Simplified Power Allocation for the DSL Multi-Access Channel through Column-wise Diagonal Dominance

Raphael Cendrillon and Marc Moonen*
SCD/ESAT, Katholieke Universiteit Leuven
raphael.cendrillon@esat.kuleuven.ac.be
marc.moonen@esat.kuleuven.ac.be

Radu Suciu
Alcatel Bell
Antwerp, Belgium
radu.suciu@alcatel.be

Abstract

In the newest generation of DSL systems crosstalk is the *dominant source of performance degradation. Many crosstalk cancellation schemes have been proposed. These schemes typically employ some form of co-ordination between modems and lead to large performance gains. The use of crosstalk cancellation means that power allocation should be viewed as a multi-user problem. In this paper we investigate optimal (ie. capacity maximizing) power allocation in DSL systems which employ co-ordination to facilitate crosstalk cancellation.*

By exploiting certain properties of the DSL channel it is shown that power allocation can be simplified considerably. The result has each user waterfilling against the background noise only, explicitly ignoring the interference from other users. We show this to be near-optimal for upstream DSL when Central Office (CO) modems are co-ordinated. Compared with conventional waterfilling which is done against the background noise and *interference, the performance gains are significant.*

1 Introduction

xDSL systems such as ADSL and VDSL offer the potential to bring truly broadband access to the mass-consumer market. The newer generations of xDSL such as VDSL aim at providing data rates up to 52 Mbps in the downstream, enabling a broad range of applications such as video-on-demand, video-conferencing and online education. In VDSL such high data-rates are supported by operating over short loop lengths and transmitting in frequencies up to 12 MHz.

Unfortunately, the use of such high frequency ranges can cause significant electromagnetic coupling between neighbouring twisted-pairs within a binder. This coupling creates interference, referred to as crosstalk, between the systems operating within a binder. Over short loop lengths crosstalk is typically 10-15 dB larger than the background noise and is *the* dominant source of performance degradation.

Many techniques have been proposed for crosstalk cancellation in DSL e.g. [1, 2]. In particular, if Discrete Multi-Tone (DMT) modulation is used, then synchronized transmission allows crosstalk to be canceled on a per-tone basis[1]. This leads to significant performance gains with a realisable complexity.

Another benefit of DMT is that it allows shaping of the transmit spectra, also known as waterfilling to be implemented in a straightforward manner. In highly non-flat channels, like those seen on the twisted-pair medium, waterfilling leads to significant data-rate gains. Waterfilling is traditionally viewed as a single user problem with each user allocating power according to the Channel Signal-to-Interference-plus-Noise-Ratio (C-SINR). That is, each user's transmit Power Spectral Density (PSD) is found by a waterfilling against the background noise and interference of other systems[3]. When crosstalk cancellation is employed however optimal power allocation requires us to examine the multi-user aspect of the DSL channel.

In this paper we describe optimal (ie. capacity maximizing) power allocations for the DSL Multi-Access Channel (MAC). The DSL-MAC is encountered in upstream transmission where receiving modems at the Central Office (CO) are co-located. This facilitates co-ordinated (ie. joint) reception and hence crosstalk cancellation.

As we will show, exploiting certain properties of the DSL channel allows us to significantly simplify the power allocation problem. The result is that each user waterfills against the background noise alone, explicitly ignoring crosstalk from other users.

This property has been noted previously where it was shown that waterfilling against the background noise alone is optimal for a particular receiver structure, namely the Zero Forcing-Decision Feedback Equalizer (ZF-DFE)[1]. Here we show that such a waterfilling scheme is optimal (to within a reasonable approximation for DSL channels) in an information theoretic sense. That is, it maximizes the capacity of the DSL-MAC when an optimal receiver structure is used.

2 The DSL Channel

2.1 DMT modulation

In this work we restrict our attention to DSL systems which employ Discrete Multi-Tone (DMT) modulation.

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This modulation scheme is currently adopted in ADSL as well as draft VDSL standards[4]. DMT is effectively a low-complexity implementation of frequency domain transmission. The main benefits of frequency domain transmission come from bitloading and powerloading:

Bitloading allows a DSL modem to dynamically vary the constellation used on a per-tone basis. The constellation employed depends on the SNR at the receiver. Through rate-adaption, the modem can keep the probability of error at a constant value. Furthermore, it can allocate large constellations to tones with high SNRs, ensuring efficient use of the channel.

Powerloading allows the modem to vary the power transmitted at each tone. Through this the modem can strike the optimal balance between transmitting on tones with the highest SNR and maximizing the transmission bandwidth. Due to the highly frequency selective nature of the DSL channel, powerloading yields significant benefit.

2.2 Crosstalk

Crosstalk is a significant problem in DSL and it's cancellation leads to large performance gains. In particular, so-called Far-End Crosstalk (FEXT) (ie. crosstalk from modems transmitting in the same direction) may be cancelled on a per-tone basis if the modems within a binder are synchronized[1]. This leads to dramatic improvements in performance with reasonable complexity. We thus adopt a channel model which describes crosstalk on a per-tone basis. Transmission of one DMT-block on tone k is modeled as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \tag{1}$$

In upstream communications the CO receivers are often co-located which facilitates co-ordinated (ie. joint) reception. In the upstream direction $\mathbf{x}_k$ is the set of QAM-symbols transmitted by each of the Customer Premises (CP) modems on tone k where $x_k^n \triangleq [\mathbf{x}_k]_n$ is the symbol transmitted by modem n. $\mathbf{y}_k$ is the set of received signals on each of the CO modems where $y_k^n \triangleq [\mathbf{y}_k]_n$ is the signal received on modem n. $\mathbf{H}_k$ is the channel matrix where $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from CP transmitter m into CO receiver n. Note that $h_k^{n,n}$ is the direct channel of user n. The transmit auto-correlation on tone k is $\mathbf{S}_k \triangleq \mathcal{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\}$ whose elements are defined $s_k^{n,m} \triangleq [\mathbf{S}_k]_{n,m}$. For convenience we also define $s_k^n \triangleq [\mathbf{S}_k]_{n,n}$

The receivers suffer from additive noise $\mathbf{z}_k$ from sources such as alien crosstalk, RFI and thermal noise. $z_k^n \triangleq [\mathbf{z}_k]_n$ is the noise seen at receiver n which we assume to be Gaussian. There are N users in the binder so $\mathbf{x}_k$, $\mathbf{y}_k$ and $\mathbf{z}_k$ are all vectors of length N, whilst $\mathbf{H}_k$ is a matrix of dimension $N \times N$.

In this paper we restrict our attention to the AWGN channel where $\mathcal{E}\left\{\mathbf{z}_k \mathbf{z}_k^H\right\} = \sigma_k^2 \mathbf{I}_N$ and $\mathbf{I}_N$ is the $N \times N$ identity matrix. Note that this is without loss of generality since in scenarios with crosstalk cancellation co-ordination is always possible between receivers. As such, any channel with a noise covariance matrix $\sigma_k^2 \mathbf{F}_k$ can be turned into an equivalent AWGN channel by application of a noise-whitening filter at the receiver $\mathbf{G}_k^{-H}$. $\mathbf{G}_k$ is related to $\mathbf{F}_k$ through the Cholesky decomposition, ie. $\mathbf{G}_k^H \mathbf{G}_k \overset{\text{chol}}{=} \mathbf{F}_k$.

One peculiar property of the DSL channel is that the channel from transmitter n to receiver n will always have a much larger magnitude than the channel from transmitter n to any other receiver. The difference is typically on the order of 15 dB. We refer to this property as *column-wise diagonal dominance* as in [1]. It ensures that a diagonal element of the channel matrix $\mathbf{H}_k$ will always be the largest element of it's column.

$$|h_k^{n,n}| \gg |h_k^{m,n}|, \ \forall m \neq n \tag{2}$$

This property will allow us to simplify power allocation considerably.

2.3 Power Constraints

The power constraint for DSL systems is on each transmitter (modem) rather than on the total power of all transmitters. Thus the constraints in power allocation are

$$\sum_{k=1}^{K} s_k^n \leq P_n, \ \forall n \tag{3}$$

where P_n is typically determined by the analog front end of modem n or by standardization/regulatory bodies. We also have the natural constraint

$$s_k^n \geq 0, \ \forall n, k \tag{4}$$

3 Conventional Power Allocation

In conventional DSL systems co-ordination is not possible between transmitters or receivers. The lack of co-ordination, and thus crosstalk cancellation is reflected in the power allocation strategies which are traditionally adopted. In the absence of crosstalk cancellation the DSL channel is a so-called *Interference Channel* from the Information theory perspective. Using a standard equalizer and slicer at the receiver the achievable rate of each user is

$$\begin{aligned} C_n &= \sum_{k=1}^{K} I(x_k^n; y_k^n) \\ &= \sum_{k=1}^{K} \log_2 \left(1 + \frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m \neq n} s_k^m |h_k^{n,m}|^2 + \sigma_k^2 [\mathbf{F}_k]_{n,n}}\right) \end{aligned}$$

where $I(a;b)$ is defined as the mutual information between a and b. Each user is detected in the presence of background noise $\sigma_k^2 [\mathbf{F}_k]_{n,n}$ and interference from other users $\sum_{m \neq n} s_k^m |h_k^{n,m}|^2$. The term $[\mathbf{F}_k]_{n,n}$ is present since the lack of receiver co-ordination prevents noise-whitening. Operating at the capacity of an interference channel corresponds to maximizing a weighted sum of the different users' rates. The weights used reflect the desired trade-off between the data-rates of the different users within the system. The optimal power allocation can found through an optimisation

$$\max_{\{\mathbf{S}_k\}_{k=1,\ldots,K}} \sum_{n=1}^{N} w_n C_n \tag{5}$$

Unfortunately this optimization is non-convex. Due to the high dimensionality of the solution space (e.g. in VDSL $K = 4096$) this problem is computationally intractable.

For this reason power allocation in conventional DSL systems has typically been based upon heuristic approaches.

The most common approach is for each user to allocate power independently, waterfilling against the background noise and the interference of the other users within the system[3]. Under this approach the power allocation for user n is defined as

$$s_k^n = \left[\frac{1}{\lambda_n} - \frac{\sum_{m \neq n} s_k^m \left| h_k^{n,m} \right|^2 + \sigma_k^2 \left[\mathbf{F}_k \right]_{n,n}}{\left| h_k^{n,n} \right|^2} \right]^+ \quad (6)$$

where the function $[x]^+ \triangleq \max(0, x)$. Here λ_n is chosen such that the total power constraint in (3) is met with equality. Here each user waterfills against the ratio of the noise plus interference term $\sum_{m \neq n} s_k^m \left| h_k^{n,m} \right|^2 + \sigma_k^2 \left[\mathbf{F}_k \right]_{n,n}$ to the channel gain $\left| h_k^{n,n} \right|^2$. Put another way, each user waterfills against the inverse channel-SINR.

A modified version of this approach was proposed in [5] where the total power constraint P_n of each user is varied based on their target data-rate. Waterfilling is done for each user in turn, and iterated across all users until convergence. The algorithm, referred to as *iterative waterfilling* is based on the proposition that with each user acting in a selfish way; attempting to maximize their own data-rate, the algorithm will converge to a point which is near-optimal from a global perspective, ie. one which maximizes (5).

Note that (6) which from now on will be referred to as *conventional waterfilling*, is based on the intrinsic assumption that crosstalk cancellation will *not* be used. Each user is encouraged to allocate power in the regions of the channel where interference is low. When crosstalk cancellation is used a different approach will be necessary.

4 Optimal Power Allocation for MACs

In this section we examine the case when co-ordination is possible between receivers at the CO. This corresponds to the upstream channel.

In information theory when co-ordination is available between receivers the channel is known as a Multi-Access Channel (MAC). We concern ourselves with maximizing the unweighted rate-sum of the system. In general finding all optimal operating points requires us to optimize a weighted rate-sum and this is the subject of ongoing research. We have however observed that in DSL channels where crosstalk cancellation is applied, varying the weights typically has little effect on the resultant data rates.

Provided an optimal receiver structure is used the achievable rate sum can be shown to be[6]

$$\begin{aligned} C &= \sum_{n=1}^{N} \sum_{k=1}^{K} I\left(x_k^n; \mathbf{y}_k \,|\, x_k^1, \ldots, x_k^{n-1}\right) \quad (7) \\ &= \sum_{k=1}^{K} I\left(\mathbf{x}_k; \mathbf{y}_k\right) \\ &= \sum_{k=1}^{K} \log_2 \left| \mathbf{I}_N + \sigma_k^{-2} \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H \right| \end{aligned}$$

where $I(a; b \,|\, c)$ is the mutual information between a and b conditioned on c. The goal is to maximize C as a function of $\{\mathbf{S}_k\}_{k=1\ldots K}$. This optimisation must be done under a total power constraint on each modem (3), plus the non-negativity constraint (4). Since co-ordination is not possible between transmitters we have an additional constraint

$$s_k^{n,m} = 0, \ \forall m \neq n \quad (8)$$

This problem was addressed in [6] where the optimal power allocation was shown to be a vector form of waterfilling which must occur simultaneously for all users within the system. The optimal power allocation is

$$s_k^n = \left[\frac{1}{\lambda_n} - \frac{1}{\left(\mathbf{h}_k^n\right)^H \left(\sum_{m \neq n} s_k^m \mathbf{h}_k^m \left(\mathbf{h}_k^m\right)^H + \sigma_k^2 \mathbf{I}_N \right)^{-1} \mathbf{h}_k^n} \right]^+ \quad (9)$$

where $\mathbf{h}_k^n \triangleq \left[\mathbf{H}_k\right]_{\text{column } n}$ and $\{\lambda_1 \ldots \lambda_N\}$ are chosen such that the power constraints in (3) are met with equality.

5 Simplified Power Allocation for DSL-MACs

No closed form solution is known for (9) although a cheap iterative algorithm has been proposed which has guaranteed convergence[7]. Whilst this algorithm allows us to find the optimum power allocation in an efficient way, we can exploit the properties of the DSL channel, specifically column-wise diagonal dominance (2) to simplify power allocation even further.

Under the condition of column-wise diagonal dominance and high SNR, the optimal power allocation is closely approximated by

$$s_k^n = \left[\frac{1}{\lambda_n} - \frac{\sigma_k^2}{\left| h_k^{n,n} \right|^2} \right]^+ \quad (10)$$

where $\{\lambda_1 \ldots \lambda_N\}$ are chosen such that the power constraints in (3) are met with equality.

Proof: See Appendix.

Using the power allocation strategy in (10) each user's PSD can be determined independently, considerably reducing complexity. In contrast to the conventional waterfilling of (6) each user waterfills against their own direct channel and the background noise as if interference were not present. In other words they waterfill against the inverse channel-SNR not the channel-SINR. This is intuitively satisfying since the high SNR and column-wise diagonal dominance of the DSL channel facilitate near-perfect crosstalk cancellation.

In contrast to (6), (10) allows power allocation to be done with much lower complexity since the power allocation problems of the different users are de-coupled.

6 Optimal Receiver Structure

With this power allocation, a low complexity DFE based receiver structure can be used to achieve the full capacity of the channel. Note that the conditioning of the mutual information in (7) on the previous user's symbols $x_k^1, \ldots, x_k^{n-1}$ reflects the successive interference cancellation nature of the optimal receiver structure. See [8] for more details.

7 Performance

We now compare the performance of conventional waterfilling (6) and simplified waterfilling (10) against the truly optimal power allocation scheme (9) for the upstream channel with co-ordinated reception.

Our simulation scenario uses VDSL modems with 4096 tones, the 998 FDD bandplan, ETSI alien noise model A, a coding gain of 3 dB, a noise margin of 6 dB and a total power constraint of 11.5 dBmW on each modem. The target error probability is $< 10^{-7}$ and all lines are 0.5 mm (24-Gauge). Empirical transfer functions are used, details can be found in [4]. Our scenario consists of 4 near-end and 4 far-end users located 300m. and 1200m. from the CO respectively.

Finding the power allocation for conventional waterfilling (6) was done using *iterative waterfilling* as described in [5] with all users set to full power. Each user waterfills against the interference of the other users in the system and the background noise. The process is repeated iteratively until convergence. This reflects what would actually occur in a real scenario as the users adapt their power allocations over time. Finding the power allocation using our simplified waterfilling scheme is done using a standard waterfilling algorithm applied independently to each user as described by (10). The optimal power allocation (9) was found efficiently using an iterative scheme[7].

The PSDs resulting from the different algorithms are shown in Fig. 1. Note that the PSDs of the *near-end* users are identical for all of the schemes. This occurs because the near-end users have high-SINR channels. The result is a flat transmit PSD since for any of the definitions of s_k^n in (6), (9) or (10)

$$\lim_{\mathrm{SINR}\to\infty} s_k^n = \frac{1}{\lambda_n}$$

We now turn our attention to the PSDs of the *far-end* users. First notice that the PSDs found using the optimal and the simplified waterfilling algorithms are virtually identical as predicted (both PSDs overlap in Fig. 1). This was the case for all scenarios we evaluated. Examining the PSD found with conventional waterfilling we see that the introduction of interference into the waterfilling equation in (6) results in a power allocation at lower frequencies. This is logical since crosstalk coupling increases with frequency. As such, the introduction of interference will tend to discourage loading at high frequencies and push the allocated far-end spectra towards DC.

To determine the performance of each of the schemes we used these power allocations along with the optimal receiver structure[8] and evaluated the achieved rates. The results are listed in Tab. 1. As can be seen, for far-end users conventional waterfilling gives less than 1/3 of the rate achieved using the optimal power allocation. Simplified waterfilling, on the other hand, leads to virtually identical performance to the optimal scheme. Note that in order to make a fair comparison crosstalk cancellation was used when evaluating the performance of *all* power allocation schemes including conventional waterfilling.

Scheme	Avg. Far-end Rate	Avg. Near-end Rate
Conv. W.f.	2.9 Mbps	59.6 Mbps
Simp. W.f.	10 Mbps	59.6 Mbps
Optimal	10 Mbps	59.6 Mbps

Table 1. Rates Achieved using Different Power Allocation Schemes

8 Conclusions

In this paper we investigated optimal power allocation for the DSL Multi-Access Channel. We showed that in the DSL environment the property of column-wise diagonal dominance simplifies the problem of power allocation considerably. The simplified power allocation scheme consists of a waterfilling against the background noise-only, explicitly ignoring crosstalk. This is intuitively satisfying since the property of column-wise diagonal dominance allows for near-perfect crosstalk cancellation.

Simulations show minimal performance degradation through the use of the simplified waterfilling scheme. Additionally we noted that power allocation using a conventional waterfilling algorithm (against interference and background noise) leads to poor performance when co-ordination is possible.

In this work we have considered co-ordination between receivers which corresponds to upstream transmission in DSL. An important extension of this work is to investigate simplified waterfilling schemes when co-ordination is available between transmitters only. This corresponds to the downstream direction of a DSL system where we suspect that the simplified waterfilling algorithm will also be near-optimal.

Appendix

We begin with the optimal power allocation for the MAC in (9). Define

$$\begin{aligned}\mathbf{Q}_k &\triangleq \sigma_k^2\mathbf{I}_N + \sum_{m\neq n} s_k^m \mathbf{h}_k^m \left(\mathbf{h}_k^m\right)^H \\ &= \sigma_k^2\mathbf{I}_N + \begin{bmatrix} \overline{\mathbf{h}}_k^1 \\ \vdots \\ \overline{\mathbf{h}}_k^N \end{bmatrix} \begin{bmatrix} \left(\overline{\mathbf{h}}_k^1\right)^H & \cdots & \left(\overline{\mathbf{h}}_k^N\right)^H \end{bmatrix}\end{aligned}$$

where

$$\begin{aligned}\overline{\mathbf{h}}_k^i \triangleq \Big[\; & h_k^{i,1}\sqrt{s_k^1} \quad \cdots \quad h_k^{i,n-1}\sqrt{s_k^{n-1}}, \\ & h_k^{i,n+1}\sqrt{s_k^{n+1}} \quad \cdots \quad h_k^{i,N}\sqrt{s_k^N} \;\Big]\end{aligned}$$

Define the ith column of the identity matrix $\mathbf{e}_i \triangleq [\mathbf{I}_N]_{\mathrm{column}\, i}$. Using the column-wise diagonal dominance property (2) we can approximate $\mathbf{h}_k^n \simeq \mathbf{e}_n h_k^{n,n}$. Hence

$$s_k^n \simeq \left[\frac{1}{\lambda_n} - \frac{1}{|h_k^{n,n}|^2 \left[\mathbf{Q}_k^{-1}\right]_{n,n}}\right]^+$$

Now $\left[\mathbf{Q}_k^{-1}\right]_{n,n} = \left|\overline{\mathbf{Q}}_k^{n,n}\right| |\mathbf{Q}_k|^{-1}$ where $\overline{\mathbf{Q}}_k^{n,n}$ is the sub-matrix formed by removing row n and column n from $\mathbf{Q}_k$. Since re-ordering of columns and rows has no effect on the

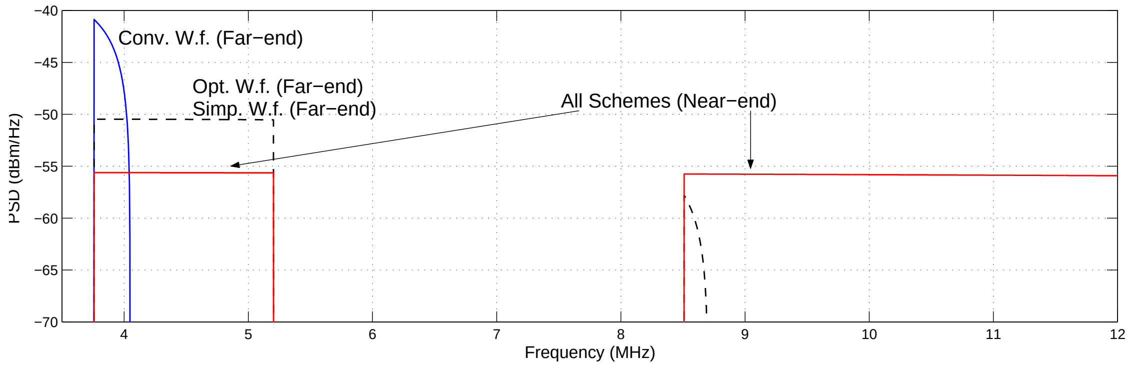

Figure 1. PSDs of Power Allocation Schemes

determinant

$$|\mathbf{Q}_k| = \left| \sigma_k^2 \mathbf{I}_N + \begin{bmatrix} \overline{\mathbf{h}}_k^n \\ \mathbf{M}^H \end{bmatrix} \begin{bmatrix} \left(\overline{\mathbf{h}}_k^n\right)^H & \mathbf{M} \end{bmatrix} \right|$$

where

$$\mathbf{M} \triangleq \begin{bmatrix} \overline{\mathbf{h}}_k^{1\ H} & \ldots & \overline{\mathbf{h}}_k^{n-1\ H}, & \overline{\mathbf{h}}_k^{n+1\ H} & \ldots & \overline{\mathbf{h}}_k^{N\ H} \end{bmatrix}$$

Divide $\mathbf{Q}_k$ into sub-matrices

$$|\mathbf{Q}_k| = \begin{vmatrix} a & \mathbf{b}^H \\ \mathbf{c} & \mathbf{D} \end{vmatrix}$$

where $a \triangleq \sigma_k^2 + \left\|\overline{\mathbf{h}}_k^n\right\|^2$, $\mathbf{b}^H \triangleq \overline{\mathbf{h}}_k^n \mathbf{M}$, $\mathbf{c} \triangleq \mathbf{M}^H \left(\overline{\mathbf{h}}_k^n\right)^H$ and $\mathbf{D} \triangleq \mathbf{M}^H\mathbf{M} + \sigma_k^2 \mathbf{I}_{N-1} = \overline{\mathbf{Q}}_k^{n,n}$. Using the Schur decomposition

$$|\mathbf{Q}_k| = \left|\overline{\mathbf{Q}}_k^{n,n}\right| \left|a - \mathbf{b}^H \mathbf{D}^{-1} \mathbf{c}\right|$$

hence

$$\left[\mathbf{Q}_k^{-1}\right]_{n,n} = \left| \sigma_k^2 + \left\|\overline{\mathbf{h}}_k^n\right\|^2 - \overline{\mathbf{h}}_k^n \mathbf{G} \left(\overline{\mathbf{h}}_k^n\right)^H \right|^{-1}$$

where

$$\mathbf{G} \triangleq \mathbf{M} \left(\mathbf{M}^H\mathbf{M} + \sigma_k^2 \mathbf{I}_{N-1}\right)^{-1} \mathbf{M}^H$$

Define the singular-value decomposition (SVD) of $\mathbf{M} \overset{\text{svd}}{=} \mathbf{U}_M \Lambda_M \mathbf{V}_M^H$. Column-wise diagonal dominance (2) assures us that $\mathbf{M}$ will have full rank hence $\mathbf{U}_M$ and $\mathbf{V}_M$ will be unitary matrices of size $N-1 \times N-1$. Thus

$$\mathbf{G} = \mathbf{U}_M \Lambda_M^2 \left(\Lambda_M^2 + \sigma_k^2 \mathbf{I}_{N-1}\right)^{-1} \mathbf{U}_M^H$$

Since the SNR in DSL is high we can approximate $\Lambda_M^2 + \sigma_k^2 \mathbf{I}_{N-1} \simeq \Lambda_M^2$ and

$$\mathbf{G} \simeq \mathbf{I}_{N-1}$$

Hence

$$\left[\mathbf{Q}_k^{-1}\right]_{n,n} \simeq 1/\sigma_k^2$$

which leads to (10).

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