Modifying is Better than Deleting: A New Approach to Base Revision

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Abstract. We present three approaches to revision of belief bases, discussing the case in which the sentences in the base are partitioned between those which can and those which cannot be changed; the approaches are shown to be semantically equivalent. A new approach is then presented, based on the modification of individual rules, instead of deletion. The revised base has the same models than those generated by the other approaches, but the revised rule part alone has fewer models, that is, it is subjected to a smaller change.

Introduction

Belief revision faces the problem of maintaining the consistency of a system of beliefs when new pieces of information are added, while preserving as many old beliefs compatible with the new data as possible. In other words, given a set of beliefs and a new (incompatible) belief, we want to find a new set of beliefs which includes the new belief and differs as little as possible from the old set. Human beings tackle this task trying to repair systems locally, and keeping modifications as small as possible. Belief revision theories are the logic tools for keeping change small. Different specifications of the problem result in different formal treatments.

Some theories deal with sets closed w.r.t. logical consequence (*belief sets*): others deal with (finite) non-closed sets (*belief bases*). Belief sets are simpler: irrelevance of syntax, for instance, is trivially satisfied, because what is important is the closure of the description w.r.t. logical consequence. Belief bases, on the contrary, are much more realistic. Our knowledge is described by finite sets of sentences, and so are the norms regulating our daily behavior and all sorts of databases. Any change affects only finite sets of sentences. This makes more difficult to satisfy (and even to state clearly) the intuitively appealing principle of irrelevance of syntax. Another distinction is between purely logical approaches and approaches assuming the existence of extra-logical properties of the sentences (these properties are often assumed for dealing with non-uniqueness of the result of revision). There may be different candidates for a revision (of a set or base), and this is a hard practical difficulty, when trying to put the theory of belief revision to work (e.g. in a deductive database system). Relying on extra-logical properties of the sentences may sometimes find a unique revision, building a unique result as a combination of the candidates.

A special case of extra-logical properties, pertaining to finite bases, is the distinction of sentences between those that can be affected by the revision and those that cannot be affected. The distinction is rather common, reminding of that between defeasible and non-defeasible rules in non-monotonic logics. As a mnemonic, we shall speak of rules (sentences that can be changed) and facts (sentences that cannot be changed).

This papers deals with approaches to finite base revision, using, as only extra-logical property, the distinction among rules and facts. We aim to find a logically principled unique solution for the revision problem.

We describe three approaches to revision, and show that they are equivalent, in the sense that they result in the same bases, and that they fail to satisfy the principle of extensionality: equivalent sets of rules may result, after revision, in non-equivalent set of rules. Then, we present a new approach to revision. It produces a base which has the same set of models than the base obtained with the other approaches, but the set of models of the resulting *rule set* is a subset of those obtained for the rule set with any other approach. Our approach is fully algorithmic, and may be easily implemented using tableaux systems.

1 Revision for belief sets and belief bases

The baseline for the modern treatment of belief revision is usually taken to be the result of the joint efforts of Alchourrón, Makinson and Gärdenfors. The theory they eventually arrived at is know as the AGM approach to belief revision (or, simply, AGM theory). This approach is fully described in [7]; later work in the same line includes [13], [8]. The central notion in AGM is that of *belief set*. A belief set is a set of sentences (of some propositional language) such that it may be rationally held by an individual, that is ([7], ch.2.2), a consistent set closed under logical consequence.

Definition 1. A set K of sentences is a (non-absurd) belief set iff (i) $\perp$ is not a logic consequence of the sentences is K and (ii) if K$\vdash$ b then b $\in$K. It holds that K=Cn(K).

If a $\in$K a is accepted in K, if $\neg a$ $\in$K a is rejected, else it is indetermined. Three basic types of changes are identified: expansion, contraction, revision. The table below summarises meaning and notation for them. We refer the reader to [7] for the full theory.

previous state	expansion $\mathrm{K}^{+}a$	contraction $\mathrm{K}^{-}a$	revision $\mathrm{K}^{*}a$
a accepted in K	a accepted in K	a indetermined in K	a accepted in K
a indetermined in K	a accepted in K	a indetermined in K	a accepted in K
a rejected in K	$\mathrm{K}_{\perp}$	a rejected in K	accepted in K

Instead of giving operational definitions for the three operations, AGM give postulates for them, intended to constrain the class of all possible operational definitions. Expansion is defined as a unique operation, that is, $\mathrm{K}^{+}a$ =Cn(K$\cup\{a\}$). Revision is linked to contraction and expansion by means of the Levi identity: $\mathrm{K}^{*}a$ =$(\mathrm{K}^{-}\neg a)^{+}a$. The postulates for contraction and revision do not define unique operations. A contraction of K by a is a belief set K' which is subset of K and such that a $\notin$K'; the principle of *minimal change* suggests that we look for the largest of such subsets. It is easily shown that, in general, there is not a unique set. In general, we may find a family of sets, which is called K$\perp a$ (for a formal definition, see the next section). Full meet contraction results from intersecting all the sets in K$\perp a$. While it seems appealing, full meet contraction has a major drawback, in that $\mathrm{K}^{-}a$=K∩Cn($\neg a$). Contraction results in a very small set, and minimal change is not respected. On the other side, if we take only one of the sets in $\mathrm{K}\perp a$ (*maxichoice* contraction), we have no logically principled way of doing it.

A different approach maintains that the assumption of closed sets is too unrealistic to be really significant as a model of how belief works. This approach is discussed e.g. in [4], [15], [10], [11], [16]. First we describe bases in which all sentences may be changed, then bases partitioned in rules and facts.

We start from the definition of three families of interesting subsets of B.

Definition 2. $\mathrm{B}\perp a$, $\mathrm{B}_{\Rightarrow}a$, $\mathrm{B} \ddagger a$ are defined as follows.

– $\mathrm{B}\perp a$ is the family of maximal subsets of B not implying a:

$$\mathrm{C} \in \mathrm{B}\perp a \iff \begin{cases} \mathrm{C} \subseteq \mathrm{B} \\ a \notin \mathrm{Cn}(\mathrm{C}) \\ \text{if } b \notin \mathrm{C} \text{ and } b \in \mathrm{B}, \text{ then } a \in \mathrm{Cn}(\mathrm{C} \cup \{b\}) \end{cases}$$

– $\mathrm{B}_{\Rightarrow}a$ is the family of minimal subsets of B implying a:

$$\mathrm{C} \in \mathrm{B}_{\Rightarrow}a \iff \begin{cases} \mathrm{C} \subseteq \mathrm{B} \\ a \in \mathrm{Cn}(\mathrm{C}) \\ \text{if D} \subset \text{C, then D} \notin \mathrm{B}_{\Rightarrow}a \end{cases}$$

– $\mathrm{B} \ddagger a$ is the family of minimal incisions of a from B such that:

$$\mathrm{I} \in \mathrm{B} \ddagger a \iff \begin{cases} \mathrm{I} \subseteq \mathrm{B} \\ \mathrm{C} \cap \mathrm{I} \neq \emptyset \text{ for each C} \in \mathrm{B}_{\Rightarrow}a \\ \text{if D} \subset \text{I, there is C} \in \mathrm{B}_{\Rightarrow}a \text{ such that C} \cap \mathrm{D} = \emptyset \end{cases}$$

It is worth noting that if a is logically true then $\mathrm{B}\perp a = \emptyset$, $\mathrm{B}_{\Rightarrow}a = \{\emptyset\}$, $\mathrm{B} \ddagger a = \emptyset$. Moreover, if B is finite any element C of one of the families above is finite, and there is a finite number of them. Different definitions of revision are obtained as functions of the above defined families. For simplicity, we define at first the contraction functions, then, according to Levi identity, the revision functions are obtained by adding the new data to the output of the contraction function. The main idea is that of simple base contraction, using $\mathrm{B}\perp a$. Contraction is defined as the intersection of all subsets in $\mathrm{B}\perp a$. (We modify the original notion given in [3], [9], [14] in order to retain the finiteness of the base). A different approach was introduced by [1] for belief sets, but may be easily adapted to belief bases. The safe contraction is the base obtained deleting all possible minimal subsets implying a. Last, deleting the elements of an incision on $\mathrm{B}_{\Rightarrow}a$ transforms B in a set B' such that a ∉B'. Deleting the elements of all possible incisions (all possible choices of elements from $\mathrm{B}_{\Rightarrow}a$) we get the excided contraction.

Definition 3. Simple base, safe, and excided contraction and revision are defined as follows:

type	*contraction*	*symbol*
simple base	$\bigcap_{\mathrm{C}\in\mathrm{B}\perp a}\mathrm{C}$	$\mathrm{B}^{\ominus_1}a$
safe	$\left\{b \mid b\in\mathrm{B} \text{ and } b\notin\bigcup_{\mathrm{C}\in\mathrm{B}_{\Rightarrow}a}\mathrm{C}\right\}$	$\mathrm{B}^{\ominus_2}a$
excided	$\left\{b \mid b\in\mathrm{B} \text{ and } b\notin\bigcup_{\mathrm{I}\in\mathrm{B}\ddagger a}\mathrm{I}\right\}$	$\mathrm{B}^{\ominus_3}a$

revision
$\mathrm{B}^{\oplus_i}a=\mathrm{B}^{\ominus_i}\neg a\cup\{a\}$ $i=1,2,3$

The three contraction (and revision) functions share a common flavour. First of all, all of them use a set-theoretic operation on all the sets of a certain family. They might be called *full* operations, in the sense in which the adjective "full" pertains to full meet contraction according to AGM.

We study now the relationships between the sets defined by these operations.

Theorem 4. *If* $a\in\mathrm{Cn}(\mathrm{B})$ *and* B *is finite, then* $\mathrm{B}^{\ominus_1}a=\mathrm{B}^{\ominus_2}a=\mathrm{B}^{\ominus_3}a$.

The problems with full approaches is essentially the same as those with full meet contraction: they usually contract too much, that is, the resulting set is too small.

Now we introduce the distinction among facts and rules. Facts are sentences which are not subject to change in the process of revision, while rules may be changed. If B is a base, the sets of facts and rules will be denoted B_φ and B_ρ. The operations studied in the previous section are extended to this case.

We first examine an extension to the $^{\ominus_1}$ operation, called prioritized base revision, the idea of which is due to [15]. In a prioritized revision, we first take all facts not implying a, then we add as many rules as we can without implying a, in all different ways.

Definition 5. $\mathrm{B}\Downarrow a$ is the family of the sets $\mathrm{C}=(\mathrm{C}_\rho,\mathrm{C}_\varphi)$, such that:

1. $\mathrm{C}_\rho\subseteq\mathrm{B}_\rho$ and $\mathrm{C}_\varphi\subseteq\mathrm{B}_\varphi$;
2. $a\notin\mathrm{Cn}(\mathrm{C}_\varphi)$ and if $\mathrm{C}_\varphi\subset\mathrm{D}\subseteq\mathrm{B}_\varphi$, then $a\in\mathrm{Cn}(\mathrm{D})$;
3. $a\notin\mathrm{Cn}(\mathrm{C}_\varphi\cup\mathrm{C}_\rho)$, and if $\mathrm{C}_\rho\subset\mathrm{E}\subseteq\mathrm{B}_\rho$, then $a\in\mathrm{Cn}(\mathrm{C}_\varphi\cup\mathrm{E})$.

Definition 6. Let B be a finite base, and let $\Phi(\mathrm{C})=\bigwedge_{b_i\in\mathrm{C}_\rho\cup\mathrm{C}_\varphi}b_i$; then the prioritized base contraction of B by a, is the set $\mathrm{B}\hat{\ominus}_1a=\{\bigvee_{\mathrm{C}\in\mathrm{B}\Downarrow a}\Phi(\mathrm{C})\}$.

If we take the assumption that the sentence we want to retract is not implied by facts alone, that is $a\notin\mathrm{Cn}(\mathrm{B}_\varphi)$, then the elements of $\mathrm{B}\Downarrow a$ have the form $(\mathrm{C}_\rho,\mathrm{B}_\varphi)$, $\Phi(C)=\bigwedge_{b_i\in\mathrm{C}_\rho}b_i\wedge\bigwedge_{\varphi_i\in\mathrm{B}_\varphi}\varphi_i=\Phi(\mathrm{C}_\rho)\wedge\bigwedge_{\varphi_i\in\mathbf{B}_\varphi}\varphi_i$ and the following holds:

$$\text{if } a\notin\mathrm{Cn}(\mathrm{B}_\varphi),\ \text{then } \mathrm{B}\hat{\ominus}_1a=\left(\left\{\bigvee_{\mathrm{C}\in\mathrm{B}\Downarrow a}\Phi(\mathrm{C}_\rho)\right\},\mathrm{B}_\varphi\right) \tag{1}$$

Another approach is related to the $^{\ominus_2}$ operation, whose idea is due to [1]. We assume that only rules have to be blamed for a, so only rules involved in deriving a are deleted from the base; no set in $\mathrm{B}_{\Rightarrow}a$ contains only facts.

Definition 7. For each $\mathrm{C}\in\mathrm{B}_{\Rightarrow}a$, let $\mathrm{C}_\rho=\mathrm{B}_\rho\cap\mathrm{C}$, such that $\mathrm{C}\not\subseteq\mathrm{B}_\varphi$. The safe contraction of a from B, $\mathrm{B}\hat{\ominus}_2a=\left(\left\{b|b\in\mathrm{B}_\rho \text{ and } b\notin\bigcup_{\mathrm{C}\in\mathrm{B}_{\Rightarrow}a}\mathrm{C}_\rho\right\},\mathrm{B}_\varphi\right)$.

A third approach may be derived extending the $^{\ominus_3}$ operation. This uses prioritized incisions. We assume, as usual, that a cannot be derived by facts alone. As above, for $\mathrm{C}\in\mathrm{B}_{\Rightarrow}a$, $\mathrm{C}_\rho=\mathrm{B}_\rho\cap\mathrm{C}$ and $\mathrm{C}_\varphi=\mathrm{B}_\varphi\cap\mathrm{C}$.

Definition 8. A prioritized incision (p-incision) on $\mathrm{B}_{\Rightarrow}a$ is a set $\mathrm{I} \subseteq \mathrm{B}_\rho$ such that $\mathrm{C}_\rho \cap \mathrm{I} \neq \emptyset$ for each $\mathrm{C} \in \mathrm{B}_{\Rightarrow}a$. A p-incision is minimal if for all $\mathrm{D} \subset \mathrm{I}$, D is not a p-incision. The family of all possible p-incisions is denoted by $\mathrm{B} \dagger a$.

Definition 9. The excided p-contraction of B by a, $\mathrm{B}\hat{\ominus}_3 a$, is the set $\{b | b \in \mathrm{B}$ and $b \notin \bigcup_{\mathrm{I} \in \mathrm{B}\dagger a} \mathrm{I}\}$.

As above, we examine the relations among these three types of operations. A result like the one obtained in the unpartitioned case holds, that is:

Theorem 10. *If* $a \in \mathrm{Cn}(\mathrm{B})$ *and* B *is finite, then* $\mathrm{B}\hat{\ominus}_1 a = \mathrm{B}\hat{\ominus}_2 a = \mathrm{B}\hat{\ominus}_3 a$*.*

2 Modifying is better than deleting

2.1 The Proposed Procedure

In this section, we show how to modify rules in order to obtain a revised base. The revised base is equivalent to that we would obtain from previously described methods, but the rule part, taken alone, has a larger set of consequences than its counterparts. That is, if RB is the revised base obtained through our procedure, and RB1 is the revised base obtained through one of the previously described procedues, Cn(RB)=Cn(RB1) but Cn(RB_ρ)$\supseteq$Cn($\mathrm{RB1}_\rho$).

Bases, Evaluations, Models Each fact φ_i may be rewritten as a disjunction of possibly negated atoms: $\varphi_i = \bigvee_k \gamma_{ik}$; similarly each rule ρ_i may be rewritten as

$$\bigvee_j \neg\alpha_{ij} \vee \bigvee_k \beta_{ik} = \alpha_i \rightarrow \beta_i$$

where $\alpha_i = \bigwedge_j \alpha_{ij}$ and $\beta_i = \bigvee_k \beta_{ik}$. Facts and rules in this form are said to be in normal form (or, simply, normal).

Let X be a set of atoms and let v be an evaluation function for X, $v : X \rightarrow \{true, false\}$. Let V be the set of evaluation functions for (X).

Definition 11. $v \in V$ is a model of $a \in \Sigma(A)$ iff $v(a) = true$. v is a model of $A = \{a_i\}$ iff v is a model of each a_i. The set of models of a base B is denoted by $V(\mathrm{B})$.

Adding formulae to a base means getting a smaller set of models. In particular, if a is a formula inconsistent with a base B, $V(B \cup \{a\}) = \emptyset$. The revision of $\mathrm{B} = (\mathrm{B}_\rho, \mathrm{B}_\varphi)$ by a should be some base B' such that $V(\mathrm{B}'_\rho) \cap V(\mathrm{B}'_\varphi) \neq \emptyset$. In the process of revision, only the rule part of the base is changed, that is, $\mathrm{B}'_\varphi = \mathrm{B}_\varphi \cup \{a\}$. So, finding a revision amounts to finding a new set of rules B'_ρ.

Models and Revisions It principle, nothing is said about the relationships between $V(\mathrm{B}'_\rho)$ and $V(\mathrm{B}_\rho)$. Some properties are however desirable. First of all, we want the new formula a to be independent from the new rules $V(\mathrm{B}'_\rho)$, that is, $V(\mathrm{B}'_\rho) \cap V(a) \neq V(\mathrm{B}'_\rho)$; then we want that the new rules do not discard any of the models for the original rules, that is, $V(\mathrm{B}'_\rho) \supseteq V(\mathrm{B}_\rho)$. This is in line with Levi's idea that a revision loses something and then adds something else; in this case, the set of models gets larger (from $V(\mathrm{B}_\rho)$ to $V(\mathrm{B}'_\rho)$) and then is (effectively) intersected with $V(a)$. This covers the main case. If a is consistent with B, that is $V(\mathrm{B}_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a) \neq \emptyset$, we simply add a to the set of facts, leaving the rules unchanged.

So the complete definition is:

Definition 12. A revision of $\mathrm{B} = (\mathrm{B}_\rho, \mathrm{B}_\varphi)$ by a is a (finite) base $\mathrm{B}' = (\mathrm{B}'_\rho, \mathrm{B}_\varphi \cup \{a\})$ such that:

1. If $V(\mathrm{B}_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a) \neq \emptyset$, then $\mathrm{B}'_\rho = \mathrm{B}_\rho$, else:
   (a) $V(\mathrm{B}'_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a) \neq \emptyset$;
   (b) $V(\mathrm{B}'_\rho) \supseteq V(\mathrm{B}_\rho)$;
   (c) $V(\mathrm{B}'_\rho) \cap V(a) \neq V(\mathrm{B}'_\rho)$;

Now we have to define a minimal inconsistent sub-base; this may be easily done using the definition of $\mathrm{B}_{\Rightarrow} a$. A minimal inconsistent sub-base for $(\mathrm{B}_\rho, \mathrm{B}_\varphi \cup \{a\})$ is simply an element of $\mathrm{B}_{\Rightarrow} \neg a$. The set of rules not included in the rule part of any minimal inconsistent sub-base will be denoted $\hat{\mathrm{B}}_\rho$. $\hat{\mathrm{B}}_\rho$ will denote the set of rules not included in the rule part of any minimal inconsistent sub-base for $(\mathrm{B}_\rho, \mathrm{B}_\varphi \cup \{a\})$.

Orderly Revision and Minimax Revision Saying that rules are not ordered by importance means, roughly, that all the rules (involved in inconsistency) should be modified, their sets of model getting larger. We say that a revision is *orderly* if it modifies all and only the rules involved in inconsistency. Rules in the new base correspond to rules in the old one, with the possible exception that some old rules are eliminated. There is no reason to modify rules that do not belong to any minimal inconsistent subbase: so a "good" revision $\mathrm{B}' = (\mathrm{B}'_\rho, \mathrm{B}_\varphi \cup \{a\})$ should have a set of rules B'_ρ such that $\hat{\mathrm{B}}_\rho \subseteq \mathrm{B}'_\rho$, that is, $V(\mathrm{B}'_\rho) \subseteq V(\hat{\mathrm{B}}_\rho)$. This means that $V(\mathrm{B}') = V(\mathrm{B}'_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a) \subseteq V(\hat{\mathrm{B}}_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a)$ and sets an upper bound on the size of $V(\mathrm{B}')$. It is immediate to see that $\hat{\mathrm{B}}_\rho$ is the set of rules obtained through the operation of safe revision: $\mathrm{B}\hat{\ominus}_2 a$.

Definition 13. Let $\mathrm{B}\top a$ be the family of sets of rules B'_ρ such that:

1. $V(\mathrm{B}_\rho) \subset V(\mathrm{B}'_\rho) \subseteq V(\hat{\mathrm{B}}_\rho)$;
2. $V(\mathrm{B}'_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a) \neq \emptyset$.

Let $m(\mathrm{B}\top a)$ be the set of $\subseteq$-minimal elements of $\mathrm{B}\top a$: $\mathrm{B}'_\rho \in m(\mathrm{B}\top a)$ iff there is no $\mathrm{B}''_\rho \in m(\mathrm{B}\top a)$ such that $V(\mathrm{B}''_\rho) \subseteq V(\mathrm{B}'_\rho)$. A *minimax* revision of B by a is $\check{B} = (\check{B}_\rho, \mathrm{B}_\varphi \cup \{a\})$ such that $V(\check{B}_\rho) = \bigcup_{\mathrm{B}'_\rho \in m(\mathrm{B}\top a)} V(\mathrm{B}'_\rho)$

Theorem 14. $V(\check{\mathrm{B}}) = V(\hat{\mathrm{B}}_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a)$

Theorem 15. $\mathrm{B}^* a = (\{\bar{\rho}_i\}, \mathrm{B}_\varphi \cup \{a\})$ *is a minimax revision where:*

$$\bar{\rho}_i = \begin{cases} \alpha_i \to \beta_i & \textit{if } \alpha_i \to \beta_i \in \hat{\mathrm{B}}_\rho \\ \{(\alpha_i \wedge \neg\varphi) \to \beta_i)\} & \textit{where } \varphi \in \mathrm{B}_\varphi \cup a \textit{ otherwise} \end{cases}$$

2.2 Minimax Revision and AGM Postulates

We are now going to examine relations between minimax base revision and the AGM theory of belief set revision. Let us say that a sentence a belongs to the belief set generated by B iff a holds in any model in $V(\mathrm{B})$, that is:

Definition 16. Given a base $\mathrm{B} = (\mathrm{B}_\rho, \mathrm{B}_\varphi)$, the belief set generated by B is denoted $\mathrm{K}(\mathrm{B})$. A sentence $a \in \mathrm{K}(\mathrm{B})$ iff $V(\mathrm{B}) \subseteq V(a)$.

It follows from the definition that $\mathrm{K}(\mathrm{B}_1) \subseteq \mathrm{K}(\mathrm{B}_2)$ iff $V(\mathrm{B}_2) \subseteq V(\mathrm{B}_1)$. From Theorem 14 we know that $V(\mathrm{B}^*a) = V(\hat{\mathrm{B}}_\rho) \cap V(\mathrm{B}_\varphi) \cap V(a)$, that is, $V(\mathrm{B}^*a) = V((\hat{\mathrm{B}}_\rho, \mathrm{B}_\varphi \cup \{a\}))$. It follows that $\mathrm{K}(\mathrm{B}_\rho^{*a}, \mathrm{B}_\varphi \cup \{a\}) = K(\hat{\mathrm{B}}_\rho, \mathrm{B}_\varphi \cup \{a\})$, $V(\mathrm{B}_\rho^{*a}) \subseteq V(\hat{\mathrm{B}}_\rho)$, and $\mathrm{K}(\hat{\mathrm{B}}_\rho) \subseteq \mathrm{K}(\mathrm{B}_\rho^{*a})$.

The importance of this fact becomes apparent if we introduce another operation, which we name *retraction* to distinguish it from contraction. Retraction applies only to (sentences implied only by) facts and modifies the fact part of a base.

Definition 17. The retraction of φ_j from $\mathrm{B} = (\mathrm{B}_\rho, \mathrm{B}_\varphi)$ is the base $\mathrm{B}_{-\varphi_j} = (\mathrm{B}_\rho, \{\varphi_i\}_{i \neq j})$.

Proposition 18. *For any* $\varphi_j \in \mathrm{B}_\varphi \cup \{a\}$, $\mathrm{K}((\hat{\mathrm{B}}_\rho, \mathrm{B}_\varphi \cup \{a\})_{-\varphi_j}) \subseteq \mathrm{K}((\mathrm{B}_\rho^{*a}, \mathrm{B}_\varphi \cup \{a\})_{-\varphi_j})$.

To show the result it suffices to show that $V((\mathrm{B}_\rho^{*a}, \mathrm{B}_\varphi \cup \{a\})_{-\varphi_j}) \subseteq V((\hat{\mathrm{B}}_\rho, \mathrm{B}_\varphi \cup \{a\})_{-\varphi_j})$. This holds because $V(\mathrm{B}_\rho^{*a}) \subseteq V(\hat{\mathrm{B}}_\rho)$.

We may now define contraction from revision and retraction.

Definition 19. The contraction of $\mathrm{B} = (\mathrm{B}_\rho, \mathrm{B}_\varphi)$ by a is $\mathrm{B}^-a = (\mathrm{B}_\rho^{*\neg a}, \mathrm{B}_\varphi) = \mathrm{B}^-a = (\mathrm{B}_\rho^{*\neg a}, \mathrm{B}_\varphi \cup \{\neg a\})_{-\neg a}$.

2.3 Discussion

Theorem 15 gives us a simple recipe for transforming rules. Not only is the procedure fully algorithmic, but it can also be decomposed: the revision is built piecewise, starting from individual rules and individual facts. The result of the revision is itself in normal form, so that iterated revision is well-defined, contrary to what results for standard AGM belief set revision. (For ways of modifying the AGM theory in order to allow iterated revision, see:[2],[6]).

Minimax revision offers a clear definition of so-called *multiple* revision, that is revision by means of a set of sentences. When contraction is considered as the main operation, using a set of sentences instead of one often results in unclear intuitions (see [5]).

Extensionality does not hold in general. Adding to a set a derivative rule does not change the set of consequences of the rule, but it may change the output of a revision operation. This suggests a stronger (not algorithmic) notion of equivalence between two bases:

Definition 20. (Strong equivalence). $\mathrm{B1} = (\mathrm{B1}_\rho, \mathrm{B1}_\varphi)$ is strongly equivalent to $\mathrm{B2} = (\mathrm{B2}_\rho, \mathrm{B2}_\varphi)$ iff

1. $V(\mathrm{B1}) = V(\mathrm{B2})$ and
2. $\forall a,\ V(\mathrm{B1}^{*}a) = V(\mathrm{B2}^{*}a)$.

Equivalent, but not strongly equivalent, bases are given in example 25.

3 Examples

Five examples are described. The first two are closely related each other, and show how the procedure described above fares when compared to standard procedures (e.g. safe revision). The third example illustrates the need for using rules in normal form. The fourth one shows that the procedure, may deal with contradiction resulting from chains of implications. The fifth one deals with strong equivalence.

Example 21. Let $\mathrm{B} = (\{a \to c, b \to c\}, \{a \vee b\})$. Let the new fact be $\neg c$.

$$\begin{aligned}\mathrm{B}^{*}\neg c &= (\{(a \wedge \neg((a \vee b) \wedge \neg c)) \to c, (b \wedge \neg((a \vee b) \wedge \neg c)) \to c\}, \{a \vee b, \neg c\}) \\ &= (\{(a \wedge c) \to c, (b \wedge c) \to c\}, \{a \vee b, \neg c\}) = (\emptyset, \{a \vee b, \neg c\})\end{aligned}$$

The same result would obtain if standard procedures had been used.

Example 22. Let $\mathrm{B} = (\{a \to c, b \to c\}, \{a \wedge b\})$. Let the new fact be $\neg c$.

$$\begin{aligned}\mathrm{B}^{*}\neg c &= (\{(a \wedge \neg((a \wedge b) \wedge \neg c)) \to c, (b \wedge \neg((a \wedge b) \wedge \neg c)) \to c\}, \{a \wedge b, \neg c\}) \\ &= (\{(a \wedge \neg b) \to c, (b \wedge \neg a) \to c\}, \{a \wedge b, \neg c\})\end{aligned}$$

The only difference between this example and the preceding one is that the fact component is $\{a \wedge b\}$ instead of $\{a \vee b\}$.

Standard procedures do not distinguish between these aspects: when rules are deleted, we always get the revision $(\emptyset, \mathrm{B}_{\varphi} \cup \{\neg c\})$. Our procedure, which modifies rules on the ground of the new set of facts, produces a different result. It could be argued that the models of the two revisions are exactly the same, so that the difference is only at a surface level. The answer is that the two revisions are indeed different as far as the rule part is concerned, because $V(\{(a \wedge \neg b) \to c, (b \wedge \neg a) \to c\}) \subset V(\emptyset)$. Were we to retract all facts, we would end up with a different set of evaluations, showing that our procedure is sensitive to differences which do not influence the behaviour of other procedures. There is a sort of monotonicity property of the procedure. The (fact component of the) base of the first example is, intuitively, weaker than the second one, and this is reflected by the corresponding sets of evaluations. The two revisions show the same trend, as the rule set resulting from the first revision (the empty set) is of course weaker than the rule set resulting from the second revision.

Example 23. Let $\mathrm{B} = (\{a \vee b \to c\}, \{a\})$. (the rule is not in normal form). Let the new fact be $\neg c$. If we did not care about the normal form, the revision would be:

$$(\{((a \vee b) \wedge \neg(a \wedge \neg c)) \to c\}, \{a, \neg c\}) = (\{(b \wedge \neg a) \to c\}, \{a, \neg c\})$$

A base equivalent to B and in normal form is the the base $\mathrm{B}' = (\{a \to c, b \to c\}, \{a\})$. As $b \to c$ is not included in any minimal inconsistent sub-base,

$$\mathrm{B}'^{*}\neg c = (\{(a \wedge \neg(a \wedge \neg c)) \to c, b \to c\}, \{a, \neg c\}) = (\{b \to c\}, \{a, \neg c\})$$

The difference between the two resulting bases is appreciated if we imagine the retraction of facts; in this case, it results that $V(b \to c) \subset V((b \wedge \neg a) \to c)$. This is a general result: rules in normal form result in smaller sets of evaluations, that is, more specific rule sets.

Example 24. Let $\mathrm{B} = (\{a \to b, b \to c, b \to d\}, \{a, d\})$. Let the new fact be $\neg c$.

The only minimal inconsistent sub-base is $(\{a \to b, b \to c\}, \{a, d, \neg c\})$.

$$\mathrm{B}^* \neg c = (\{a \wedge \neg d \to b, a \wedge c \to b, b \wedge \neg a \to c, b \wedge \neg d \to c, b \to d\}, \{a, d, \neg c\})$$

This should be contrasted with the result of standard procedures, which delete rules involved in contradiction: $(\{b \to d\}, \{a, d, \neg c\})$. Our procedure produces a more specific rule set, which allows more inferences. This example shows also that the simple procedure automatically deals with contradictions deriving from chains of conditionals.

Example 25. Let

$$\mathrm{B1} = (\{a \to b, b \to c\}, \{b\}) \qquad \mathrm{B2} = (\{a \to b, b \to c, a \to c\}, \{b\})$$

be two equivalent bases. It is easy to see that $V(\mathrm{B1}_\rho) = V(\mathrm{B2}_\rho)$. Let the new fact be $\neg c$, therefore the revisions are

$$\mathrm{B1}^* \neg c = (\{a \to b\}, \{b, \neg c\}) \qquad \mathrm{B2}^* \neg c = (\{a \to b, a \to c\}, \{b, \neg c\})$$

and $V(\mathrm{B2}^* \neg c) \subset V(\mathrm{B1}^* \neg c)$. This example shows that in dealing with bases, *syntax matters*.

4 Directions for future work

In this paper we have described a simple procedure for modifying knowledge bases expressed by finite sets of formulae of a propositional language, where each formula is a rule (which can be changed) or a fact (which cannot), to accommodate new facts; the main difference between our approach and other approaches is that rules are modified, instead of being deleted.

This procedure may be extended in many directions. The first possibility is the extension to bases with more than two degrees of importance for sentences. This is the case with normative systems, where the hierarchy of importance stems directly from the hierarchy of normative sources. The second direction deals with the meaning of the rules. While the belief revision literature usually employs a propositional language, speaking about rules and facts suggests that rules might be rather seen as axiom schemes, describing universally quantified relationships between variables. How do belief revision procedures behave in this new setting? This is, we believe, a general questions, to which few answers are given in the literature. The third field for extending the framework is that of (propositional) modal logic. The motivation is again that of representing some features of the evolution of normative systems, which are usually represented by means of systems of modal (deontic) logic. It is indeed the mechanism of derogation (specifying exceptions) which originally suggested to modify rules instead of deleting them.

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