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CONTRIBUTION

TITLE: Optimal Spectrum Management

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PROJECT: **T1E1.4, DSM**

ABSTRACT

This document contains a detailed description of the *Optimal Spectrum Management* (OSM) algorithm for inclusion in the DSM report. The concept to add the OSM was agreed in the February meeting, but specific text was requested for the exact addition to the DSM Report, T1E1.4/2003-018R9.

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INTRODUCTION

In the next section a detailed description of the *Optimal Spectrum Management* (OSM) algorithm is presented. Main modification compared to **T1E1.4/2003-365** are the following :

- Notation from the DSM report has been used
- Power constraint includes the tone spacing and could also be different per user (index i included)
- Inclusion of the assumptions under which the algorithm was derived

TEXT PROPOSAL

A.1.3 Optimal Spectrum Management (OSM)

This section provides informative text that addresses a basic spectrum-management objective to maximize the rate of a user (in this case user 2), subject to minimum service rates for the other users within the network (in this case user 1). Mathematically, the OSM procedure of this section maximizes the rate of user 2 over all of the possible transmit PSDs for user 1 and user 2

$$
\begin{aligned}
&\max_{S_1,S_2} R_2(S_1,S_2)\\
&\text{s.t. } R_1(S_1,S_2) \geq R_1^{\text{target}}\\
&\qquad \sum\nolimits_n S_1(n)\cdot\Delta f \leq P_{1,\max}\\
&\qquad \sum\nolimits_n S_2(n)\cdot\Delta f \leq P_{2,\max}
\end{aligned}
$$

Target rate constraint for user 1

Power constraints for users 1 and 2

where $S_i(n)$ is the transmit PSD of user i on tone n, $P_{\text{i, max}}$ is the maximum transmit power supported by modem i , and R_1^{target} is the target service data-rate for user 1. Also, $S_i = [S_i(1), S_i(2), \cdots S_i(N)]$ is a vector containing the transmit PSD of user i on all N tones, Δf is the tone spacing and. $R_i(S_1,S_2)$ is the data-rate achieved by user i when transmit spectra S_1 and S_2 are used by user 1 and user 2 respectively..

The OSM procedure assumes that discrete multi-tone (DMT) modulation is employed and that the capacity on each tone can be modelled independently. So, the inter-symbol and inter-carrier interference is neglected.

Unfortunately this is a non-convex optimization and requires complexity $O(e^{NM})$ to solve where N is the number of tones in the system and M the number of users. With N=256 in ADSL and N=4096 in VDSL this leads to a computationally intractable problem.

The procedure uses a technique from optimization theory known as *dual decomposition* to solve the spectrum management problem with a linear complexity in N. This leads to the OSM algorithm that is computationally tractable.

1. The 2-User Case

   The OSM algorithm is based on maximizing the so-called *Lagrangian* on each tone. This section first provides a 2-user version of the OSM algorithm for ease of explanation. The Lagrangian on tone n is then defined

$$L(n) = w \cdot b_1\big(n, S_1(n), S_2(n)\big) + (1-w) \cdot b_2\big(n, S_1(n), S_2(n)\big) - l_1 \cdot S_1(n) - l_2 \cdot S_2(n)$$

   where $b_i\big(n, S_1(n), S_2(n)\big)$ denotes the bit loading achieved by user *i* on tone *n* when user 1 and user 2 adopt transmit PSDs $S_1(n)$ and $S_2(n)$ respectively. The optimal transmit spectra on tone *n* are found by maximising *L(n)*

$$S_1(n)^{\mathrm{opt}}, S_2(n)^{\mathrm{opt}} = \arg \max_{S_1(n), S_2(n)} L(n)$$

   The weight *w* determines the desired trade-off of data-rates between user 1 and user 2. Setting *w* = 1 gives full priority to user 1 and user 2's data rate is then ignored. Setting *w* = 0 instead gives full priority to user 2 and user 1's data rate is ignored. The variables λ_1 and λ_2 are the *Lagrangian multipliers,* and enforce the power constraints on modems 1 and 2 respectively.

   During operation the OSM algorithm adjusts *w* such that the target data-rate of user 1 is just achieved. The algorithm does not give more priority to user 1 than is necessary to achieve their target data-rate, thereby maximizing the data-rate of user 2. In a similar fashion λ_1 and λ_2 are adjusted such that the power constraints on both modems are enforced.

   The complete algorithm is listed below:

Algorithm A.1.3.1: Optimal Spectrum Management – 2 Users

initialise *w*, λ_1, λ_2

while $R_1 \neq R_1^{\mathrm{target}}$

    while ($\sum_n S_1(n) \cdot \Delta f \neq P_{1,\max}$) and ($\lambda_1 > 0$)

        while ($\sum_n S_2(n) \cdot \Delta f \neq P_{2,\max}$) and ($\lambda_2 > 0$)

            for each tone *n*: find PSD pair ($S_1(n)$, $S_2(n)$) which maximizes *L(n)*

            if $\sum_n S_2(n) \cdot \Delta f > P_{2,\max}$ increase λ_2, else decrease λ_2

        end

        if $\sum_n S_1(n) \cdot \Delta f > P_{1,\max}$ increase λ_1, else decrease λ_1

    end

    if $R_1 < R_1^{\mathrm{target}}$ increase *w*, else decrease *w*

end

The M-User Case

The general OSM algorithm for M users maximizes the rate of a user (in this case user $i=M$), subject to minimum service rates for the other users within the network (in this case users 1...M-1). Specifically, the M-ary OSM algorithm maximizes the rate of user M over all of the possible transmit PSDs for users 1...M

$$\max_{S_1,\ldots,S_M} R_M(S_1,\ldots,S_M)$$
$$\text{s.t. } R_i(S_1,\ldots,S_M) \geq R_i^{\text{target}} \; \forall i < M$$
$$\sum\nolimits_n S_i(n)\cdot\Delta f \leq P_{\text{i, max}} \forall i$$

Target rate constraints for users 1...M-1

Power constraints for users 1...M

With M users, M-1 weights $w_1 \ldots w_{M\text{-}1}$ are required. These enforce the target rates on users 1...M-1. The weight for user M is related to the other weights as

$$w_M = 1 - \sum_{i=1}^{M-1} w_i$$

A Lagrangian multiplier λ_i is required for each user i to enforce the total power constraint. The Lagrangian on tone n is then defined

$$L(n) = \sum_{i=1}^{M} \left(w_i \cdot b_i\left(n, S_1(n), \ldots, S_M(n)\right) - l_i \cdot S_i(n) \right)$$

where $b_i\left(n, S_1(n), \ldots, S_M(n)\right)$ denotes the bitloading achieved by user i on tone n when the users adopt transmit PSDs $S_1(n), \ldots, S_M(n)$. The optimal transmit spectra on tone n are found by maximising *L(n)*

$$(S_1(n)^{opt}, \ldots, S_M(n)^{opt}) = \arg \max_{S_1(n),\ldots,S_M(n)} L(n)$$

During operation the OSM algorithm adjusts $w_1 \ldots w_{M\text{-}1}$ such that the target data-rates of users 1...M-1 are just achieved. The algorithm does not give more priority to users 1...M-1 than is necessary to achieve their target data-rates, thereby maximising the data-rate of user M. In a similar fashion $\lambda_1 \ldots \lambda_M$ are adjusted such that the power constraints are enforced on each modem.

The complete algorithm is listed below. For more details see [1], [2] and [3].

Algorithm A.1.3.2: Optimal Spectrum Management – *M* Users

initialise $w_1,\ldots, w_{M-1}$, $\lambda_1,\ldots,\lambda_M$

while $R_1 \neq R_1^{\text{target}}$

$\vdots$

while $R_{M-1} \neq R_{M-1}^{\text{target}}$

while ($\sum_n S_1(n)\cdot\Delta f \neq P_{1,\max}$) and ($\lambda_1 > 0$)

$\vdots$

while ($\sum_n S_M(n)\cdot\Delta f \neq P_{\mathrm{M},\max}$) and ($\lambda_M > 0$)

$$w_M = 1 - \sum_{i=1}^{M-1} w_i$$

for each tone *n*:

find PSD tuple ($S_1(n),\ldots,S_M(n)$) which maximises

$$L(n) = \sum_i w_i \cdot b_i\left(n, S_1(n),\ldots,S_M(n)\right) - l_i \cdot S_i(n)$$

if $\sum_n S_M(n)\cdot\Delta f > P_{\mathrm{M},\max}$ increase λ_M, else decrease λ_M

end

$\vdots$

if $\sum_n S_1(n)\cdot\Delta f > P_{1,\max}$ increase λ_1, else decrease λ_1

end

if $R_{M-1} < R_{M-1}^{\text{target}}$ increase w_{M-1}, else decrease w_{M-1}

end

$\vdots$

if $R_1 < R_1^{\text{target}}$ increase w_1, else decrease w_1

end

PROPOSAL

We propose that the text proposal from the previous section is included in the DSM report as paragraph A.1.3.

We want that the references are also mentioned in the DSM report, since they can give extra information for the interested reader (proof of optimality under the mentioned assumptions).

References

[1] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, T. Bostoen, "Optimal Multiuser Spectrum Management for Digital Subscriber Lines," *submitted to IEEE Transactions on Communications, accepted for IEEE Intl. Conf. on Communications (ICC) 2004*.

Available at http://www.esat.kuleuven.ac.be/~rcedrill/research/publications.html

[2] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, T. Bostoen, "On the Optimality of Iterative Waterfilling in DSL," ANSI T1E1.4 Working Group (DSL Access) Meeting, contrib. 2003-325, San Diego, December 2003.

[3] W. Yu, G. Ginis, J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," in *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105 – 1115, June 2002.