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A DOUBLE-SURVEY ESTIMATE OF POPULATION SIZE FROM INCOMPLETE COUNTS

The problem of estimating the size of a population from "total counts" known to be inaccurate has been approached from several directions (Table 1). The first 3 are applicable when the entities being counted cannot be distinguished individually, but each of these methods suffers from the requirement that the population is counted, albeit incompletely, on numerous occasions. The effort required is daunting.

Caughley (1974) showed that only 2 counts were needed when entities could be identified individually such that the tallies of 2 equally skilled observers could be dissected into those seen by one or other observer and those seen by both. The method was illustrated by Eltringham's (1972) data on groups of elephants (*Loxodonta africana*) counted and mapped independently by 2 observers during a "complete count" from the air. This note extends that method by dispensing with the requirement that the probability of a given entity being seen by one observer is the same as its being seen by the other. Hence the method is now generalised to allow for the 2 counts being made by different methods of survey. We will give an example of its use in which nests of the crocodile (*Crocodylus porosus*) are counted and mapped from the air and from the ground.

When it is possible to map the location of an entity it is possible also to determine how many were found by both surveys (B),

Table 1. Methods of estimating population size from incomplete counts.

Method	Counts required	Data	Reference
Binomial estimate	Many	Mean and variance of counts at 1 level of sightability	Hanson (1967)
Parabolic estimate	Many	Mean and variance of counts at 2 levels of sightability	Caughley and Goddard (1972)
Bounded counts	Many	Largest and second largest count	Robson and Whitlock (1964)
Replicate-observers binomial estimate	Two	Numbers seen by both observers and number seen by only one or other observer	Caughley (1974)
Disparate-observers binomial estimate	Two	Numbers seen by both observers, number seen only by the first and number seen only by the second.	This report

how many by survey 1 but not survey 2 (S_1), and how many by survey 2 but not survey 1 (S_2). If M is the unknown number missed by both surveys and N is the total number of entities, also unknown, then the exhaustive frequencies and the probabilities associated with them are

$$\begin{array}{ccccccccc} B & + & S_1 & + & S_2 & + & M & = & N \\ P_1P_2 & + & P_1(1\text{-}P_2) & + & P_2(1\text{-}P_1) & + & (1\text{-}P_1)(1\text{-}P_2) & = & 1 \end{array}$$

P_1 being the probability of an entity being seen by the first survey and P_2 the probability of its being seen by the second. Hence the unknown parameters can be estimated from the known frequencies B, S_1 and S_2, by

$$\hat{P}_1 = B/(B + S_2)$$
$$\hat{P}_2 = B/(B + S_1)$$
$$\hat{M} = S_1S_2/B$$
$$\hat{N} = (B + S_1)(B + S_2)/B \qquad (1)$$

The model is logically equivalent to that of the Petersen estimate. On the first survey a sample is mapped (marked), the sample of the second survey comprising some entities previously mapped (recaptures), others unmapped. The difference lies in the symmetry of the present model: the first and second surveys are interchangeable. Nonetheless, the well-explored mathematics of the Petersen estimate can be adapted easily to this model. Chapman (1951) has given a correction for the Petersen estimate. Applying this to equation 1 our estimate becomes

$$N = \frac{(S_1 + B + 1)(S_2 + B + 1)}{(B + 1)} - 1 \qquad (2)$$

This is, in contrast to the estimate of eqn. (1), exactly unbiased when $S_1 + S_2 + 2B \geqq N$. Its variance can be estimated by a translation of Seber's (1973:60) formula which is also exactly unbiased when $S_1 + S_2 + 2B \geqq N$:

$$\text{Var}(\hat{N}) = \frac{S_1S_2(S_1 + B + 1)(S_2 + B + 1)}{(B + 1)^2(B + 2)} \qquad (3)$$

Our example uses counts of crocodile nests in the swamps of the Liverpool River System, Northern Australia. The frequencies are too low for a precise estimate, but they serve, notwithstanding, to demonstrate the method. $S_1 = 2$ nests were seen only from the air, $S_2 = 5$ only from the ground, and $B = 1$ nest was located by both surveys. Hence the probability of seeing a nest from the air is estimated as $P_1 = 1/(1 + 5) = 0.167$, and from the ground as $P_2 = 1/(1 + 2) = 0.333$. The number missed by both surveys is estimated as $\hat{M} = 2 \times 5/1 =$

10 and the total number, both counted and uncounted, is estimated by eqn. (2) as $\hat{N} = 13$. It has an approximate variance (eqn. 3) of 23 and hence a standard error of $\sqrt{23} = \pm 5$.

The use of this method assumes that the counts of the 2 surveys are independent and that there is a constant probability of seeing each nest by a given method of survey. The first assumption is critical: obviously one does not show the locations mapped at the first survey to the people who will run the second. The other assumption is not critical. We simulated pairs of surveys in which the probability of seeing a nest, rather than being a constant for a survey, was a random draw from a beta distribution of fixed mean and variance, different distributions being used for the 2 surveys. These produced estimates similar to those of control simulations in which probabilities were set at the means of the beta distributions.

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