 1995, OHTSU et al. (1995), IAHR Congress, London). Another important flow feature is the deviation of the pressure distribution from hydrostatic. Figure 2 presents depth-averaged pressure gradients measured at wave crests and troughs for fully-developed inflow conditions. The data show that the mean pressure gradient may differ by up to 20% from the hydrostatic gradient. The pressure gradient is greater than hydrostatic



Undular hydraulic jump with partially-developed (or developing) inflow



Development of small roller(s) over free-surface undulations

Fig. 1. Sketch of undular jump flow patterns.

at trough and less than hydrostatic at wave crests as predicted by irrotational flow theory (ROUSE 1938, LIGGETT 1994, MONTES and CHANSON 1998). As a result, neither backwater calculations nor St Venant equations are applicable in the downstream undular region.

While the authors' Equation (7) provides a criterion for the transition from undular hydraulic jump to breaking jump with formed roller, they did not elaborate on the free-surface undulation disappearance. The discusser conducted numerous experiments with developing and fully-developed inflows (B = 0.25 & 0.5 m) (Table 1). He found that the disappearance of the undulations did not occur suddenly but took place in stages. The findings are in agreement with RYABENKO's (1990) observations. For very-low supercritical Froude numbers, the free-surface was perfectly two-dimensional. With increasing inflow Froude numbers, lateral shock waves developed for $F_1 > F^a$. There is a general agreement between all data including the authors' results that $F^a \sim 1.2$, but for some experiments with rough sidewalls for which shock waves were always observed (Table 1). For large Froude numbers (i.e. $F^a < F^b < F_1 < F^c$), small breaking was observed above some, but not all wave crests. Small rollers associated with breaking were seen 'riding' near the top of the undulations (Fig. 1). The surface area of breaking increased with increasing Froude numbers until the undulations disappeared completely for $F_1 > F^c$. Experimental results are summarised in Table 1, the inflow conditions being listed in column 2. For undular jumps with fully-developed inflow, the characteristic Froude number F^b is about 1.69 in average (Table 1, column 4). That is, equal to the upper limit $F_{1\text{limit}}$ proposed by the authors. For practicing engineers, however, the disappearance of free-surface undulations (i.e. $F_1 > F^c$) may be a more important, meaningful criterion. Experimental results yield $F^c \sim 1.5$ to 2.9 for fully-developed inflows and $F^c \sim 1.7$ to 2.1 for partially-developed inflows (Table 1).

Overall the authors must be congratulated for their authoritative paper. Their contribution on the topic was needed after years of discussions and argumentation between researchers.

References

BAKHMETEFF, B.A., and MATZKE, A.E. (1936). 'The Hydraulic Jump in Terms of Dynamic Similarity.' *Transactions,* ASCE, Vol. 101, pp. 630-647. Discussion : Vol. 101, pp. 648-680.

CHANSON, H. (1995). 'Flow Characteristics of Undular Hydraulic Jumps. Comparison with Near-Critical Flows.' *Report CH45/95*, Dept. of Civil Engineering, University of Queensland, Australia, June, 202 pages.

CHANSON, H. (2000). 'Boundary Shear Stress Measurements in Undular Flows : Application to Standing Wave Bed Forms.' *Water Res. Res.*, Vol. 36, No. 10, pp. 3063-3076.

DARCY, H.P.G., and BAZIN, H. (1865). 'Recherches Hydrauliques.' *Imprimerie Impériales*, Paris, France, Parties 1ère et 2ème.

FAWER, C. (1937). 'Etude de Quelques Ecoulements Permanents à Filets Courbes.' *Thesis*, Lausanne, Switzerland, Imprimerie La Concorde, 127 pages.

IMAI, S., and NAKAGAWA, T. (1992). 'On Transverse Variation of Velocity and Bed Shear Stress in Hydraulic Jumps in a Rectangular Open Channel.' *Acta Mechanica*, Vol. 93, pp. 191-203.

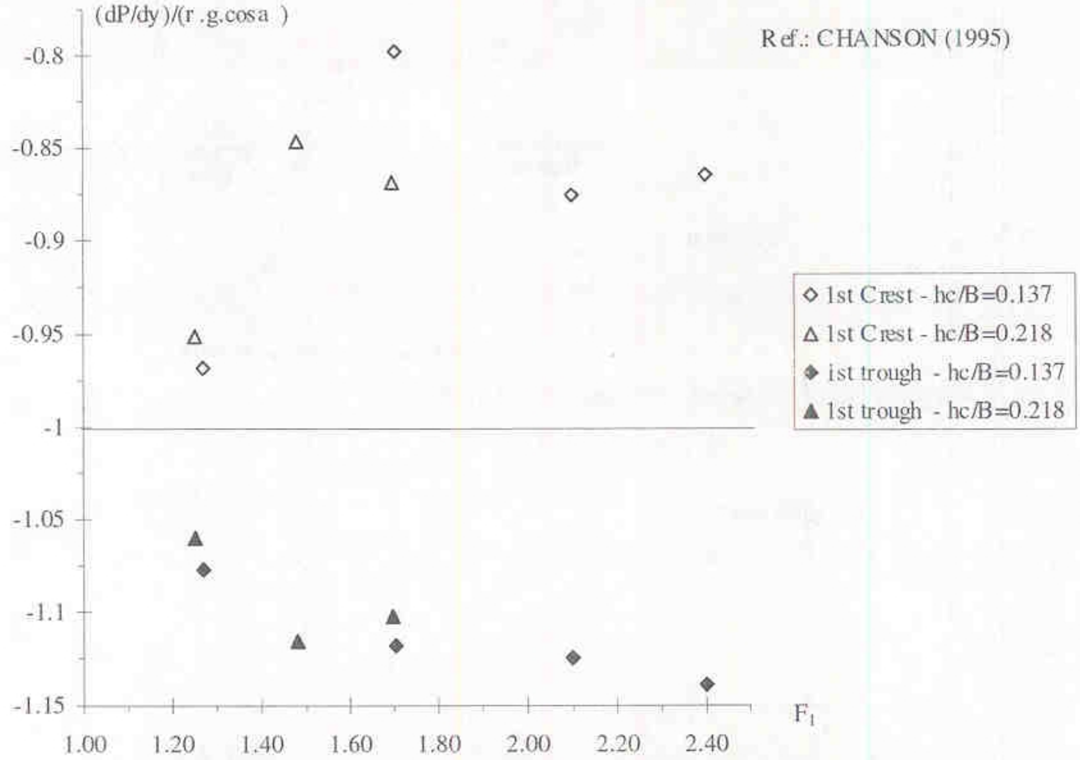


Fig. 2. Dimensionless pressure gradient $\partial P/\partial y/(\rho\ g \cos\alpha)$ in undular hydraulic jumps as a function of the upstream Froude number F_1 (Centreline data) - Uniform equilibrium, fully-developed upstream.

LIGGETT, J.A. (1994). 'Fluid Mechanics.' *McGraw-Hill*, New York, USA.

MONTES, J.S. (1979). 'Undular Hydraulic Jump - Discussion.' *Jl of Hyd. Div.*, ASCE, Vol. 105, No. HY9, pp. 1208-1211.

MONTES, J.S. (1986). 'A Study of the Undular Jump Profile.' *Proc. 9th Australasian Fluid Mechanics Conference AFMC*, Auckland, New Zealand, pp. 148-151.

MONTES, J.S., and CHANSON, H. (1998). 'Characteristics of Undular Hydraulic Jumps. Results and Calculations.' *Jl of Hyd. Engrg.*, ASCE, Vol. 124, No. 2, pp. 192-205.

OVALLE, A., and DOMINGUEZ, A. (1934). *Civil Engineering Thesis*, Catholic University, Santiago, Chile.

ROUSE, H. (1938). 'Fluid Mechanics for Hydraulic Engineers.' *McGraw-Hill Publ.*, New York, USA (also *Dover Publ.*, New York, USA, 1961, 422 pages)

SERRE, F. (1953). 'Contribution à l'Etude des Ecoulements Permanents et Variables dans les Canaux.' *Jl La Houille Blanche*, Dec., pp. 830-872.

YASUDA Y., OHTSU I., and GOTOH H. (1993). 'A Few Experiments on Undular Hydraulic Jump.' *48th Annual Meeting JSCE*, Japan, Sept., II-159, pp. 416-417.

Notation

F^a characteristic inflow Froude number associated with the appearance of sidewall shock waves upstream of the first wave crest;
F^b characteristic inflow Froude number associated with the appearance of wave breaking at some wave crest;
F^c characteristic inflow Froude number associated with the complete disappearance of freesurface undulations;
P pressure (Pa);
α channel slope with horizontal.

Table 1. Characteristic Froude numbers for undular jump formations

Reference (1)	Inflow conditions (2)	F_{1limit}	F^a (3)	F^b (4)	F^c (5)	Remarks (6)
Theoretical results						
SERRE (1953)		1.455				Theory.
IWASA (1955)		1.553				Theory.
Experimental observations						
OVALLE and DOMINGUEZ (1934) (a)			--	--	1.59	
			--	--	3.63	With special precautions.
BAKHMETEFF and MATZKE (1936) (b)	P/D		--	--	2.1	B = 0.152 m.
FAWER (1937)	P/D		--	1.82	2.46	B = 0.303 m. h_c/B = 0.1 to 0.26.
BINNIE and ORKNEY (1955)	P/D		--	--	1.74	
IWASA (1955)			1.5	--	1.9	
RYABENKO (1990)						B = 1 m.
		F/D	--	--	2.08	h_c/B = 0.090
		Hydrostatic	--	--	1.90	h_c/B = 0.143
		pressure	--	--	1.95	h_c/B = 0.147
		distribution	--	--	1.82	h_c/B = 0.185
		upstream of	--	--	1.86	h_c/B = 0.187
		jump	--	--	1.92	h_c/B = 0.192
			--	--	1.69	h_c/B = 0.224
			--	--	1.77	h_c/B = 0.231
			--	--	1.88	h_c/B = 0.239
			--	--	1.94	h_c/B = 0.244
			--	2.02	--	h_c/B = 0.094
			--	1.68	--	h_c/B = 0.083
			--	1.99	--	h_c/B = 0.094
			--	1.82	--	h_c/B = 0.110
			--	1.51	--	h_c/B = 0.105
			--	1.71	--	h_c/B = 0.142
			--	1.58	--	h_c/B = 0.160

		--	1.54	--	$h_c/B = 0.184$
		--	1.56	--	$h_c/B = 0.210$
		--	1.48	--	$h_c/B = 0.276$
	P/D Non-hydrostatic pressure distribution upstream of jump	--	--	1.01	$h_c/B = 0.060$
		--	--	1.34	$h_c/B = 0.109$
		--	--	1.02	$h_c/B = 0.121$
		--	--	1.0	$h_c/B = 0.010$
		--	--	1.03	$h_c/B = 0.041$
		--	--	1.25	$h_c/B = 0.043$
		--	--	1.42	$h_c/B = 0.044$
		--	--	1.61	$h_c/B = 0.124$
		--	1.07	--	$h_c/B = 0.083$
		--	1.04	--	$h_c/B = 0.103$
		--	1.28	--	$h_c/B = 0.130$
		--	1.08	--	$h_c/B = 0.126$
		--	1.22	--	$h_c/B = 0.171$
		--	1.09	--	$h_c/B = 0.170$
		--	1.19	--	$h_c/B = 0.180$
		--	1.44	--	$h_c/B = 0.155$
		--	1.12	--	$h_c/B = 0.216$
IMAI and NAKAGAWA (1992)	F/D	--	--	2.5	B = 0.3 m.
CHANSON (1995)	F/D				B = 0.25 m. Smooth sidewalls.
		1.22	2.10	> 2.6	$h_c/B = 0.075$
		1.22	1.80	2.91	$h_c/B = 0.137$
		1.2	1.7	--	$h_c/B = 0.172$
		1.26	1.65	2.83	$h_c/B = 0.219$
		--	1.65	--	$h_c/B = 0.224$
		1.14	--	--	$h_c/B = 0.286$
		1.10	1.4	1.6	$h_c/B = 0.347$
		1.16	--	1.6	$h_c/B = 0.403$
		1.20	1.35	1.5	$h_c/B = 0.454$
	F/D	1.0	~ 1.25	> 1.50	B = 0.25 m. Rough sidewalls (stucco pattern). $h_c/B = 0.35$ to 0.40
Present study	P/D	--	> 1.7	2.05	$h_c/B = 0.146$, $d/h_1 = 0.4$. B = 0.5 m.

Notes : Inflow conditions : P/D = partially-developed (or developing) inflow; F/D = fully-developed (or developed) inflow.

([a]): as cited by MONTES (1986).

([b]): BAKHMETEFF and MATZKE (1936) indicate an upper limit for undular jump of $F_1 = 1.732$. But a re-analysis of their data indicate that the upper limit for their experiments was 2.1.

Reply by the authors

The authors would like to thank the discusser for the information about the lower limit of the Froude number for the disappearance of free-surface undulations F^C.

Recently, the authors have investigated the characteristics of a breaking undular jump under a wide range of experimental conditions. In a breaking undular jump, a roller is formed at the first wave crest, and steady free-surface undulations continue downstream.

For a fully developed inflow condition, hydraulic conditions for undular jump formations have been shown on the basis of the