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MODAL TABLEAUX FOR NONMONOTONIC REASONING

1 Introduction

Artosi and Governatori (1994) and Artosi, Cattabriga and Governatori (1994) presented a tableau-like proof system, called KEM, which has been proven to be able to cope with a wide variety of (normal) modal logics. KEM is based on D'Agostino and Mondadori's (1994) classical proof system KE, a combination of tableau and natural deduction inference rules which allows for a restricted ("analytic") Use of the cut rule. The key feature of KEM, besides its being based neither on resolution nor on standard sequent/tableau inference techniques, is that it generates models and checks them using a label scheme to bookkeep "world" paths. Governatori (1995) and Artosi, Governatori and Sartor (1996) showed how this formalism can be extended to handle various system of multimodal logic devised for dealing with nonmonotonic reasoning, by relying in particular on Meyer and van der Hoek's (1992) logic for actuality and preference. In this paper we shall be concerned with developing a similar extension this time by relying on Schwind and Siegel's (1993, 1994) system $\mathcal{H}$, another multimodal logic devised for dealing with nonmonotonic inference. This logic will be introduced in Section 2. Section 3 will provide a description of KEM method for dealing with $\mathcal{H}$. Section 4 will present an example application. Finally, the last section will provide concluding remarks.

2 The Modal Logic $\mathcal{H}$

We assume a standard modal language consisting of: propositional variables; the usual logical operators $\neg, \wedge, \vee, \rightarrow, \equiv, \Box, \Diamond$ for negation, conjunction, disjunction, conditionality, biconditionality, necessity and possibility respectively. Formulas are defined in the usual way. We shall use $A, B, C, \dots$ to denote arbitrary formulas. The modal logic $\mathcal{H}$ is obtained by enlarging the basic modal language with a modal operator $[H]$. Thus the set of $\mathcal{H}$-formulas includes all the formulas of the form $[H]A$. A "hypothesis" operator H is then defined as the dual of $[H]$. HA means that A is a hypothesis (accordingly $[H]A$ means that $\neg A$ is not a hypothesis). In addition to the axioms of the standard T system we have the following axioms:

1. $\Box A \rightarrow [H]A$

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2. $[H](A \to B) \to ([H]A \to [H]B)$.

It turns out that this setting is that of a multimodal K/T system with $\Box$ (and $\Diamond$) and $[H]$ (and H) behaving as normal T and K modalities respectively. A model for $\mathcal{H}$ is thus a structure

$$\langle S, R_h, R_k, \upsilon \rangle$$

where S is a (non empty) set of worlds; $R_k \subset S \times S$ is the standard T accessibility relation on S; $R_h \subset S \times S$ is a K accessibility relation on S; $R_h \subset R_k$, and υ is as usual with the following clauses for $[H]$ and $\Box$ respectively:

$$\upsilon([H]A, u) = T \iff \forall z \in W : uR_h z, \upsilon(A, z) = T,$$

$$\upsilon(\Box A, u) = T \iff \forall z \in W : uR_k z, \upsilon(A, z) = T.$$

2.1 Representing defaults in $\mathcal{H}$

A hypothesis theory is a pair $\mathcal{HT} = (F, HY)$ where F is a set of formulas and HY is a set of hypotheses. Since $\mathcal{H}$ is monotonic, nonmonotonicity follows from defining an extension of F in HY as a set $Th_{\mathcal{H}}(F \cup HY')$ where HY' is a subset of HY such that $F \cup HY'$ is maximal consistent according to HY (this means: if any other hypothesis of HY is added the resulting theory is inconsistent). Let $\Delta = (W, D)$ be a (propositional) default theory. This can be translated to a hypothesis theory

$$(\Box W \cup \Box D, HY)$$

where

- $\Box W = \{\Box A : A \in W\}$;
- $\Box D = \{\Box A \wedge HB \to \Box C : \frac{A:B}{C} \in D\}$; and
- $HY = \{HB : B \in Just(D)\}$,

where $Just(D)$ is the set of justifications of the set of defaults D. Schwind and Siegel (1994) showed the correspondance between default logic and hypothesis theory.

3 The system KEM

In this section we describe the modal proof system KEM. We first recall some basic notions.

3.1 Preliminaries

As usual (see Smullyan 1968) by a *signed formula* (*S-formula*) we mean an expression of the form SA where A is a formula and $S \in \{T, F\}$. Thus TA if $v(A, u) = T$ and FA if $v(A, u) = F$ for some model $\langle S, R_h, R_k, v\rangle$ and $u \in S$. We shall use $X, Y, Z, \ldots$ to denote arbitrary S-formulas. For ease of exposition we shall use a generalized "$\alpha, \beta, \nu_x, \pi_x$" ($x \in \{h, k\}$) form of Smullyan-Fitting's (Smullyan 1968, Fitting 1983) "α, β, ν, π" unifying notation as exposed in the following tables

α	α_1	α_2
$TA \wedge B$	TA	TB
$FA \vee B$	FA	FB
$FA \rightarrow B$	TA	FB
$F\neg A$	TA	TA

β	β_1	β_2
$FA \wedge B$	FA	FB
$TA \vee B$	TA	TB
$TA \rightarrow B$	FA	TB
$T\neg A$	FA	FA

ν_h	ν_k	ν_0
$T[H]A$	$T\Box A$	TA
FHA	$F\Diamond A$	FA

π_h	π_k	π_0
$F[H]A$	$F\Box$	FA
THA	$T\Diamond A$	TA

By the *conjugate* X^C of a S-formula X we shall mean the result of changing S to its opposite, with the exception of the S-formulas listed in the left column of the following tables which have both the S-formulas listed in the other columns as their conjugates.

X	X^C	
$T\Box A$	$F\Box A$	$T\Diamond\neg A$
$F\Diamond A$	$T\Diamond A$	$F\Box\neg A$
$F\Box A$	$T\Box A$	$F\Diamond\neg A$
$T\Diamond A$	$F\Diamond A$	$T\Box\neg A$

X	X^C	
$T[H]A$	$F[H]A$	$TH\neg A$
FHA	THA	$F[H]\neg A$
$F[H]A$	$T[H]A$	$FH\neg A$
THA	FHA	$T[H]\neg A$

For example, $T\Box A$ has both $F\Box A$ and $T\Diamond\neg A$ as its conjugates. Two S-formulas X, Z such that $Z = X^C$, will be called *complementary.*

3.2 Informal explanation

As we have said KEM approach wants we work with "world" labels. A "world" label is either a constant or a variable "world" symbol or a "structured" sequence of world-symbols we shall call a "world-path". Intuitively, constant and variable world-symbols stand for worlds and sets of worlds respectively, while a world-path conveys information about access between the worlds in it. We attach labels to S-formulas to yield *labelled signed formulas* (*LS-formulas*), i.e., pairs of the form X, i where X is a S-formula and i is a label. A LS-formula SA, i means, intuitively, that A is true (false) at the (last) world (on the path represented by) i. In the course of proof search, labels are manipulated in a way closely related to the semantics of modal operators and "matched" using a (specialized, logic-dependent) unification algorithm. That two world-paths i and k are unifiable means, intuitively, that they virtually represent the same path, i.e., any world which you could get by the path i could be reached by the path k and vice versa. LS-formulas whose labels are unifiable turn out to be true (false) at the same world(s) relative to the accessibility relation that holds in the appropriate class of models. In particular two LS-formulas X, X^C whose labels are unifiable stand for formulas which are contradictory "in the same world". These ideas are formalized as follows.

***Remark* 1** The idea of using a label scheme to bookkeep "world" paths in modal theorem proving goes back at least to Fitch (1966). Similar, or related, ideas have been proposed by Fitting (1972, 1983) Wrightson (1985) and, more recently, by Catach (1991), Jackson and Reichgelt (1989), Tapscott (1987), Wallen (1990) and also in the "translation" tradition of Auffray and Enjalbert (1992), Ohlbach (1991, 1993), and in Gabbay's (1996) Discipline of Labelled Deductive Systems (see also D'Agostino and Gabbay (1994) tableau extension with labels).

3.3 Label formalism

Let $\Phi_C^k = \{w_1, w_2, \cdots\}$, $\Phi_C^h = \{h_1, h_2, \cdots\}$ and $\Phi_V^k = \{W_1, W_2, \cdots\}$, $\Phi_V^h = \{H_1, H_2, \cdots\}$ be (non empty) sets respectively of constant and variable world-simbols. Let us define $\Phi_C = \Phi_C^k \cup \Phi_C^h$ and $\Phi_V = \Phi_V^k \cup \Phi_V^h$. We can now define the set $\Im$ of labels as:

$$\begin{aligned} \Im \;\; = \;\; & \bigcup_{1 \le i} \Im_i \text{ where } \Im_i \text{ is :} \\ & \Im_1 = \Phi_C \cup \Phi_V; \\ & \Im_2 = \Im_1 \times \Phi_C; \\ & \Im_{n+1} = \Im_1 \times \Im_n, \;\; n > 1 \; . \end{aligned}$$

That is a world-label is either (i) an element of the set Φ_C, or (ii) an element of the set Φ_V, or (iii) a path term (k', k) where (iiia) $k' \in \Phi_C \cup \Phi_V$ and (iiib) $k \in \Phi_C$ or $k = (m', m)$ where (m', m) is a label. According to the above informal explanation, we may think of a label $i \in \Phi_C$ as denoting a (given) world, and a label $i \in \Phi_V$ as denoting a set or worlds (any world) in some Kripke model. A label $i = (k', k)$ may be viewed as representing a path from k to a (set of) world(s) k' accessible from k. For instance, $(w_2^x, (W_1^x, w_1^x))$, $x \in \{h, k\}$ represents a path which takes us to a world w_2^i accessible via any world accessible from w_1^x (i.e., accessible from the subpath (W_1^x, w_1^x)) according to R_x.

A bit of terminology. For any label $i = (k', k)$ we call k' the *head* of i, k the *body* of i, and denote them by $h(i)$ and $b(i)$ respectively. Notice that these notions are recursive: if $b(i)$ denotes the body of i, then $b(b(i))$ will denote the body of $b(i)$, $b(b(b(i)))$ will denote the body of $b(b(i))$; and so on. For example, if i is $(w_4, (W_3, (w_3, (W_2, w_1))))$, then $b(i) = W_3, (w_3, (W_2, w_1)))$, $b(b(i)) = (w_3, (W_2, w_1))$, $b(b(b(i))) = (W_2, w_1)$, $b(b(b(b(i)))) = w_1$. We call each of $b(i)$,$b(b(i))$, etc., a *segment* of i. Let $s(i)$ denote any segment of i (obviously, by definition every segment $s(i)$ of a label i is a label); then $h(s(i))$ will denote the head of $s(i)$, $b(s(i))$ will denote the body of $s(i)$, and so on. For any label i, we define the length of i, $l(i)$, as the number of world-symbols in i, i.e., $l(i) = n \Leftrightarrow i \in \Im_n$. $s^n(i)$ will denote the segment of i of lenght n, i.e., $s^n(i) = s(i)$ such that $l(s(i)) = n$. The *countersement-n* of i, i.e., $c^n(i)$ identifies the sub-label obtained from i after have identified s^n with a dummy label w_0. For example given the label $i = (w_4, (W_3, (w_3, (W_2, w_1))))$, $l(i) = 5$; its segment of length 3 is $s^3(i) = (w_3, (W_2, w_1)$, and $c^3(i) = (w_4, (W_3, w_0))$, where $w_0 = (w_3, (W_2, w_1)$. We shall call a label i *restricted* if $h(i) \in \Phi_C$, otherwise we call it *unrestricted*.

3.4 Unification scheme

We define a substitution in the usual way as a function

$$\begin{aligned} \sigma \;\; &: \;\; \Phi_V^k \longrightarrow \Im^- \\ &: \;\; \Phi_V^h \longrightarrow \Phi_C^h. \end{aligned}$$

where $\Im^- = \Im - \Phi_V$. For two labels i, k and a substitution σ, if σ is a unifier of i and k then we shall say that i and k are σ-unifiable. We shall (somewhat unconventionally) use $(i, k)\sigma$ to denote both that i and k are σ-unifiable and the result of their unification. On this basis we define several specialised, logic-dependent notions of both σ "high" (σ^M) and σ "low" (σ_L) unification. In general "high" unifications are meant to mirror specific accessibility constraints. They are used to build "low" unifications, which account for the full range of conditions governing the appropriate accessibility relation. For example, in the case of a logic with multiple independent but interacting modalities, the "high" unifications characterizing each modality are combined into the "low", overall unification which characterizes this logic. In the present case, we need the following "high" unifications which account for the modal operators $[H]$ and $\Box$ respectively.

$$\begin{aligned} (i, k)\sigma^{\mathcal{H}} =& (c^{l(b(i))}(i), c^{l(b(k))}(k))\sigma \iff \\ & \text{either } h(i) \text{ or } h(k) \in \Phi_C^h, \text{ or} \\ & h(i), h(k) \in \Phi_V^k \end{aligned} \qquad (\sigma^{\mathcal{H}})$$

where $w_0 = (b(i), b(k))\sigma^{\mathcal{H}}$.

Moreover we need another substitution $\sigma^\Box$ isolating the behavior of the worlds of type Φ^k:

$$\sigma^\Box = \sigma/_{\Phi_V^k}$$

$\sigma^{\Box}$ is σ restricted to varaibles of type k.

$$(i,k)\sigma^{\mathcal{T}} = \begin{cases} (s^{l(k)}(i),k)\sigma^{\mathcal{H}} & l(i) > l(k), \text{ and} \\ & \forall m \geq l(k), (i^m, h(k))\sigma^{\Box} = (h(i), h(k))\sigma^{\Box} \\ (i, s^{l(i)}(k))\sigma^{\mathcal{H}} & l(k) > l(i), \text{ and} \\ & \forall m \geq l(i), (h(i), k^m)\sigma^{\Box} = (h(i), h(k))\sigma^{\Box} \end{cases} \qquad (\sigma^{\mathcal{T}})$$

We can now define an high unification correponding to the combination of the two unifications for the modalities of $\mathcal{H}$.

$$(i,k)\sigma^{\mathcal{HT}} = \begin{cases} (i,k)\sigma^{\mathcal{H}} \\ (i,k)\sigma^{\mathcal{T}} \end{cases} \qquad (\sigma^{\mathcal{HT}})$$

from which the low unification for $\mathcal{H}$ follows.

$$(i,k)\sigma_{\mathcal{H}} = \begin{cases} (c^n(i), c^m(k))\sigma^{\mathcal{HT}} \\ (i,k)\sigma^{\mathcal{HT}} \end{cases} \qquad (\sigma_{\mathcal{H}})$$

where $w_0 = (s^n(i), s^m(k))\sigma_{\mathcal{H}}$.

Examples and discussion about the above unifications have been provided by Governatori (1997).

3.5 Inferences rules

Artosi and Governatori (1994) proved that the following rules give a sound and complete system for a wide variety of normal modal logics.

Propositional rules

$$\frac{\alpha, i}{\alpha_1, i} \qquad \frac{\alpha, i}{\alpha_2, i} \qquad (\alpha)$$

$$\frac{\begin{array}{c}\beta, i \\ \beta_1^C, k\end{array}}{\beta_2, (i,k)\sigma_{\mathcal{H}}}[(i,k)\sigma_{\mathcal{H}}] \qquad \frac{\begin{array}{c}\beta, i \\ \beta_1^C, k\end{array}}{\beta_2, (i,k)\sigma_{\mathcal{H}}}[(i,k)\sigma_{\mathcal{H}}] \qquad (\beta)$$

Modal rules

$$\frac{\nu_x, i}{\nu_0, (m,i)}[m \in \Phi_V^x \text{ and new}, x \in \{h,k\}] \qquad (\nu_n)$$

$$\frac{\pi_x, i}{\pi_0, (m,i)}[m \in \Phi_C^x \text{ and new}, x \in \{h,k\}] \qquad (\pi_n)$$

Structural rules

$$\frac{}{X,i \qquad X^C,i}[i \text{ restricted}] \tag{PB}$$

$$\frac{\begin{array}{c} X,i \\ X^C,k \end{array}}{\times(i,k)\sigma_{\mathcal{H}}}[(i,k)\sigma_{\mathcal{H}}] \tag{PNC}$$

Here the α-rules are just the familiar linear branch-expansion rules of the tableau method, while the β-rules correspond to such common natural inference patterns as *modus ponens*, *modus tollens*, etc. (i, k, m stand for arbitrary labels). The rules for the modal operators are as usual. "m new" in the proviso for the ν_x- and π_x-rule means: m must not have occurred in any label yet used. Notice that in all inferences via an α-rule the label of the premise carries over unchanged to the conclusion, and in all inferences via a β-rule the labels of the premises must be $\sigma_{\mathcal{H}}$-unifiable, so that the conclusion inherits their unification. PB (the "Principle of Bivalence") represents the (LS-version of the) semantic counterpart of the cut rule of the sequent calculus (intuitive meaning: a formula A is either true or false in any *given* world, whence the requirement that i should be restricted). PNC (the "Principle of Non-Contradiction") corresponds to the familiar branch-closure rule of the tableau method, saying that from the occurrence of a pair of LS-formulas X, i, X^C, k such that $(i,k)\sigma_{\mathcal{H}}$ (let us call them *$\sigma_{\mathcal{H}}$-complementary*) on a branch we may infer the closure ("$\times$") of the branch. The $(i,k)\sigma_{\mathcal{H}}$ in the "conclusion" of PNC means that the contradiction holds "in the same world". Soundness and completeness for the modal logic $\mathcal{H}$ follow by an obvious modification of the proofs given by Governatori (1995).

3.6 Proof search

Let $\Gamma = \{X_1, \ldots, X_m\}$ be a set of S-formulas. Then $\mathcal{T}$ is a *KEM-tree for* Γ if there exists a finite sequence $(\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_n)$ such that (i) $\mathcal{T}_1$ is a 1-branch tree consisting of $\{X_1, i \ldots, X_m, i\}$, where i is an arbitrary constant label; (ii) $\mathcal{T}_n = \mathcal{T}$, and (iii) for each $i < n, \mathcal{T}_{i+1}$ results from $\mathcal{T}_i$ by an application of a rule of KEM. A branch τ of a KEM-tree $\mathcal{T}$ of LS-formulas is said to be *$\sigma_{\mathcal{H}}$-closed* if it ends with an application of PNC, open otherwise. As usual with tableau methods, a set Γ of formulas is checked for consistency by constructing a KEM-tree for Γ. It is worth noting that each KEM-tree is a (class of) Hintikka's model(s) where the labels denote worlds (i.e., Hintikka's modal sets), and the unifications behave according to the conditions placed on the appropriate accessibility relations. Moreover we say that a formula A is a *KEM-consequence of a set of formulas* Γ if A occurs in all the open branches of a KEM-tree for Γ. We now describe a sistematic procedure for KEM. First we define the following notions.

Given a branch τ of a KEM-tree, we shall call an LS-formula X, i *E-analysed in* τ if either (i) X is of type α and both α_1, i and α_2, i occur in τ; or (ii) X is of type β and one of the following conditions is satisfied: (a) if β_1^C, k occurs in τ and $(i,k)\sigma_{\mathcal{H}}$, then also $\beta_2, (i,k)\sigma_{\mathcal{H}}$ occurs in τ, (b) if β_2^C, k occurs in τ and $(i,k)\sigma_{\mathcal{H}}$, then also $\beta_1, (i,k)\sigma_{\mathcal{H}}$ occurs in τ; or (iii) X is of type ν_x and $\nu_0, (m,i)$ occurs in τ for some $m \in \Phi_V$ not previously occurring in τ, or (iv) X is of type π_x and $\pi_0, (m,i)$ occurs in τ for some $m \in \Phi_C$ not previously occurring in τ.

We shall call a branch τ of a KEM-tree *E-completed* if every LS-formula in it is E-analysed and it contains no complementary formulas which are not $\sigma_{\mathcal{H}}$-complementary. We shall say a branch τ of a KEM-tree *completed* if it is E-completed and all the LS-formulas of type β in it either are analysed or cannot be analysed. We shall call a KEM-tree *completed* if every branch is completed.

The following procedure starts from the 1-branch, 1-node tree consisting of $\{X_1, i \ldots, X_m, i\}$ and applies the rules of KEM until the resulting KEM-tree is either closed or completed.

At each stage of proof search (i) we choose an open non completed branch τ. If τ is not E-completed, then (ii) we apply the 1-premise rules until τ becomes E-completed. If the resulting branch τ' is neither closed nor completed, then (iii) we apply the 2-premise rules until τ becomes E-completed. If the resulting branch τ' is neither closed nor completed, then (iv) we choose an LS-formula of type β which is not yet analysed in the branch and apply PB so that the resulting LS-formulas are β_1, i' and β_1^C, i' (or, equivalently β_2, i' and β_2^C, i'), where $i = i'$ if i is restricted (and already occurring when $h(i) \in \Phi_C^h$), otherwise i' is obtained from i by instantiating $h(i)$ to a constant not occurring in i; (v) ("Modal PB") if the branch is not E-completed nor closed, because of complementary formulas which are not $\sigma_{\mathcal{H}}$-complementary, then we have to see whether a restricted label unifying with both the labels of the complementary formulas occurs previously in the branch; if such a label exists, or can be built using already existing labels and the unification rules, then the branch is closed, (vi) we repeat the procedure in each branch generated by PB.

The above procedure is based on on a (deterministic) procedure working for *canonical* KEM-tree. A KEM-tree is said to be canonical if it is generated by applying the rules of KEM in the following fixed order: first the α-, ν_x- and π_x-rule, then the β-rule and PNC, and finally PB. Two interesting properties of canonical KEM-trees are (i) that a canonical KEM-tree always terminates, since for each formula there are a finite number of subformulas and the number of labels which can occur in the KEM-tree for a formula A (of $\mathcal{H}$) is limited by the number of modal operators belonging to A, and (ii) that for each closed KEM-tree a closed canonical KEM-tree exists. Proofs of termination and completeness for canonical KEM-trees have been given by Artosi and Governatori (1994) and Governatori (1995).

4 *An Example*

In what follows we assume a straightforward modal extension of the propositional fragment of Reiter's default logic. Let us consider the following default theory

$$\left\{p, \frac{p : \Diamond q}{r}, \frac{p : \Diamond s}{\neg r}\right\}$$

According to the translation from default logic to hypothesis theory (see Section 2.1) this is translated into

$$(\{\Box p, \Box p \wedge H\Diamond q \rightarrow \Box r, \Box p \wedge H\Diamond s \rightarrow \Box\neg r\}, \{H\Diamond q, H\Diamond s\}).$$

This theory has the following two alternative extensions:

$$Th_{\mathcal{H}}(\{\Box p, \Box p \wedge H\Diamond q \rightarrow \Box r, \Box p \wedge H\Diamond s \rightarrow \Box\neg r\} \cup Hq)$$

and

$$Th_{\mathcal{H}}(\{\Box p, \Box p \wedge H\Diamond q \rightarrow \Box r, \Box p \wedge H\Diamond s \rightarrow \Box\neg r\} \cup Hs),$$

the first containing $\Box r$ and the second containing $\Box\neg r$. We provide a KEM-computation of the first extension.

1. $T\Box p$	w_1		
2. THq	w_1		
3. $T\Box p \wedge H\Diamond q \rightarrow \Box r$	w_1		
4. Tp	(W_1, w_1)		
5. Tq	(h_1, w_1)		
6. $T\Box p \wedge H\Diamond q$	w_1	7. $F\Box p \wedge H\Diamond q$	w_1
8. $TH\Diamond q$	w_1	13. $FH\Diamond q$	w_1
9. $T\Box r$	w_1	14. $F\Diamond q$	(H_1, w_1)
10. $T\Diamond q$	(h_2, w_1)	15. Fq	$(W_3, (H_1, w_1))$
11. Tq	$(w_2, (h_2, w_1))$	16. $\times$	
12. Tr	(W_2, w_1)		

Notice that the left branch is open, and thus all the formulas in it (in particular $\Box r$) are consequences of the extension. Moreover this branch displays the model where such an extension holds. The right branch is $\sigma_{\mathcal{H}}$-closed because (5) and (15) are $\sigma_{\mathcal{H}}$-complementary. The argument for the other extension is similar.

5 *Final Remarks*

It was not the objective of this paper to develop a theory of defeasible reasoning. Our motivation was rather practical. We sought for computationally tractable and easily implementable theorem proving techniques suitable for dealing with nonmonotonic forms of inference in a modal setting. The discussion in Section 4 was thus mainly aimed at showing the potential scope of application of the method. In effect, we believe that the method for computing extensions outlined in this section nicely exploits the computational and proof-theoretical advantages offerred by the modal theorem proving system KEM. As we have argued elsewere (see Artosi, Cattabriga and Governatori 1994, Artosi and Governatori 1994), this system enjoy most of the features a suitable proof search system for modal (and in general non-classical) logics should have. In contrast with both resolution and translation-based methods it works for the full modal language (thus avoiding any preprocessing of the input formulas, such as transforming either in clausal form or in some "intermediate" logic); it provides a uniform treatment of a wide variety of modal logics, and it is flexible enough to be extended to cover almost any setting having a Kripke-model based semantics (this is clearly shown by our treatment of a multimodal system arising from the combination of Meyer and van der Hoek's (1992) modal default logic with a deontic logic of the Jones and I. Pörn (1985) type given by Artosi, Governatori and Sartor (1996)). From this perspective it is similar to sequent or tableau proof methods (e.g., Fitting 1983, Catach 1991). Nevertheless, it has several advantages over most tableau/sequent based theorem proving methods: being based on D'Agostino an Mondadori's (1994) classical KE, it eliminates the typical redundancy of the standard cut-free methods, which makes them unsuitable for computational treatment, and, thanks to its label unification scheme, it offers a simple and efficient solution to the permutation problem which arises at the level of the usual tableau-sequent rules for the modal operators. However, unlike e.g., Wallen's (1990) connection method, it uses a natural and easily implementable style of proof construction, and so it appears to provide an adequate basis for combining both efficiency and naturalness. (As to the implementation the reader is referred to Artosi, Cattabriga and Governatori (1994), where a Prolog implementation is provided, and to Artosi, Cattabriga and Governatori (1995) where some related issues are discussed).

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