EFFICIENT EQUALIZERS FOR SINGLE AND MULTI-CARRIER ENVIRONMENTS WITH KNOWN SYMBOL PADDING

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ABSTRACT

The use of a cyclic prefix (CP) to mitigate inter-symbol interference is a technique commonly applied in both single and multi-carrier systems. Recently it has been suggested that the CP be replaced by a pre-defined sequence of known symbols. This technique, referred to as 'Known Symbol Padding' (KSP) inserts a short training sequence (TS) at the beginning of each transmission block for equalizer adaption. This allows for fast tracking of changes in the channel and simple synchronization.

In this paper we present purely deterministic equalizers for both single and multi-carrier environments with KSP. We show that, through better utilization of the CP overhead, these equalizers exhibit superior performance to those in a conventional CP system. Low-complexity implementations, particularly for the multi-carrier case are also given.

1. INTRODUCTION

The use of a cyclic prefix (CP) to mitigate inter-symbol interference (ISI) is a technique commonly applied in both single and multi-carrier systems. In the single carrier context the CP allows complete ISI cancelation using a finite length Frequency Domain Equalizer (FDE) whilst in Discrete Multi-tone (DMT), a multi-carrier context, the CP allows the division of a single, frequency selective channel into a number of independent, flat sub-channels[1]. Equalizers in CP systems are typically trained using a long training sequence (TS) before data transmission.

Recently it has been suggested that the CP be replaced by a pre-defined sequence of known symbols[2]. This technique, referred to as 'Known Symbol Padding' (KSP) inserts a short TS at the beginning of each transmission block to maintain the cyclic (ISI mitigating) structure of the data and allow equalizer adaptation. This allows for fast tracking of changes in the channel, which is extremely important in rapidly time-varying environments such as HiperLAN[3]. KSP has also been shown to allow for simple synchronization[2].

Semiblind equalization is typically proposed for KSP systems[4]. By exploiting both statistical constraints on the transmitted data and the TS itself, semiblind techniques achieve good performance however they typically have complex implementations and require a large number of blocks before convergence. This latter point is of particular concern since it affects the semiblind equalizer's tracking ability in a rapidly time-varying environment.

In this paper we present purely deterministic equalizers for both single and multi-carrier (SC and MC) environments with KSP. We show that these equalizers exhibit superior performance to those based on a long-training sequence in a SC-CP system. Furthermore, low-complexity implementations, particularly for the MC case, are given.

2. SINGLE CARRIER KSP

2.1. Channel Model

A linear, convolutive channel with additive noise can be modeled as

$$\mathbf{y}^{(i)} = \mathbf{H}\begin{bmatrix} \mathbf{x}_{prev} \\ \mathbf{x}^{(i)} \end{bmatrix} + \mathbf{n}^{(i)}$$

where $\mathbf{y}^{(i)}$, $\mathbf{x}^{(i)}$ and $\mathbf{n}^{(i)}$ are respectively the i-th received block, i-th transmitted block and a noise vector all of dimension $N \times 1$. The vector $\mathbf{x}_{prev} = \mathbf{x}^{(i-1)}(N-L+1:N)$ indicates the last L elements of the previously transmitted block. $\mathbf{H}$ is the $N \times (N+L)$ Toeplitz filtering (Sylvester) matrix constructed from the channel impulse response

$$\mathbf{h} = [\ \mathbf{h}(0) \quad \ldots \quad \mathbf{h}(L)\ ]$$

In the case where each block can be formed by the concatenation of an $M \times 1$ block of data symbols $\mathbf{s}^{(i)}$ and a $v \times 1$ training sequence $\mathbf{b}$ which is constant over all blocks

$$\mathbf{x}^{(i)} = \begin{bmatrix} \mathbf{s}^{(i)} \\ \mathbf{b} \end{bmatrix}$$

$N = M + v$ and we assume that $L \leq v$, the received block becomes a function of the transmitted block only

$$\begin{aligned} \mathbf{y}^{(i)} &= \mathbf{H}\begin{bmatrix} \mathbf{b} \\ \mathbf{s}^{(i)} \\ \mathbf{b} \end{bmatrix} + \mathbf{n}^{(i)} \\ &= \mathbf{H}_{circ}\begin{bmatrix} \mathbf{s}^{(i)} \\ \mathbf{b} \end{bmatrix} + \mathbf{n}^{(i)} \end{aligned}$$

where $\mathbf{H}_{circ}$ is the circulant Toeplitz matrix with first column $\widetilde{\mathbf{h}}$, the channel impulse response $\mathbf{h}$ zero padded to length N. Here we have assumed that $L = v$ but a similar result if obtained for any $L \leq v$.

Exploiting the circulant nature of $\mathbf{H}_{circ}$

$$\mathbf{y}^{(i)} = \mathcal{I}_N diag\left\{\mathcal{F}_N \widetilde{\mathbf{h}}\right\} \mathcal{F}_N \begin{bmatrix} \mathbf{s}^{(i)} \\ \mathbf{b} \end{bmatrix} + \mathbf{n}^{(i)} \quad (1)$$

where $\mathcal{F}_N$ and $\mathcal{I}_N$ represent N-point DFT and inverse DFT matrices respectively.

2.2. Equalization

The circulant structure of $\mathbf{H}_{circ}$ causes the channel to perform a circular convolution on the transmitted data $\mathbf{s}^{(i)}$. This operation can be completely reversed using a frequency domain equalizer (FDE) resulting in the cancelation of all ISI (in the noiseless case). In the following sections we derive a M-MSE equalizer for the channel which uses the training sequence $\mathbf{b}$ over several received blocks to adapt it's parameters.

We desire to find some set of N FDE parameters $\underline{\mathbf{w}}$ that satisfy

$$\begin{aligned} \mathbf{x}^{(i)} &= \mathcal{I}_N diag\left\{\underline{\mathbf{w}}\right\} \mathcal{F}_N \mathbf{y}^{(i)} \\ &= \mathcal{I}_N diag\left\{\mathcal{F}_N \mathbf{y}^{(i)}\right\} \underline{\mathbf{w}} \end{aligned} \quad (2)$$

Using $\underline{\mathbf{w}}$ and the first M rows of (2) we can find an estimate of the transmitted data

$$\widehat{\mathbf{s}^{(i)}} = \mathcal{I}_N(1:M,:)\, diag\left\{\underline{\mathbf{w}}\right\} \underline{\mathbf{y}}^{(i)} \quad (3)$$

which can be implemented with a complexity $o(Nlog(N))$.

2.3. Equalizer Training

We now describe the calculation of $\underline{\mathbf{w}}$ by examining the received TS over several blocks. Since $\mathbf{s}^{(i)}$ is unknown, we take the last v rows of equation (2) to yield

$$\mathcal{I}_N(M+1:N) diag\left\{\mathcal{F}_N \mathbf{y}^{(i)}\right\} \underline{\mathbf{w}} = \mathbf{b} \quad (4)$$

In the noiseless case both conditions are satisfied exactly with $\underline{\mathbf{w}} = \mathbf{1}_{N\times 1} \oslash (\mathcal{F}_N \widetilde{\mathbf{h}})$ where $\oslash$ represents component-wise division. When noise is present we wish to satisfy condition (4) as closely as possible (in a least squares sense). Applying condition (4) over k blocks results in

$$\mathbf{A}^{(k)} \underline{\mathbf{w}} \overset{\text{LS}}{=} \mathbf{B}^{(k)}$$

where

$$\mathbf{A}^{(k)} = \begin{bmatrix} \mathcal{I}_N(M+1:N,:)\, diag\left\{\underline{\mathbf{y}}^{(1)}\right\} \\ \vdots \\ \mathcal{I}_N(M+1:N,:)\, diag\left\{\underline{\mathbf{y}}^{(k)}\right\} \end{bmatrix} \quad (5)$$

$$\mathbf{B}^{(k)} = [\underbrace{\mathbf{b}^T \quad \ldots \quad \mathbf{b}^T}_{\text{k times}}]^T \quad (6)$$

$\underline{\mathbf{y}}^{(i)} = \mathcal{F}_N \mathbf{y}^{(i)}$ and the notation $\mathbf{Q}(n:m,:)$ represents rows n to m of matrix $\mathbf{Q}$. If $k \geq \frac{N}{v}$ we can solve this to find

$$\underline{\mathbf{w}} = ({\mathbf{A}^{(k)}}^H \mathbf{A}^{(k)})^{-1} {\mathbf{A}^{(k)}}^H \mathbf{B}^{(k)} \quad (7)$$

where $(.)^H$ denotes complex conjugation and transposition. Using $\underline{\mathbf{w}}$ and equation (3) we can now estimate the transmitted data. The complete Single Carrier KSP system is depicted in Figure 1.

2.4. Efficient Implementation

In this section we derive an efficient recursive implementation for the Single Carrier-KSP (SC-KSP) equalizer. Whilst explicit re-calculation of the equalizer co-efficients requires $o(vN^2)$ multiplications per received block using conventional recursive least squares (RLS), it is possible to track changes in the channel using just N^2 multiplications per received block as we shall show.

Using (5) and (6) we can show that

$${\mathbf{A}^{(k)}}^H \mathbf{B}^{(k)} = diag\left\{\Psi^{(k)}\right\}^H \mathcal{I}_N(M+1:N,:)^H \mathbf{b}$$

where $\Psi^{(k)} = \sum_{i=1}^{k} \lambda^{k-i} \underline{\mathbf{y}}^{(i)}$. Since

$$\Psi^{(k+1)} = \Psi^{(k)} + \lambda \underline{\mathbf{y}}^{(k+1)}$$

tracking Ψ requires only N additions. Here λ is a forgetting factor which equals 1 in equation (5) and can be set < 1 to track time varying channels. We can also show that

$$\begin{aligned} {\mathbf{A}^{(k)}}^H \mathbf{A}^{(k)} &= \left(\mathbf{\Omega}^{(k)}\right)^* \odot \\ &\quad \left(\mathcal{I}_N(M+1:N,:)^H \mathcal{I}_N(M+1:N,:)\right) \end{aligned}$$

where $\odot$ represents component-wise multiplication, $(.)^*$ denotes complex conjugation and

$$\mathbf{\Omega}^{(k)} = \sum_{i=1}^{k} \lambda^{k-i} \underline{\mathbf{y}}^{(i)} {\underline{\mathbf{y}}^{(i)}}^H$$

Since

$$\mathbf{\Omega}^{(k+1)} = \mathbf{\Omega}^{(k)} + \lambda \underline{\mathbf{y}}^{(k+1)} {\underline{\mathbf{y}}^{(k+1)}}^H$$

tracking $\mathbf{\Omega}$ requires only N^2 multiplications per received block. It can be shown that given Ψ and $\mathbf{\Omega}$, updating $\underline{\mathbf{w}}$ requires $\sim N^3$ multiplications. As a result, when implemented in a batch-update procedure (where we track $\mathbf{\Omega}$ and Ψ and only update $\underline{\mathbf{w}}$ after every k received blocks) the SC-KSP equalizer offers a low complexity technique for channel equalization.

2.5. Performance

The performance of the SC-KSP equalizer was evaluated against a Single Carrier-Cyclic Prefix (SC-CP) equalizer which uses a long training sequence for parameter initialization. $M = 48$ and $v = 16$ were used for both systems,

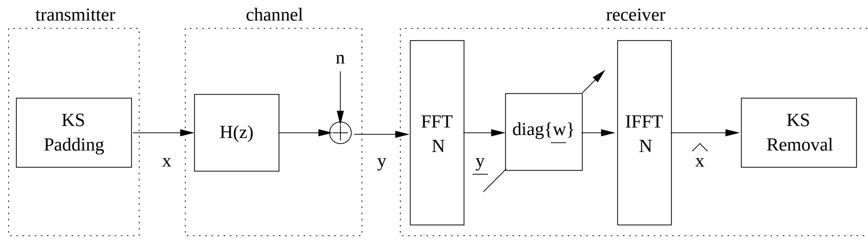

Figure 1: Single Carrier KSP System

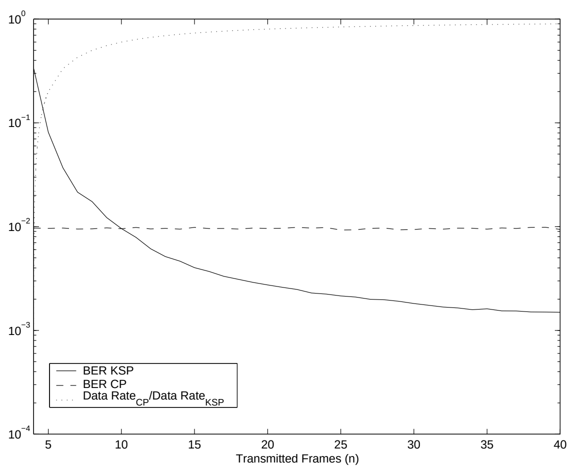

Figure 2: BER vs. Transmitted Frames (SNR=20 dB)

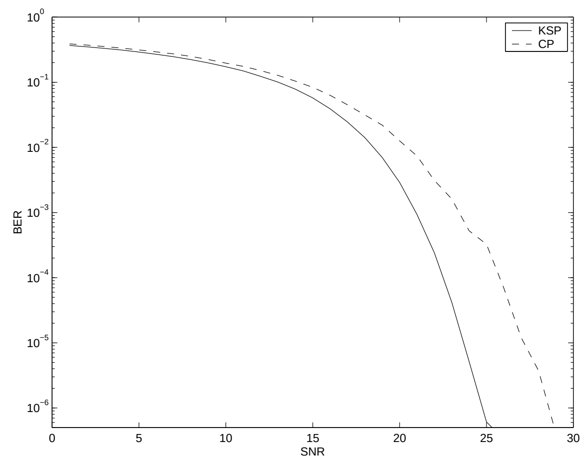

Figure 3: BER vs. SNR ($n = 30$)

and an impulse response $\mathbf{h}$ similar to those found in HiperLAN environments (with $L \le v$) was used. Noise was assumed to be circular AWGN and both systems used QPSK modulation.

The TS for the SC-KSP system was a sequence of random QPSK symbols of length v. The SC-CP system was initialized using $n_{CP} = 4$ blocks of random QPSK symbols (with CP) which were transmitted prior to any data. Both systems had equal average energy per block.

Assuming that the SC-CP equalizer must re-initialize every n_D blocks to track channel changes, the SC-CP system will achieve a data-rate only a fraction $\left(\frac{n_D}{n_D+n_{CP}}\right)$ of that of the SC-KSP system.

Plotted in Figure 2 is the BER of both systems vs. the number of transmitted blocks n for a SNR of 20 dB. Note that the SC-KSP system uses all received frames for adaption ($n = n_D$), whilst the SC-CP system only uses the first 4 ($n = n_D + n_{CP}$). As we would expect, the SC-KSP system exhibits superior performance when the number of received training symbols exceeds $M \times n_{CP}$ which corresponds to 12 blocks. Also depicted is the ratio of the data-rates of each system.

Plotted in Figure 3 are the BER vs. SNR curves of both systems with $n = 30$.

3. DISCRETE MULTI-TONE KSP

3.1. Efficient Implementation

Since Discrete Multi-tone (DMT) modulation encodes data as frequency domain symbols, it is possible to implement a KSP FDE in the domain of the transmitted symbols themselves. This leads to efficient receiver structures such as the one depicted in Figure 4. In this system every $\frac{N}{v}$-th tone is selected as a pilot-tone (PT). The idea is that non-PTs (data tones) will be loaded with data, and the PTs will be varied to force the last v time-samples of $\mathbf{x}_{KSP}$ (see figure 4) to match the TS. Since our choice of PTs form an orthogonal basis in the time-space of the TS, this should be possible for any particular choice of data. We will now derive an efficient implementation for this.

Let us denote the basis formed by the PTs as $\mathbb{B}$

$$\mathbb{B} = \frac{N}{\sqrt{v}} \mathcal{I}_N \left(:, 1 : \frac{N}{v} : 1 + \frac{N}{v}(v-1) \right)$$

Here the scaling factor $\frac{N}{\sqrt{v}}$ is chosen to ensure that the basis is normal in the time-space of the TS. Exploiting the peri-

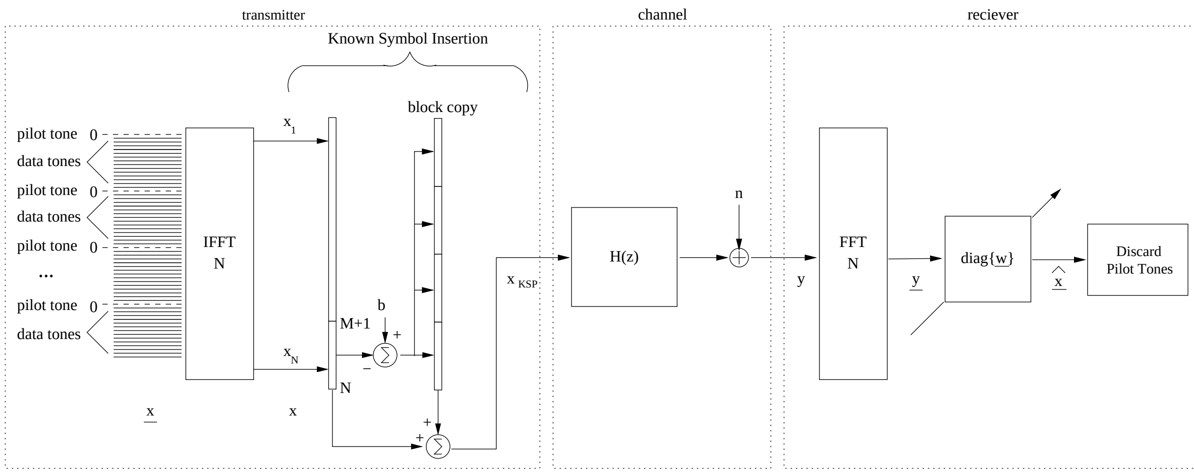

Figure 4: DMT KSP System

odic nature of every $\frac{N}{v}$-th column of $\mathcal{I}_N$

$$\mathbb{B} = \sqrt{v}\,[\ \mathbf{I}_v \quad \ldots \quad \mathbf{I}_v\ ]^T \mathcal{I}_v$$

where $\mathcal{I}_v$ represents the v-point inverse-DFT matrix and $\mathbf{I}_v$ represents the $v \times v$ identity matrix.

Let $\underline{\mathbf{x}}$ be a frequency domain vector with zeros on the PTs and data in the data-tones and let $\mathbf{x}$ be it's inverse DFT. Forcing the last v time-samples of $\mathbf{x}_{KSP}$ to match the TS through a projection on $\mathbb{B}$ yields

$$\begin{aligned}
\mathbf{x}^{(i)}_{KSP} &= \mathbf{x}^{(i)} - \mathbb{B}\mathbb{B}^H \begin{bmatrix} \mathbf{0}_{M\times 1} \\ \mathbf{x}^{(i)}(M+1:N) - \mathbf{b} \end{bmatrix} \\
&= \mathbf{x}^{(i)} - \begin{bmatrix} \mathbf{I}_v \\ \vdots \\ \mathbf{I}_v \end{bmatrix} \Big(\mathbf{x}^{(i)}(M+1:N) - \mathbf{b}\Big)
\end{aligned}$$

This operation can be implemented through a simple block copy as depicted in Figure 4.

At the receiver we simply DFT the received block, apply the FDE to the received frequency-domain symbols and discard the pilot tones to yield $\widehat{\mathbf{s}^{(i)}}$. All DFTs and IDFTs are of order N (which can be chosen a power of 2) and $\underline{\mathbf{w}}$ can be tracked and updated with N^2 and N^3 multiplications respectively using the algorithm described in section 2.4. This leads to an efficient implementation, particularly in the receiver.

3.2. Performance

It was found that the DMT implementation of KSP was highly sensitive to the choice of TS $\mathbf{b}$. Strategies for choosing $\mathbf{b}$ such that the performance of DMT-KSP exceeds that of DMT-CP are currently under investigation.

4. DISCUSSION

In this paper we presented deterministic equalizers for single and multi-carrier environments with Known Symbol Padding (KSP). KSP exploits the CP for both ISI mitigation and equalizer adaption. As a result we observed superior performance to conventional CP equalizers both in data transmission efficiency and BER.

The fact that each transmission block has a training sequence (TS) also suggests that we can track channel changes more rapidly with KSP than with a conventional CP equalizer. Low-complexity implementations for the equalizers were presented which can track channel changes with N^2 multiplications per block

In summary, the deterministic KSP equalizer offers a reduced complexity alternative to semi-blind equalization schemes such as [4], with a lower BER and higher data efficiency than conventional CP systems.

5. REFERENCES

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