# IMPROVED LINEAR CROSSTALK PRECOMPENSATION FOR DSL

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**ABSTRACT**

Crosstalk is *the* major source of performance degradation in next generation DSL systems such as VDSL. In downstream communications transmitting modems are co-located at the central office. This allows crosstalk precompensation to be employed. In crosstalk precompensation the transmitted signal is pre-distorted such that the pre-distortion destructively interferes with the crosstalk introduced by the channel.

Existing crosstalk precompensation techniques either give poor performance or require modification of customer premises equipment (CPE). This is impractical since there are millions of legacy CPE modems already in use.

We present a novel crosstalk precompensation technique based on a diagonalization of the crosstalk channel matrix. This technique does not require modification of CPE. Furthermore, certain properties of the DSL channel ensure that this *diagonalizing precompensator* achieves near-optimal performance.

## 1. INTRODUCTION

Next generation DSL systems such as VDSL aim at providing extremely high data-rates, up to 52 Mbps in the downstream. Such high data rates are supported by operating over short loop lengths and transmitting in frequencies up to 12 MHz. Unfortunately, the use of such high frequency ranges causes significant electromagnetic coupling between neighbouring twisted pairs within a binder group. This coupling creates interference, referred to as *crosstalk*, between the systems operating within a binder. Over short loop lengths crosstalk is typically 10-15 dB larger than the background noise and is *the* dominant source of performance degradation.

In upstream communications the receiving modems are co-located at the *central office* (CO) or at an *optical network unit* (ONU) located at the end of the street. This allows joint reception of the signals transmitted on the different lines, thereby enabling *crosstalk cancellation*[1].

In *downstream* (DS) communications the receiving modems reside within different *customer premises* (CP) so crosstalk cancellation is not possible. However since the transmitting modems are co-located at the CO it is possible to do transmission in a joint fashion. This allows some pre-distortion to be introduced into the signals on the different lines before transmission. This pre-distortion is designed to destructively interfere with the crosstalk introduced in the binder, a technique known as *crosstalk precompensation*[2].

Several techniques have been proposed for crosstalk precompensation. Unfortunately these lead to either poor performance or require a change of *customer premises equipment* (CPE). This is highly undesirable since there are already millions of CPEs in place all owned and operated by different customers. Replacing *CO equipment* (COE) is much easier since it is typically managed by a single operator. In addition, COE and CPE are typically manufactured by different hardware vendors, which makes joint design more difficult.

In this paper we present a novel technique for crosstalk precompensation which works with existing CPE. This technique is also shown to give near-optimal performance.

## 2. SYSTEM MODEL

Through the use of *discrete multi-tone* (DMT) transmission and synchronized transmission it is possible to model crosstalk independently on each tone

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k$$

The vector $\mathbf{x}_k \triangleq [x_k^1, \cdots, x_k^N]$ contains the transmitted signals on tone $k$. There are $N$ lines in the binder and $x_k^n$ is the signal transmitted onto line $n$ at tone $k$. $\mathbf{y}_k$ and $\mathbf{z}_k$ have similar structures. $\mathbf{y}_k$ is the vector of received signals on tone $k$. $\mathbf{z}_k$ is the vector of additive noise on tone $k$ and contains thermal noise, alien crosstalk, RFI etc. We denote the noise PSD on line $n$ as $\sigma_k^n \triangleq \mathcal{E}\{|z_k^n|^2\}$. $\mathbf{H}_k$ is the $N \times N$ channel transfer matrix on tone $k$. $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from transmitter (TX) $m$ to receiver (RX) $n$ on tone $k$. The diagonal elements of $\mathbf{H}_k$ contain the direct-channels whilst the off-diagonal elements contain the crosstalk channels. We denote the transmit PSD of user $n$ on tone $k$ as $s_k^n \triangleq \mathcal{E}\{|x_k^n|^2\}$.

In DSL spectral masks are used to ensure spectral compatibility with other systems operating within the binder[3]. We denote the spectral mask on tone $k$ as $s_{k,\text{mask}}$. Spectral masks impose the constraint $s_k^n \leq s_{k,\text{mask}},\ \forall n$.

In DS transmission the TX modems are co-located. As a result $\mathbf{H}_k$ is *row-wise diagonally dominant* (RWDD). This means that on each row of $\mathbf{H}_k$, the diagonal element has the largest magnitude

$$|h_k^{n,n}| \gg |h_k^{n,m}|,\ \forall m \neq n \tag{1}$$

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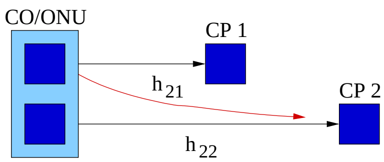


**Fig. 1**. Row-wise Diagonal Dominance $|h_{22}| \gg |h_{21}|$

The physical reason behind this is that the crosstalk signal must propagate through the full length of the victim's line, as depicted in Fig. 1. This together with the attenuation which results from shielding between twisted pairs ensures RWDD of $\mathbf{H}_k$. We can measure the degree of RWDD with the parameter $\alpha_k$

$$|h_k^{n,m}| \leq \alpha_k \, |h_k^{n,n}| \tag{2}$$

RWDD has been verified through extensive measurement campaigns of real binders. In 99% of lines $\alpha_k$ is bounded

$$\alpha_k \leq K_{\text{fext}} f_k \sqrt{l}$$

where $K_{\text{fext}} = -22.5$ dB, $l$ is the line length in kilometers, and $f_k$ is the frequency on tone $k$ in MHz[4]. On typical lines $\alpha_k$ is less than -11.3 dB.

RWDD implies that the rows of $\mathbf{H}_k$ are approximately orthogonal. Define through the SVD

$$\mathbf{H}_k = \mathbf{U}_k \Lambda_k \mathbf{V}_k^H$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are orthogonal matrices containing the left and right singular vectors and $\Lambda_k \triangleq \text{diag}\{\lambda_1, \ldots, \lambda_N\}$ where $\lambda_n$ is the $n$th singular value. RWDD implies $\mathbf{U}_k \simeq \mathbf{I}_N$. Hence we can approximate $\mathbf{H}_k \simeq \Lambda_k \mathbf{V}_k^H$. This leads to

$$\mathbf{H}_k^{-1} \simeq \mathbf{V}_k \Lambda_k^{-1} \tag{3}$$

and

$$\mathbf{H}_k^H \simeq \mathbf{V}_k \Lambda_k \tag{4}$$

Taking (3) and (4) together yields

$$\mathbf{H}_k^{-1} \simeq \mathbf{H}_k^H \Lambda_k^{-2} \tag{5}$$

Since $\mathbf{H}_k \mathbf{H}_k^H \simeq \Lambda_k^2$ we can approximate

$$\begin{aligned} \lambda_{k,n}^2 &\simeq \sum_m |h_k^{n,m}|^2 \\ &\simeq |h_k^{n,n}|^2 \end{aligned} \tag{6}$$

where we use (1) in the second line.

## 3. CROSSTALK PRECOMPENSATION

Several techniques have been proposed for crosstalk precompensation. They are all based on the concept of pre-distorting the signals before transmission such that the pre-distortion and crosstalk annihilate.

### 3.1. Zero Forcing Precompensator

The *zero forcing precompensator* (ZFP) pre-distorts the transmitted signals with the inverse of the channel matrix[5]. So

$$\overline{\mathbf{x}}_k = \mathbf{P}_{k,\text{zf}} \mathbf{x}_k$$

where the vector of pre-distorted signals $\overline{\mathbf{x}}_k = [\overline{x}_k^1, \ldots, \overline{x}_k^N]$ and

$$\mathbf{P}_{k,\text{zf}} \triangleq \beta_{k,\text{zf}} \mathbf{H}_k^{-1}$$

This is depicted in Fig. 2. The parameter $\beta_{k,\text{zf}}$ ensures that the precompensation operation does not increase the transmit power. Note that

$$\overline{x}_k^n = \sum_m \beta_{k,\text{zf}} \left[\mathbf{H}_k^{-1}\right]_{n,m} x_k^n$$

Hence

$$\begin{aligned} \mathcal{E}\{|\overline{x}_k^n|^2\} &= \beta_{k,\text{zf}}^2 \sum_m \left| \left[\mathbf{H}_k^{-1}\right]_{n,m} \right|^2 s_k^n \\ &\leq \beta_{k,\text{zf}}^2 \left\| \left[\mathbf{H}_k^{-1}\right]_{\text{row } n} \right\|^2 s_{k,\text{mask}} \end{aligned}$$

To ensure that $\overline{\mathbf{x}}_k$ does not exceed the spectral masks we require

$$\beta_{k,\text{zf}} = \min_n \left\| \left[\mathbf{H}_k^{-1}\right]_{\text{row } n} \right\|^{-1} \tag{7}$$

From (5)

$$\begin{aligned} \left\| \left[\mathbf{H}_k^{-1}\right]_{\text{row } n} \right\|^2 &\simeq \sum_m |h_k^{m,n}|^2 \lambda_{k,m}^{-4} \qquad (8) \\ &\simeq |h_k^{n,n}|^{-2} + \sum_{m \neq n} |h_k^{m,n}|^2 \, |h_k^{m,m}|^{-4} \\ &\simeq |h_k^{n,n}|^{-2} \end{aligned}$$

where we use (6) in line 2, and (1) in line 3. Combining this with (7) yields

$$\beta_{k,\text{zf}} \simeq \min_n |h_k^{n,n}| \tag{9}$$

Now, with the ZFP

$$\begin{aligned} \mathbf{y}_k &= \mathbf{H}_k \mathbf{P}_{k,\text{zf}} \mathbf{x}_k + \mathbf{z}_k \\ &= \beta_{k,\text{zf}} \mathbf{x}_k + \mathbf{z}_k \\ &\simeq \min_n |h_k^{n,n}| \, \mathbf{x}_k + \mathbf{z}_k \end{aligned}$$

hence the data-rate of user $n$ on tone $k$ can be approximated

$$c_{k,\text{zf}}^n \simeq \log_2 \left( 1 + \frac{1}{\Gamma} \min_n |h_k^{n,n}|^2 \, s_k^n \sigma_{k,n}^{-2} \right)$$

with the approximation becoming exact as $\alpha_k \to 0$. $\Gamma$ denotes the SNR-gap to capacity and is a function of the target BER, coding gain and noise margin[6].

So we see that with the ZFP all modems see the channel of the worst line within the binder. This leads to very poor performance, especially when the lines are of varying length or when one of the lines contains a bridged tap.

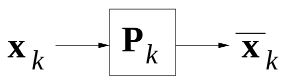


**Fig. 2**. Linear Precompensator

### 3.2. Multi-user Tomlinson-Harashima Precoder

Similar to the ZFP, the *Multi-user Tomlinson-Harashima Precoder* (MU-THP) pre-distorts the transmitted signal with the inverse of the channel. However in contrast to the ZFP, the MU-THP uses non-linear modulo operations to ensure that the TX power is not increased. As such no normalization parameter $\beta_k$ is required. This leads to significantly improved performance with only a modest increase in complexity[2].

Define the QR decomposition of the conjugate transpose of the channel

$$\mathbf{H}_k^H = \mathbf{Q}_k \mathbf{R}_k$$

The structure of the MU-THP is shown in Fig. 3. It consists of a feed-forward filter $\mathbf{F}_k \triangleq \mathbf{Q}_k$ and a feedback filter $\mathbf{B}_k \triangleq \mathbf{I}_N - \mathrm{diag}\{\mathbf{R}_k^H\}^{-1}\mathbf{R}_k^H$. With the MU-THP

$$\begin{aligned}\mathbf{y}_k &= \mathbf{H}_k \mathbf{F}_k \left(\mathbf{I}_N - \mathbf{B}_k\right)^{-1} \mathbf{x}_k + \mathbf{z}_k \\ &= \mathrm{diag}\{\mathbf{R}_k^H\}\mathbf{x}_k + \mathbf{z}_k \\ &\simeq \mathrm{diag}\{\mathbf{H}_k\}\mathbf{x}_k + \mathbf{z}_k\end{aligned}$$

The approximation on line 3 is based on (1), see [2] for details. Hence the data-rate of user $n$ on tone $k$ can be approximated

$$c_{k,\mathrm{th}}^n \simeq \log_2\left(1 + \frac{1}{\Gamma}\left|h_k^{n,n}\right|^2 s_k^n \sigma_{k,n}^{-2}\right)$$

with the approximation becoming exact as $\alpha_k \to 0$. So the MU-THP allows crosstalk to be completely removed without decreasing the channel gains seen by the individual modems.

Unfortunately the MU-THP required a modulo operation at the RX to make the modulo operation at the TX transparent. This requires a hardware modification to CPE which can be extremely difficult due to the millions of legacy DSL modems which are already in use.

Additionally, the MU-THP is non-linear which makes it difficult to apply partial crosstalk precompensation techniques[7, 8]. These techniques are crucial since in binders containing hundreds of lines, full crosstalk cancellation has an impractically large computational complexity.

### 3.3. Diagonalizing Precompensator

To overcome the problems of the ZFP and the MU-THP we propose the *diagonalizing precompensator* (DP). This technique requires no modification of CPE, is linear, and can be easily combined with partial crosstalk cancellation. As we shall show, the DP is near-optimal in RWDD channels and gives very similar performance to the MU-THP.

Similar to the ZFP, the DP pre-distorts the transmitted signals with a linear matrix multiplication. However in contrast to the ZFP, the DP attempts not to invert $\mathbf{H}_k$ but to diagonalize it instead. So the pre-distorted signals

$$\overline{\mathbf{x}}_k = \mathbf{P}_{k,\mathrm{diag}}\mathbf{x}_k$$

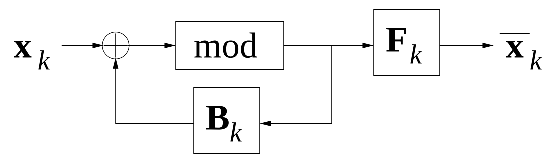


**Fig. 3**. Multi-user Tomlinson-Harashima Precoder

where

$$\mathbf{P}_{k,\mathrm{diag}} \triangleq \beta_{k,\mathrm{diag}}\mathbf{H}_k^{-1}\mathrm{diag}\{\mathbf{H}_k\}$$

The normalizing factor $\beta_{k,\mathrm{diag}}$ ensures that the spectral mask is not exceeded on any line

$$\beta_{k,\mathrm{diag}} \triangleq \min_n \left\| \left[\mathbf{H}_k^{-1}\mathrm{diag}\{\mathbf{H}_k\}\right]_{\mathrm{row}\,n}\right\|^{-1}$$

From (8)

$$\begin{aligned}\left\| \left[\mathbf{H}_k^{-1}\mathrm{diag}\{\mathbf{H}_k\}\right]_{\mathrm{row}\,n}\right\|^{2} &\simeq \sum_m \left|h_k^{m,n}\right|^2 \left|h_k^{m,m}\right|^2 \lambda_{k,m}^{-4} \\ &\simeq 1 + \sum_{m\neq n} \left|h_k^{m,n}\right|^2 \left|h_k^{m,m}\right|^{-2} \\ &\simeq 1\end{aligned}$$

where we use (6) in line 2 and (1) in line 3. Hence

$$\beta_{k,\mathrm{diag}} \simeq 1 \tag{10}$$

Now, with the DP

$$\begin{aligned}\mathbf{y}_k &= \mathbf{H}_k \mathbf{P}_{k,\mathrm{diag}}\mathbf{x}_k + \mathbf{z}_k \\ &= \beta_{k,\mathrm{diag}}\mathrm{diag}\{\mathbf{H}_k\}\mathbf{x}_k + \mathbf{z}_k \\ &\simeq \mathrm{diag}\{\mathbf{H}_k\}\mathbf{x}_k + \mathbf{z}_k\end{aligned}$$

Hence the data-rate of user $n$ on tone $k$ can be approximated

$$c_{k,\mathrm{diag}}^n \simeq \log_2\left(1 + \frac{1}{\Gamma}\left|h_k^{n,n}\right|^2 s_k^n \sigma_{k,n}^{-2}\right)$$

with the approximation becoming exact as $\alpha_k \to 0$. So we see that, as with the MU-THP, the DP removes crosstalk perfectly without affecting the direct channels of the individual modems. In contrast to the MU-THP this can be done without modifying CPE.

### 3.4. Theoretical Capacity

It is also interesting to compare the performance of different crosstalk precompensation techniques with a theoretical upper bound. In the downstream the DSL channel is a multi-user broadcast channel. As such there is no single capacity but rather a *rate region* which is achievable[9]. However if we use all TXs to communicate with a single CPE RX, the data-rate can be upper bounded by

$$\begin{aligned}c_k^n &= I(x_k^n; y_k^n) \\ &\leq I(\overline{\mathbf{x}}_k; y_k^n) \\ &\leq \log_2\left(1 + \frac{1}{\Gamma}\left\|\left[\mathbf{H}_k\right]_{\mathrm{row}\,n}\right\|^2 s_k^n \sigma_k^{-2}\right) \\ &\leq \log_2\left(1 + \frac{1}{\Gamma}\left[1 + (N-1)\alpha_k^2\right]\left|h_k^{n,n}\right|^2 s_k^n \sigma_k^{-2}\right)\end{aligned} \tag{11}$$

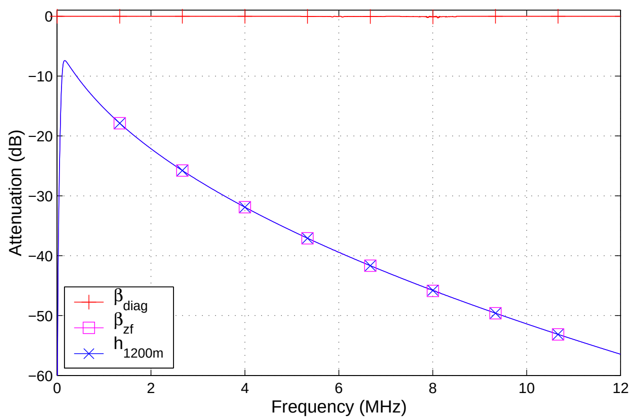


**Fig. 4**. Value of normalizing factor $\beta$ vs. frequency

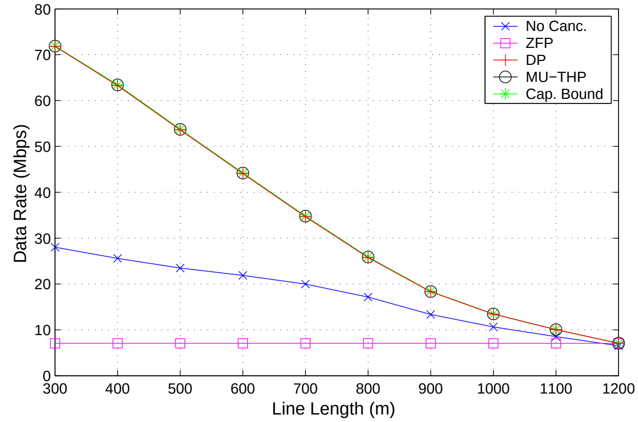


**Fig. 5**. Data-rates with different crosstalk precomp. techniques

where $I(a;b)$ denotes the mutual information between $a$ and $b$. We use (2) to get to line 4. Equality in (11) achieved when we use a maximum-ratio combining precompensator with

$$\overline{\mathbf{x}}_k = [\mathbf{H}_k]^H_{\text{row } n}\, x^n_k$$

All other RXs must be disabled to achieve this data rate for RX $n$.

## 4. PERFORMANCE

To demonstrate the performance of the different precompensation techniques we ran simulations in a binder consisting of 10 VDSL lines. The lines have a diameter of 0.4mm and vary in length from 300 m. to 1200 m. in 100 m. increments. Each modem has a coding gain of 3 dB, a noise margin of 6 dB and a target error probability of $10^{-7}$ or less which results in $\Gamma = 12.9$ dB. The modems use 4096 tones, the 998 FDD bandplan and transmit at -60 dBm/Hz. We use ETSI noise model A and the semi-empirical transfer functions of [4].

Fig. 4 shows the values of $\beta_{k,\text{zf}}$ and $\beta_{k,\text{diag}}$ versus frequency. As we predicted from (9), $\beta_{k,\text{zf}}$ is closely approximated by the magnitude of the weakest channel in the binder, in this case the channel of the 1200 m. line. As predicted by (10), $\beta_{k,\text{diag}}$ is close to unity.

Shown in Fig. 5 are the data-rates achieved by the various lines with the different precompensation techniques. Note that with the ZFP all lines receive the same performance as the 1200 m. line. In this binder this results in a performance that is worse than with no crosstalk cancellation. Both the DP and MU-THP give near-optimal performance, closely approximating the theoretical bound.

## 5. CONCLUSIONS

In this paper we presented a novel technique for crosstalk precompensation which attempts to diagonalize the crosstalk channel. This technique which we term the *diagonalizing precompensator* (DP) is linear and has a low computational complexity.

The row-wise diagonal dominance of the downstream DSL channel ensures that the DP gives near-optimal performance. Unlike other precompensation techniques, the DP does not require a modification of CPE and can be easily combined with partial techniques.

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