# A Sensitivity Method for Assessing the Impact of Generator/Transformer Impedance on Power System Fault Behaviour

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***Abstract*—An analytical method is presented for determining the potential impact that the connection of a single new generator may have upon the performance of a transmission system under balanced three-phase fault conditions. Expressions are derived that demonstrate the dependence of the system-wide fault behaviour of the modified network upon both the sub-transient or short circuit impedance of the new generator/transformer and the configuration of the original network. From these expressions sensitivity factors are developed that characterise the potential impact that the connection of the new generator can have upon network fault behaviour. The procedure is derived in general terms and verified with results from simulation of a 17-bus test system. The results emphasize that the impact of the generator/transformer is often more contingent upon the network characteristics at the point of connection rather than the sub-transient/short circuit impedances of the new elements.**

***Index Terms*-- Power system faults, Sensitivity analysis, Fault currents, Overvoltages, Power generation planning**

## I. Nomenclature

$I_k^{(f)}$ – balanced three phase fault current at bus *k*

$\Delta V_l^{(k)}$ – voltage change at bus *l* due to fault at bus *k*

$Z_G$ – sub-transient impedance of synchronous generator

$Z_T$ – short circuit impedance of three-phase transformer

$Z_{kl}$ – transfer impedance between bus *k* and bus *l*

## II. Introduction

THE increasing demand for electricity ensures that power transmission networks must be augmented regularly with extra generating capacity. Although this enhances network performance under steady state conditions, the impact of new generators upon network fault performance also must be assessed. Changes to network fault conditions will impact upon relay settings and component selection and may necessitate further modification of the existing power system.

While techniques have been developed to consider the impact of alterations to transmission line impedances on system behaviour under fault conditions [1], the impact of a generator design has been examined less thoroughly. Historical network behaviour also will be less applicable given the increased use of new generators such as gas turbines, embedded or distributed generators, sustainable supplies or even novel generators such as Powerformer™ [2] – the high voltage directly connected generator developed by ABB corporate research – that characterize modern power systems.

This paper presents an analytical method for determining the influence that the connection of single generator and its step-up transformer exerts on network fault performance. The impact is quantified by considering the fault currents, fault voltage disturbances and generator fault in-feeds that will be produced in the modified network. Sensitivity factors are derived, which can be used for determining points in the network where there exists the greatest potential for change in network behaviour upon the addition of a new generator.

Although the method was developed specifically for comparison of conventional and directly connected generators [3], the technique derived in this paper is focused primarily on conventional generator configurations

## III. Network Modelling

In this investigation system fault behaviour was characterised using quasi-steady state fault analysis techniques as described in [4], allowing the network to be modelled in a linear manner, and described by the admittance or impedance matrix. In either case however it can be difficult to determine the impact of a single component on network performance.

### A. *Impedance Matrix Representation*

When performing fault analysis it is often convenient to model the network using the impedance matrix representation as once constructed fault currents and all subsequent voltage disturbances caused by a fault at any point in the network can be obtained directly. The main drawback of this technique is the difficulty of constructing the impedance matrix. It must be determined either from the inverse of the admittance matrix as described in [5] – a technique that can be computationally expensive and prone to matrix singularities – or by using algorithms in which network elements are added sequentially to the impedance matrix [6, 7]. Modification to network

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configuration is performed by either re-applying steps from the construction algorithm or utilizing techniques such as compensation currents. In either case the impact of individual network elements is usually lost during matrix construction.

An alternative technique uses LU decomposition methods to obtain the required elements of the impedance matrix from only the relevant components of the admittance matrix. Changes to network configuration are made on the admittance matrix and the new impedance matrix elements are calculated subsequently. When combined with sparsity methods and optimal ordering, this technique is generally much faster than using impedance matrix methods and utilizes less computer storage [8], however the impact of specific network elements still cannot be identified easily.

## IV. Network Modification

The limitation of the proceeding techniques is that several operations are usually required before fault parameters can be extracted after network configuration is modified. By recognizing that under balanced fault conditions the fault impedance of a new generator can be modelled as a single radial connection, with no mutual impedances, to an existing power system it is possible to develop a method for assessing the impact that the design of this new generator will have upon the fault behaviour of the modified power system.

The connection of a new generator and its unit transformer to an existing power system is shown in Fig. 1.

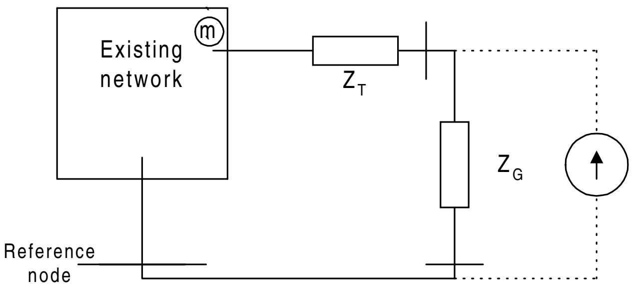


Fig. 1. Connection of generator and transformer to pre-existing network

The new generator is represented by its Norton's equivalent utilizing the sub-transient reactance of the generator. The transformer is represented by its short-circuit impedance.

The original or existing network can be modelled by an impedance matrix of the form shown in (1).

$$Z_{BUS} = \begin{bmatrix} Z_{11} & \cdots & Z_{1m} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{m1} & \cdots & Z_{mm} & \cdots & Z_{mN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{Nm} & \cdots & Z_{NN} \end{bmatrix} \quad (1)$$

The connection of the new generator and transformer to the network at bus $m$, however, is represented by applying the relevant step from the impedance matrix construction algorithm outlined in [6]. This process consists of first creating an augmented impedance matrix containing the connection of a new node to the original network. Gaussian elimination (Kron reduction) is then applied to produce the impedance matrix for the modified network. The general form of all elements in the new matrix is illustrated in (2).

$$\begin{aligned} Z_{kl,new} &= Z_{kl} - \frac{Z_{km}Z_{ml}}{(Z_G + Z_T) + Z_{mm}} \\ &= \frac{Z_{kl}(Z_G + Z_T) + Z_{kl}Z_{mm} - Z_{km}Z_{ml}}{(Z_G + Z_T) + Z_{mm}} \end{aligned} \quad (2)$$

A new impedance matrix has now been derived that describes the modified network in terms of elements of the impedance matrix of the original network and the impedance of the new generator and its unit transformer. Expressions illustrating the relationship between these parameters and fault behaviour throughout the modified network can be developed.

Finally (2) also illustrates that, apart from the Thevenin's impedance at the terminals of a new conventional generator, the modified network would be identical irrespective of whether the generator and transformer were modelled separately or if the generator was directly connected with a sub-transient reactance of ($Z_g + Z_t$).

## V. Impact of Generator Design on fault behaviour

### A. *Fault Currents*

Initially the impact of the fault impedance of the new generator/transformer on balanced fault currents produced throughout the network was considered. In this study only balanced three-phase faults were considered although further work is already being completed to adapt this technique to unbalanced fault analysis. Three phase faults are the simplest to analyse analytically and, as they often represent the most severe faults within the network, provide a good point for comparison of network fault current behaviour.

The three-phase fault current at bus $k$ in the modified network can be determined from (3) as:

$$\begin{aligned} I_k^{(f)} &= \frac{V_k(0)}{Z_{kk,new}} \\ &= V_k(0)\left(\frac{(Z_G + Z_T) + Z_{mm}}{(Z_G + Z_T)Z_{kk} + Z_{mm}Z_{kk} - Z_{km}Z_{mk}}\right) \\ &= \frac{V_k(0)}{Z_{kk}}\left(\frac{(Z_G + Z_T) + Z_{mm}}{(Z_G + Z_T) + \left(Z_{mm} - \frac{Z_{km}Z_{mk}}{Z_{kk}}\right)}\right) \end{aligned} \quad (3)$$

where $V_k(0)$ designates the pre-fault voltage at bus $k$.

Equation (3) illustrates clearly that while the fault current produced in the modified network will be controlled by the specific sub-transient or short circuit impedance of the new generator/transformer, the potential variation in fault behaviour that can be produced in the modified network will be constrained by the configuration of the original network at the point of connection of the new generator, i.e. bus *m*.

This is emphasized further by recognizing the correspondence of (3) with the PZ form of a transfer function as described in [9].

$$T(s) = K\frac{(s+z_1)(s+z_2)\cdots(s+z_n)}{(s+p_1)(s+p_2)\cdots(s+p_m)} \quad (4)$$

The potential variation in fault current that can be produced in the modified network, across a particular range of generator/transformer impedances, will then depend upon the placement of the break points of (3). These consist of a single complex zero at $Z_G + Z_T = -Z_{mm}$ and a single complex pole at $Z_G + Z_T = -\left(Z_{mm} - ((Z_{km}Z_{mk})/Z_{kk})\right)$. The position of these break points is controlled by the configuration of the original network and the point of connection of the new generator.

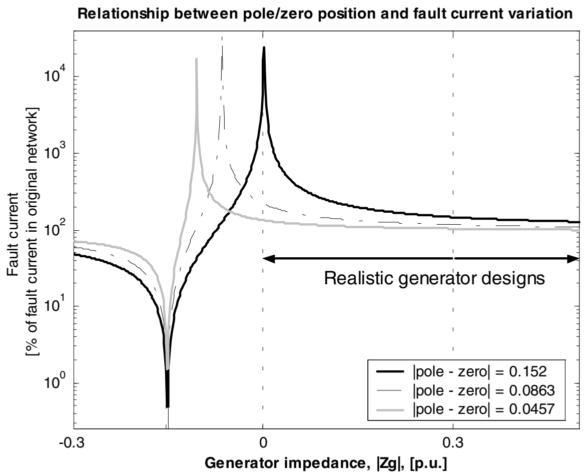


Fig. 2. Relationships between pole/zero separation and fault current variation

The degree to which the configuration of the existing network governs the potential variation in fault current produced at any point in the modified network, irrespective of the design of the new generator/transformer, is illustrated in Fig. 2. This diagram represents the variation in fault current that can be produced by changing the impedance of a single new radial branch for a common point of generator connection but three different fault locations. As can be seen, a greater distance between the relevant break points corresponds to a larger potential increase in fault current that can be produced for realistic generator designs in the modified network, when considering the proportional variation of this fault parameter with the respect to the corresponding performance of the original system.

The relationship between break point positions and the potential variation in fault current produced in the modified network can be formalized by defining the "current sensitivity factor", $\Delta_{IF}$, according to:

$$\Delta_{IF} = pole - zero$$
$$= \left(-\left(Z_{mm} - \frac{Z_{km}Z_{mk}}{Z_{kk}}\right)\right) - \left(-Z_{mm}\right) = \frac{Z_{km}Z_{mk}}{Z_{kk}} \quad (5)$$

The magnitude of the current sensitivity factor, $\Delta_{IF}$, will indicate the **comparative range** of fault currents that could be produced in the modified network if either generator or transformer impedance were allowed to vary across a large range of impedances. The specific fault current in the new network will still be dependent upon the exact values of generator/transformer impedance but the current sensitivity factor describes the degree with which these new elements will control the fault current produced at any given bus.

For a common point of connection of the new generator, the current sensitivity factor can be used to identify quantitatively the fault locations at which the fault current produced is either extremely sensitive to generator design or else relatively impervious to modifications in the design of the new generator and thus comparatively unchanged from that in the original network.

The extent to which the generator/transformer impedances control fault current behaviour will also be dependent upon the absolute position of the calculated pole and zero. These values provide a good indication of the range across which network behaviour will be highly sensitive to the total impedance of the new radial connection. Unless these break points are within the range of realistic values of generator sub-transient reactance and transformer short-circuit impedances, fault currents will be controlled by network parameters rather than the impedance of the new branch. Provided however that the network break points are within the range of realistic parameters then the current sensitivity factor should provide a good indication of the potential change in fault behaviour that can be produced in the modified network.

Finally, the current sensitivity factor, $\Delta_{IF}$, may be used to gain a preliminary understanding of the relationship between physical network configuration and fault behaviour. Equation (5) highlights that for a common point of generator connection, the impact of the new generator's design is likely to more pronounced at fault located close to the point of connection. Although this is a logical conclusion, the method outlined provides a way of verifying this behaviour numerically.

*B. Impact of Generator Impedance upon Voltage Disturbance*

The impact of generator/transformer impedance upon the voltage disturbances produced throughout the network by a given balanced fault can also be determined from knowledge of the configuration of the original network only. The voltage change produced at bus *l* in the modified network due to a balanced three-phase fault at bus *k* is given by:

$$\Delta V_l^{(k)} = -I_f^{(k)} Z_{lk,new}$$

$$= -\frac{V_k(0) Z_{lk}}{Z_{kk}} \left( \frac{(Z_G + Z_T) + \left( Z_{mm} - \frac{Z_{lm} Z_{mk}}{Z_{lk}} \right)}{(Z_G + Z_T) + \left( Z_{mm} - \frac{Z_{km} Z_{mk}}{Z_{kk}} \right)} \right) \quad (6)$$

If it is assumed that the potential influence of the connection of a new generator upon voltage disturbance can be indicated by the distance between the pole and zero in (6), then a "voltage sensitivity factor" can also be defined as:

$$\Delta_{dV} = pole - zero$$
$$= Z_{mk} \left( \frac{Z_{km}}{Z_{kk}} - \frac{Z_{lm}}{Z_{lk}} \right) \quad (7)$$

Similar relationships to those outlined for the current sensitivity factor would be expected between the calculated voltage sensitivity factor and network fault voltage disturbance. More importantly, (7) establishes a clear link between the potential impact of the impedance of the new generator/transformer on variation in system-wide voltage disturbance and the point of connection of the new generator as well as the configuration of the **original network** between the fault location and the point at which voltage disturbance is to be assessed.

### *C. Generator Contribution to Fault Current*

The contributions of the individual generator to fault currents will also vary as the impedance of the new generator/transformer is adjusted. The current contributed to a balanced three-phase short on bus $k$ by an original network generator located at bus $n$ is given by:

$$I_n^{(f)} = I_n + \frac{\Delta V_n^{(k)}}{Z_{g,n}} \quad (8)$$

where $I_n$ is the load current produced by the generator at bus $n$ in the modified network,
$Z_{g,n}$ is the sub-transient reactance of the generator at bus n.

As the fault in-feeds of the original generators are dependent upon the voltage disturbance produced at their terminal, the impact of the new branch on generator fault in-feeds can be determined from the relevant voltage sensitivity factors.

The variations in the contribution of the newly added generator at bus $m$ however will have the form shown in (9)

$$\frac{\Delta V_m^{(k)}}{Z_G + Z_T} = -\frac{V_k(0) Z_{mk}}{Z_{kk}} \left( \frac{1}{(Z_G + Z_T) + \left( Z_{mm} - \frac{Z_{km} Z_{mk}}{Z_{kk}} \right)} \right) \quad (9)$$

The sensitivity of this parameter to generator/transformer impedance will depend upon the proximity of the single pole at $Z_G + Z_T = -\left(Z_{mm} - ((Z_{km}Z_{mk})/Z_{kk})\right)$ to the origin of the complex impedance plane. For a fixed point of generator connection, the potential variation in the fault in-feed of the new generator will then depend upon $(Z_{km}Z_{mk})/Z_k$, i.e. the relevant current sensitivity factor.

## VI. Experimental Verification

The relationships developed were verified by considering the fault performance of the 17-bus network shown in Fig. 3., simulated with the PowerWorld Simulator[1]. This network is based on the 14-bus test system of [10], with some modifications made to enhance system realism. The new synchronous generator and transformer parameters were taken from [11] and [12] respectively and are listed in Table I. The techniques outlined previously were used to assess the impact on all relevant fault parameters when the illustrated generator at bus 15 was reconnected.

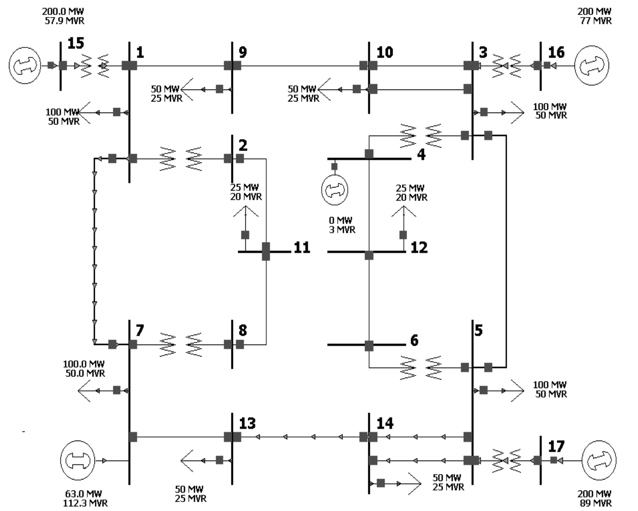


Fig. 3. Modified 14 Bus Test System [11].

A form of secondary voltage control was implemented using PowerWorld. The voltage at the HV side of each unit transformer was fixed and the generator set points were adjusted accordingly ensuring that the variation in fault performance reflected only changes in network impedance.

TABLE I
MACHINE PARAMETERS (BASED ON 100MVA)

| Generator Bus | Sub-transient Reactance [p.u.] | Transformer Impedance [p.u] |
|---|---|---|
| 15 | Variable | 0.05 |
| 16 | 0.0019 + j0.117 | 0.05 |
| 17 | 0.00091 + j0.106 | 0.05 |
| 7 | 0.00069 + j0.105 | 0 |

[1] PowerWorld Simulator, 7.0 ed. PowerWorld Corporation, 2004 South Wright Street Suite #102 Urbana, IL 61801 2001.

The relevant impedance matrix was constructed including network loads represented as constant impedances. Fault parameters were calculated as the generator reactance was varied continuously from 0.05 to 0.5 p.u. (based on 100 MVA) representing the extremes of realistic machine parameters, while the resistance of new generator was fixed at 0.00069 p.u. Faults were simulated on each of the 17 network buses and also at six in-line fault positions, three of which were placed equidistantly along lines 1-7 and 3-5. These fault locations were labelled buses 18-20 or 21-23 respectively.

### *A. Fault Current Variation*

The following diagram details the comparison of the current sensitivity factor, $\Delta_{IF}$, to fault current variations that occur as the reactance of the generator connected to bus 15 was varied from 0.5 – 0.05 p.u.

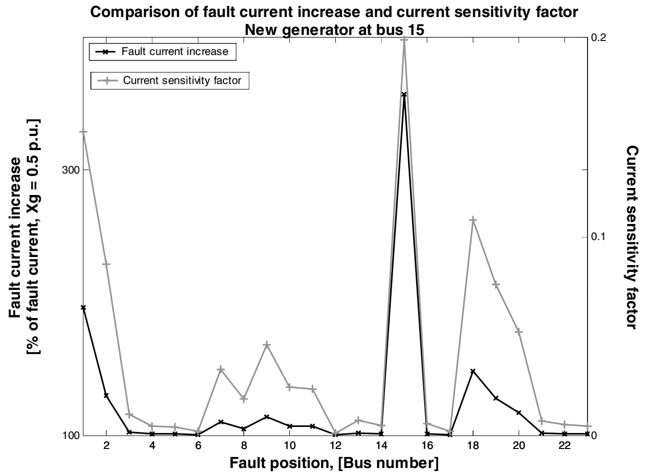


Fig. 4. Comparison of fault current variation to current sensitivity factor

The results in Fig. 4. show good correspondence between the calculated current sensitivity factor and the measured **proportional** change in fault current for the variation in generator parameters considered.

The results also demonstrate that the greater potential impact of generator parameters on fault behaviour for fault located close to the terminal of the generator of interest, i.e. buses 15, 1, 18, 9, while also highlighting that at many points system fault behaviour is relatively insensitive to variations in bus 15 generator design. Importantly, the results allow this classification to be made numerically, rather than qualitatively.

### *B. Voltage Disturbance Variation*

Although the correlation between network voltage change and the voltage sensitivity factor was considered at each bus in the network for a balanced three phase fault at every bus in the system, only the results for a fault at bus 2 are presented as they are indicative of the general behaviour.

As can be seen in Fig. 5., the results also show good consistency between the network sensitivity factor $\Delta_{dV}$ and the actual voltage disturbances measured using the simulator and presented as the proportional increase in voltage disturbance produced at each point in the network by a fault at bus 2 as generator reactance is increased from 0.05 – 0.5 p.u.

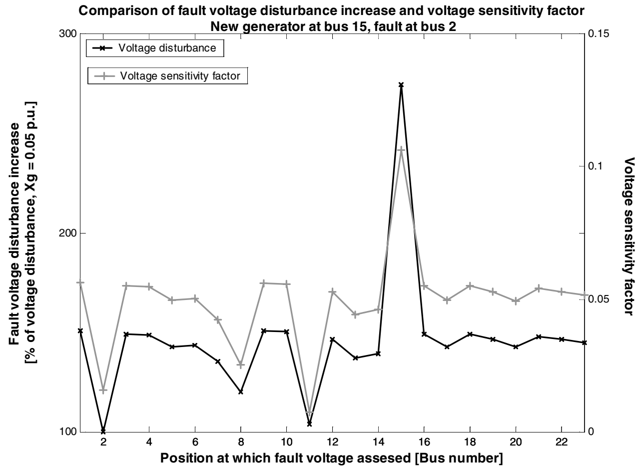


Fig. 5. Comparison of fault voltage disturbance variation and voltage sensitivity factor

The numerical values of sensitivity factor obtained however are valid only for the single generator position - fault location considered. A normalization process, which will be presented in future work, is required for direct numerical comparison of different generator positions - fault location combinations.

### *C. Generator Contribution*

Finally, Fig. 6. – 7. illustrate the correspondence between the fault in-feeds of an existing generator and the new generator to the relevant calculated voltage and current sensitivity factors as the fault location is varied.

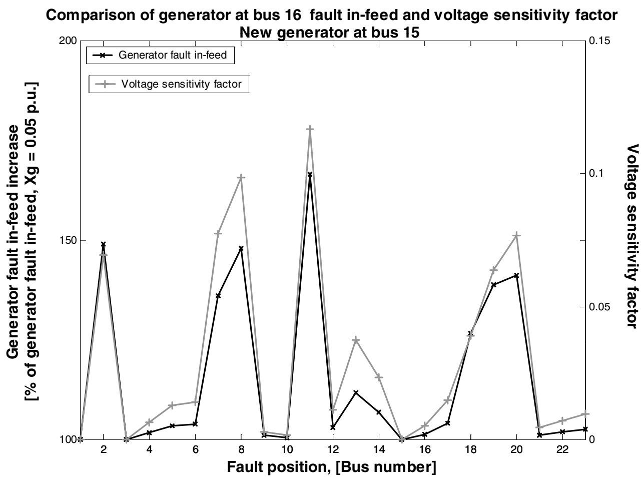


Fig. 6. Comparison of generator fault in-feed and voltage sensitivity factor

The correlation between the proportional variation in fault in-feed of the generator at bus 16 measured as the reactance of the new generator is allowed to vary between 0.05 and 0.5 p.u. and the voltage sensitivity factor calculated at the generator terminals is clearly visible in Fig. 6, suggesting that the sensitivity factor alone could be used to identify point at the generator exert significant control on fault behaviour.

Finally, Fig. 7. highlights the effectiveness of the current sensitivity factor for predicting the potential impact that the design of the new generator will have upon its fault in-feeds for different fault locations throughout the network. It would appear that the current sensitivity factor is quite responsive highlighting changes in generator contribution that are not as easily identified on the scale used.

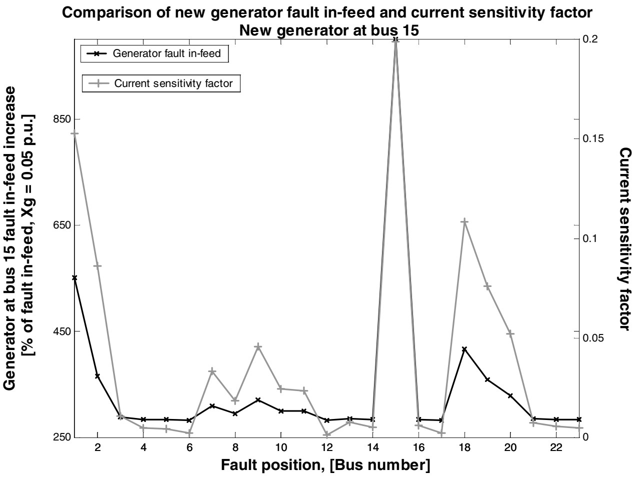


Fig. 7. Comparison of new generator fault in-feed and current sensitivity factor

## VII. Conclusions

This paper has presented a sensitivity technique for determining the impact of a new generator and its unit transformer upon the fault performance of an existing power system. It was demonstrated that the variations in fault current, network voltage disturbances and the contributions of the respective generators in the network are dependent upon both the impedance of the newly connected generator and the configuration of the pre-existing network. The influence that new or even existing generator exerts on network fault behaviour can be quantified through calculation of the sensitivity parameters $\Delta_{IF}$ and $\Delta_{dV}$, allowing a quantitative comparison between proposed generator augmentations to an existing system.

Future work will attempt to assess the robustness of the method by applying it to a larger, realistic power system, as it is possible that this technique may be somewhat reliant upon the accuracy of estimates of network components and the precision to which the calculations can be completed. It is also hoped that the relationship between the magnitude of the sensitivity factors and the variation in actual network parameters can be determined. This would allow the assessments of network variations to be made on a more general scale, eg considering only large variations in the sensitivity factors, making the method more suitable for industrial applications.

Finally, as highlighted above, work is also being completed to extend this technique to consider unbalanced fault conditions and also the impact of multiple generator additions.

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## IX. Biographies

**John McDonald** (M'2001) was born in Brisbane, Queensland, Australia, on October 21, 1977. He obtained at BE (Hons – Elec)/BA (Chinese) from the University of Queensland in 1999 and at present he is undertaking his PhD investigation at the University of Queensland entitled "Investigations into the design of Powerformer$^{TM}$ for optimal generator and system performance under fault conditions." His fields of interest include power systems analysis; system fault performance and equipment condition monitoring..

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