LOSS ALLOCATION BASED ON NETWORK REDUCTION IN DEREGULATED ELECTRICITY MARKET

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Abstract

Loss allocation of electricity has been in the limelight since the introduction of deregulation in the electricity market. For a fair and transparent market, method to allocate appropriate loss to any customer is necessary. This paper proposes an approximated loss allocation method based on network reduction. Reduction of the network is dependent upon bilateral contract between any one generator (seller) and one load (customer/buyer). Gaussian elimination is applied to reduce the bus admittance matrix of the network. The reduced network will contain only buses of primary interest – that is of the analysed contract. IEEE 14 bus test system is used to illustrate the proposed methodology. The results from this analysis are presented in this paper.

1. INTRODUCTION

Deregulation in recent years has decomposed the traditional vertically integrated system into separate generation, transmission and distribution entities. One of the main objectives is to ensure competition. Thus the end result would mean reduction of customer electricity prices.

Present pricing methods are based on either estimation of long term averages or DC flow marginal cost approximations. Deregulation of the electricity markets has called for fairer and transparent methods to be introduced. This can be achieved if all market participants satisfactorily shares costs involved in utilising the network. It is necessarily to take account of issues such as generation costs, geographic distance and transmission losses.

Transmission power losses correspond to a considerable amount of total system costs. Under past vertical system, it is acceptable to allocate approximately 4% to 10% of generated power as power loss. These losses were viewed as extra load in the system.

Present competitive market requires that these losses to be shared in a nondiscriminatory manner. An important issue to note is that electricity is an indistinguishable entity. It is almost impossible to allocate losses accurately. Hence, choosing the most appropriate loss allocation method is dependent upon a trade off between accuracy, computational time and fairness.

Available methods include *pro rata*, proportional sharing, bilateral contract, incremental loss, circuit theory, and loss formula [1]. Preliminary investigations on several of these methods have been outlined in a previous work [2].

Pro rata method is based on proportional allocation, where losses are dependent on the active generation or load of each market participant. It does not take account of the geographic location of network. This technique is used in England, Spain and Brazil [1].

Proportional sharing method is introduced to trace the geographical flow of electricity. It assumes that power at nodal inflows is shared proportionally between nodal outflows. That is, network node works as a perfect 'mixer' of inflow and outflow. This method has neither been proved nor disproved [3].

Looking from a different angle, methods based on bilateral contracts take into account of actual transactions taking place in the market. Electricity is sold by generators to customers based on agreed contracts [4].

Incremental loss method assigns losses to generator and/or loads through incremental transmission loss coefficients. It looks at the effect of a slight change in losses in relation to bus injections. This method is used in Australia, based on DC approximation [5]. It is outlined in the next section of this paper.

More recent methods are based on circuit theory and loss formula. Circuit theory method focuses on impedance matrix of the network. Flow distribution is then determined. Loss formula method expresses the system losses based on loss coefficients [6].

Next section of this paper looks at nodal pricing in Australia and New Zealand. A new method for loss allocation based on network reduction is then introduced. It is based on the bilateral contracts concept. The method will be elaborated and results tested on the IEEE 14 bus test system will be discussed [7].

2. NODAL PRICING

2.1 Australia [8]

National Electricity Market (NEM) in Australia commenced its operation on 13 December 1998 as part of deregulation process in Australia. In the NEM, a set of nodal (locational) prices for an electricity network is computed simultaneously. The computed locational prices represent marginal cost of supplying a very small increment in demand at each location. The prices also account for influential factors such as costs of producing electricity, transmission loss and capacity limitations.

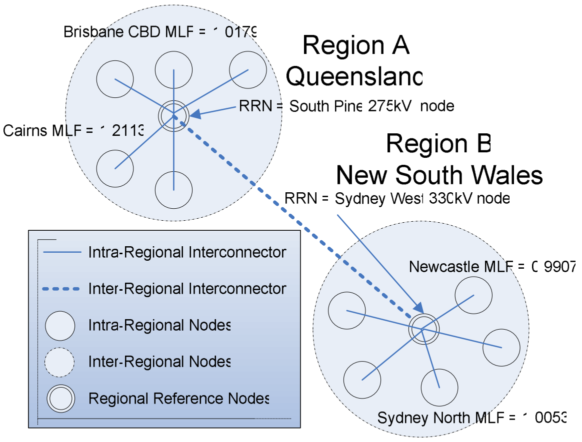

Figure 1. Nodal pricing concept

An approximation to full nodal pricing is employed. This minimises computation time; where full nodal pricing is calculated for inter-regions and within each region, the pricing is based on static marginal loss factors (MLF). The division of locational pricing is as shown in figure 1.

Intra-regional pricing: Pricing between a regional reference node (RRN) and a transmission network connection point within a network.

Previously, it is based on historical network flow data from previous 12 months for each load and generation bus relative to the RRN analysed. MLF are then calculated and averaged to arrive to a single weighted average MLF for each load and generation connection point [9]. It is calculated from equation (1).

$$MLF = 1 + \frac{\partial loss}{\partial load.increment} \qquad (1)$$

$$MLF \ at \ RRN = 1$$

Limitation of this method is that there is a two year delay between a change in load or generation data and the relevant effect on transmission loss factors.

As of 1 January 2004, a forward looking loss factor methodology is employed throughout the NEM. This method is based on the principle of "minimal extrapolation" [10]. This method takes into account of the effect of load growth. Changes in load impact transmission flows and the dispatch of generation. The static MLF for node "i" in region "j" is then defined as:

$$\mathrm{MLF} = \frac{\sum_k \left(d_i^k \left(\frac{\alpha_i^k}{\alpha_{rj}^k} \right) \right)}{\sum_k d_i^k} \qquad (2)$$

where,

α_i^k is the MLF of node "i" with respect to swing bus for trading period "k"

α_{rj}^k is the MLF of the reference node for region "j" with respect to the swing bus for trading period "k"

d_i^k is the demand for node "i" for trading period "k".

Loss factors are updated annually. Some of the loss factors for 2004/05 financial year for Queensland (Qld) and New South Wales (NSW) are included in figure 1.

Inter-regional pricing: Pricing between two RRNs. This pricing is necessary to accommodate for the large and variable flows between RRNs. It is obtained by applying linear regression to the set of hourly MLFs, based on DC approximations. These equations are also changed based on forward-looking concept. Testings have shown that there are no significant changes between the new and predecessor method [10]. Example of loss factor equation and losses are shown in equations (3) and (4) respectively.

Loss factor equation (South Pine referred to Sydney West) = $(1.0156+2.1819^{-04})(NQ_t) - (3.3981^{-06})(N_d) + (9.3783^{-06})(Q_d)$ (3)

Since Losses = ∫ (Loss factor – 1) dFlow

South Pine referred to Sydney notional link average losses = $(0.0156 - 3.3981^{-06} * N_d + 9.3783^{-06} * Q_d) * NQ_t + 1.0910^{-04} * NQ_t$ (4)

where, Q_d = Qld sent out demand
N_d = NSW sent out demand
NQ_t = transfer from Qld to NSW

Nodal spot price at a particular location within a region is then calculated by multiplying spot price at the RRN by appropriate MLF [8].

2.2 New Zealand

New Zealand Electricity Market (NZEM) has employed full nodal pricing (FNP) since its commencement in October 1996. Unlike NEM, determination of nodal pricing is based on actual power flow for every node in the transmission network. Marginal cost of meeting a change in load or generation at each node within the network is calculated separately. It incorporates the effects of power losses, line constraints and price of reserve. Equations (5) to (7) summarise the calculation of losses in the NZEM [11].

$$P_{t,i} = PG_{t,i} - PD_{t,i} - Plss_{t,i} - Flss_{t,i} \qquad (5)$$

where

$PG_{t,i}$ = MW generation at bus i at time t

$PD_{t,i}$ = MW demand at bus i at time t

$Plss_{t,i}*$ = Equivalent branch losses at bus i at time t

$$= \sum_{t(l)=i} BLoss_{tl}^{1} - \sum_{f(l)=i} BLoss_{tl}^{2} \qquad (6)$$

$Flss_{t,i}$ = Equivalent fixed losses at bus i at time t

Through piecewise linear approximation, branch losses are determined from:

$$BLoss_{tl} = \sum_{bs=1} BFlow_{tl}(bs) \text{ x } LF_{l}(bs) \qquad (7)$$

$BFlow_{tl}(bs)$ = Segment bs flow of branch flow component

$LF_{l}(bs)$ = Loss factor of segment bs of branch l

After five years of operation, a review on the outcomes of nodal pricing was conducted. Approach of the review is based on statistical analysis of spot price outcomes, and analysis of other factors such as market participants' comments and key factors that have contributed to the outcomes.

It was concluded that no strong evidence of significant benefits were identified related to the accurate signalling of full nodal pricing. It was found that there is an evident increase in price separation between nodes and day-to-day price variations at individual nodes.

Other conclusions from the review are:

- The review has suggested that marginal loss pricing results in a more efficient dispatch than average loss pricing.
- FNP (as well as a number of factors unrelated to FNP) contributes to the lack of contract market liquidity.
- Looking at a different perspective, if the pricing regime is based on fewer nodes (zonal model), it offers benefits from risk management and retail competition. Downfall of this model is that investment incentives provided by FNP model will be distorted [12].

3. PROPOSED METHODOLOGY

3.1 Network Reduction

The proposed network reduction method is used to allocate transmission losses to generators (sellers) and loads (buyers). The underlying concept is based on bilateral contracts formed between sellers and buyers. It is assumed that the full load is supplied by only one generator.

Bilateral contracts are usually long-term agreements between the two parties. The agreed price is based on market forces as well as transmission losses associated to each proposed contract. Understanding the impact of losses on bilateral contracts is important. It allows both parties to incorporate losses into their negotiations as well as optimise costs.

This method views the network as two parts; internal and external network, as shown in figure 2. The path between the contract generator and load forms the internal network. The external network represents the remaining network. Several existing reduction technique are available. They include Ward reduction, REI (derived from words: "Radial", "Equivalent" and "Independent"), and Linearisation [13].

For this proposed method, Gaussian elimination is chosen for its simplicity and efficiency of computation. The external network is reduced to an equivalent admittance in figure 2. Objective of reducing the external network is to focus on the network which is of primary interest – the contract path network. Allocation of loss of the contract buses is then determined from the reduced network.

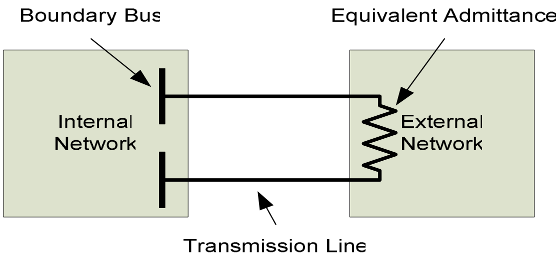

Figure 2. Network equivalence

3.2 Mathematical Formulation

Calculation for the system is based on load flow solution. A contract between generator a and load b

in a network with *n* buses is used for explanation. From load flow, the network can be represented by nodal equation (8). Y is the admittance bus matrix, V is the voltage bus vector and I is the injected current vector, **Y V = I** (8)

Gaussian elimination is then employed to successively remove one bus-voltage variable from the system at one time. All the external buses are eliminated leaving only the two contract buses, *a* and *b*.

Triangular factorisation is applied to ease computation for large-scale power system [14],
where **L U = Y** (9)

$$\mathbf{L} = \begin{bmatrix} Y_{11} & . & . & . & . \\ Y_{21} & Y_{22}^{(1)} & . & . & . \\ Y_{31} & Y_{32}^{(1)} & Y_{33}^{(2)} & . & . \\ \vdots & \vdots & \vdots & \ddots & . \\ Y_{n1} & Y_{n2}^{(1)} & \cdots & \cdots & Y_{nn}^{(a)} \end{bmatrix} \qquad (10)$$

$$\mathbf{U} = \begin{bmatrix} 1 & \frac{Y_{12}}{Y_{11}} & \frac{Y_{13}}{Y_{11}} & \cdots & \frac{Y_{1n}}{Y_{11}} \\ . & 1 & \frac{Y_{23}^{(1)}}{Y_{22}^{(1)}} & \cdots & \frac{Y_{2n}^{(1)}}{Y_{22}^{(1)}} \\ . & . & 1 & \cdots & \frac{Y_{3n}^{(2)}}{Y_{33}^{(2)}} \\ . & . & . & \ddots & \vdots \\ . & . & . & . & 1 \end{bmatrix} \qquad (11)$$

Substitute (2) into (1) gives
L U V = I (12)

Let
U V = V’ (13)
In expanded form:

$$\begin{bmatrix} 1 & \frac{Y_{12}}{Y_{11}} & \frac{Y_{13}}{Y_{11}} & \cdots & \frac{Y_{1n}}{Y_{11}} \\ . & 1 & \frac{Y_{23}^{(1)}}{Y_{22}^{(1)}} & \cdots & \frac{Y_{2n}^{(1)}}{Y_{22}^{(1)}} \\ . & . & 1 & \cdots & \frac{Y_{3n}^{(2)}}{Y_{33}^{(2)}} \\ . & . & . & \ddots & \vdots \\ . & . & . & . & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ \vdots \\ V_n' \end{bmatrix}$$

Substituting (13) into (12), thus
L V’ = I (14)
In expanded form:

$$\begin{bmatrix} Y_{11} & . & . & . & . \\ Y_{21} & Y_{22}^{(1)} & . & . & . \\ Y_{31} & Y_{32}^{(1)} & Y_{33}^{(2)} & . & . \\ \vdots & \vdots & \vdots & \ddots & . \\ Y_{n1} & Y_{n2}^{(1)} & \cdots & \cdots & Y_{nn}^{(a)} \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ \vdots \\ V_n' \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix}$$

Current injection is given by equation (15). It is zero at buses which has no external load or generating source connected.

$$I_i = \sum \frac{V_i - V_j}{Z_{ij}} \quad \text{where } j \neq i, \qquad (15)$$

Admittance of reduced equivalent network can be found from equation (16).

$$Y_{jk}^{(a)} = Y_{jk} - \frac{Y_{ja} - Y_{ak}}{Y_{aa}} \quad \text{a, j, and k = 1 to } n \qquad (16)$$

The calculated voltage, V, is the same as load flow solution. Through ohm’s law, current is calculated from the reduced equivalent admittance and bus voltage of the contract buses, *a* and *b*.

Subsequently losses for the contract path with respect to the rest of the system are calculated as shown by equations (17) and (18).
$P_{loss} = |I|^2 * R$ (17)
$Q_{loss} = |I|^2 * X$ (18)
where R and X are the resistance and reactance of the equivalent admittance respectively.

Losses calculated for each of the contracts represents the losses associated to the contract buses. These losses take into account the losses of the rest of the system, as represented by the equivalent admittance. It does not provide the exact loss allocation for only the contracted generator and load. However, it provides an indicative measure of how loss allocation changes when different contracts are of interest. The advantage of this method is that it does not specifically trace the flow of electricity since electricity is indistinguishable.

4. TEST RESULTS AND DISCUSSIONS

The network reduction loss allocation method presented in this paper was tested on the IEEE 14 bus test system as shown in figure 3. There are two main generators that generate both real and reactive power, located on buses 1 and 2. Slack bus of the system is located on bus 1. Total generation of real and reactive power is 272.5MW and 105.4Mvar respectively. Total consumption is 259MW and 73.5Mvar. Hence losses within the system are 13.53MW and 31.87Mvar. Results obtained are in per unit (p.u.) for 100MVA base.

Several contracts, as listed in table 1, were investigated to verify the fairness of the proposed methodology. Each contract is independent of another and is based on a single operating point. Load flow has to be iterated again for different operating points.

The analysis assumed that for an isolated contract between one generator and one load, the generator will supply the full load. Therefore, it is not possible to linearly add each contract losses, and the losses are expected to be not equivalent to load flow solution.

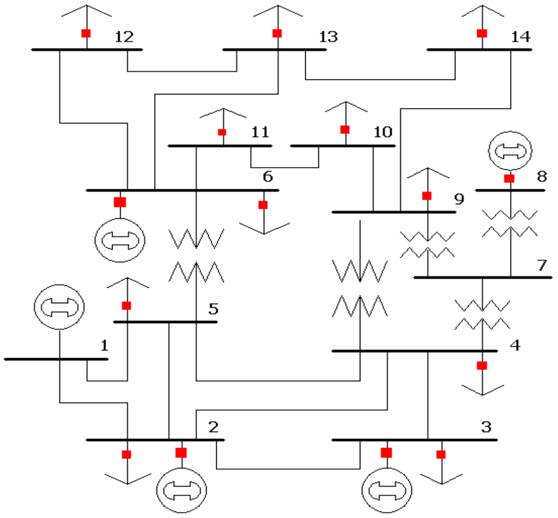

Figure 3. IEEE 14 bus test system [7]

Contract no.	Load bus no.	Equivalent admittance (p.u.)	
		Generator bus 1 (G1)	Generator bus 2 (G2)
1	2	5.8059 -18.3163i	-
2	3	1.8327 - 6.1208i	2.4225 - 8.2879i
3	4	2.8940 - 9.6024i	4.3327 - 13.7946i
4	5	3.0921 - 10.5917i	4.3849 -14.2126i
5	6	0.6052 - 3.8876i	0.6343 - 4.3315i
6	9	0.6447 - 4.1523i	0.6558 - 4.7265i
7	10	0.7032 - 3.4259i	0.7529 - 3.8044i
8	11	0.7321 - 3.0044i	0.7947 - 3.2791i
9	12	0.7935 - 2.3622i	0.8636 - 2.5189i
10	13	0.7544 - 3.0021i	0.8224 - 3.2670i
11	14	0.7364 - 2.5176i	0.799 - 2.7124i

Table 1. Equivalent admittances for each contract, 100 MVA base

Two main generators analysed were generators on bus 1 (G1) and bus 2 (G2). Contracts are formed between each of the generators with various loads in the system. The network reduction method presented was implemented to find the real and reactive power losses allocation for each independent contract.

Analysis of each contract starts from base-case load flow solution. The 14 bus network was then reduced down to the two contract buses. Thus, it is expected that the voltage calculated through the reduced network is similar to the load flow voltages. Resultant equivalent admittances for each of the contracts are listed in table 1.

The results of loss allocation for each individual contract were graphed in figures 4 and 5. From the graphs the slack bus (bus 1), which was the main generator in the system, gets a greater share of loss allocation.

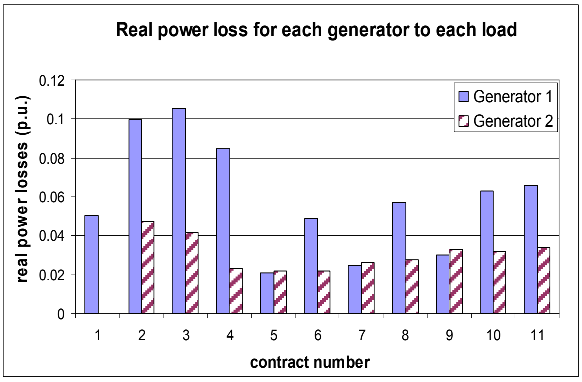

Figure 4. Real power losses, 100MVA base

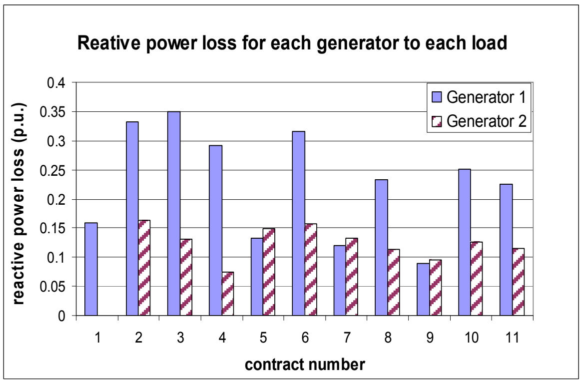

Figure 5. Reactive power losses, 100MVA base

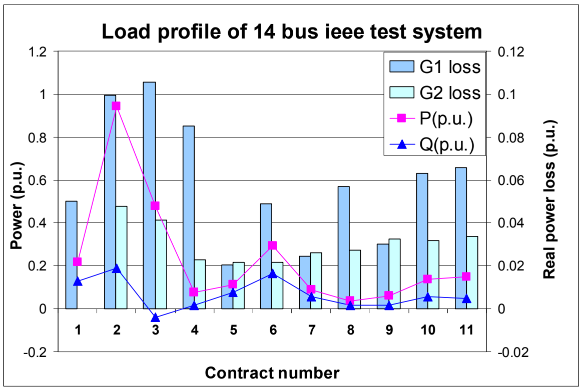

Figure 6. Load profile versus real power losses, 100 MVA base

Figure 6 graphs the load profile of each contract load buses and the calculated real power losses. The losses calculated were comparable to the load power profile. For example, contract 1 and 2 for generator 1. As expected, losses for each independent contract were higher for heavier loading. The correlation would not be linear as losses were also dependent on geographical location of each bus.

The accountability of topology through the network reduction system is shown by the following comparison. Looking at contract 1 and 3 for generator 1, where the loads consume 0.4p.u. and 0.48p.u. real power respectively. Even though both loads consume comparable amount of real power, the losses calculated is double the amount due to geographical location.

The results have also shown that the network characteristics have been preserved in the reduction process. Comparison of each independent contract has shown that the network reduction method is able to provide reasonable indicative measures of the extent of contract loss. This information may assist in contract negotiations. For example from figure 6, a buyer from contract 5 would expect to pay less compared to a buyer from contract 3. However, the calculated loss does not represent associated to only the contract buses.

5. CONCLUSIONS

Introduction of deregulation has changed many operational aspects of the electricity industry, including loss allocation. Fair and transparent allocation of losses is important as it affects the pricing of electricity. Electricity is transacted either through the pool market and/or bilateral contracts. The new method proposed, based on network reduction, analyses the loss allocated to bilateral contracts. It was tested on the IEEE 14 bus test system.

The proposed method follows the convention where electricity is indistinguishable, thus not specifically tracing the flow through any transmission lines. Although this method is path independent, the geographical location of each bus is taken into account as highlighted in the discussions section.

The losses calculated for each case account for the impact of system losses, relative to the contract buses of interest. These losses are based on a single steady state operating point, and the analysis is limited to one transaction at a time. It is not justifiable to directly associate losses calculated to loss of only the contract buses. It serves more as an indicative measure for buyers and sellers to understand the impact of losses on each contract. This allows both parties to incorporate losses into their negotiations as well as optimise costs.

6. ACKNOWLEDGEMENTS

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