

# Effects Of Material Parameters On The Stresses In High Temperature Weldments

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**ABSTRACT:** An integral equation is presented that relates time to equivalent stresses at the interface of two regions in a weldment operating within the creep regime. Taking the creep constitutive equation as the Norton power law, the paper investigates the effects of the stress index and the stress coefficient on the equivalent stresses at the critical interfaces in the weldment.

## INTRODUCTION

Weldments are by far the most common location for creep failures in the thick-walled components employed in various industries. In spite of substantial advancement in modelling and understating the behaviour of the weldments operating within creep regime, currently there are no generally accepted and efficient models to prevent their premature failures. The use of ever more powerful computers to perform time dependent finite element analysis (FEA) is now permitting predictions of stresses in a weldment more accurately than before [1,2]. However the creep FEA is usually expensive and time-consuming. This study describes the development of an integral equation for predicting the variation of equivalent stresses at the critical interface of two zones within a high temperature weldment; see below.

## ANALYSIS

A weldment typically consists of three zones, viz., the weld material (WM), the heat-affected zone (HAZ) and the parent material (PM). There are many factors that make the prediction of the behaviour of the weldments at elevated temperatures difficult. For example, when a welded joint is subjected to elevated temperature and sustained loading, WM, HAZ and PM each exhibit different creep characteristics with time. As a result, complex stress patterns that are time-dependent will set up at the interface between these materials, contributing to creep damage, cracking and failure of the weldment. Although the literature on welded joints is voluminous [1,2], the published results are mainly based on numerical and/or experimental investigations. To the author's knowledge, the analytical and/or semi-analytical solutions that reduce the cost of the analysis and give better understanding of variation of stresses with time in a weldment are scarce. The present study concentrates on the effects of material mismatch between WM, HAZ and PM on the stresses in the weldment only and describes a simple and semi-analytical model for predicting stresses in a butt-welded plate.

Koundy et al [3] have experimentally shown that the equivalent von Mises stress is the main stress component that causes creep failure in butt-welded joints. Therefore, the present study concentrates in predicting the equivalent von Mises stresses in such weldments. As it was explained above, the present study concentrates on material discontinuities at a butt-welded joint and ignores the geometrical discontinuities with the objectives to ascertain the significance of various material parameters on the stresses. Consider a plate that contains a butt-welded joint and is subjected to a uniform and constant traction as shown in Fig. 1. The assumptions used in the analysis are. the plate, weld and HAZ have a constant and uniform thickness permitting elimination of the stress concentration due to variation in the weld thickness; the plate is subjected to a constant temperature ( $T$ ) and a constant and uniform traction ( $\sigma_0$ ) where  $\sigma_0$  is applied: (a) in the  $x$  - direction, (b) in the

$y$  – direction, (c) in both the  $x$  and  $y$  - directions simultaneously; the analysis ignores residual stresses [3]; the small displacement and strain conditions are assumed to prevail. This may be justified as in practice the weldment cracking usually occurs when the material experiences relatively small values of strain.

Although the HAZ is depicted as a uniform material in Fig. 1, as will become clear shortly, the proposed model allows for various grades of material within the HAZ. Let us consider two adjacent points at a critical material interface away from the plate boundaries. Note that at the plate boundaries the stresses are equal to applied tractions for all values of time ( $t$ ) to satisfy the force equilibrium conditions. These two points can, for example, be at the interface of the weld and the HAZ, or alternatively at the interface of two different grades of the HAZ, etc. In all cases, these two points must refer to the interface region of interest in the weldment. To satisfy the equilibrium conditions, initially, at  $t=0$ , the stresses acting on these points must be equal, noting that the loading is assumed to be uniform and constant. With time, because of different creep rates, the stresses are redistributed and stresses acting at point 1 will differ from those acting at point 2. Fig. 2 shows two differential elements of dimensions  $dx$  by  $dy$  by  $dz$  (where  $z$  defines the direction perpendicular to the plane of the plate) corresponding with points 1 and 2 at  $t=0$ . One point is inside material 1 (e.g., the weld) and the other inside material 2 (e.g., the HAZ). Fig. 2 also shows the acting equivalent von Mises stress (which is equal to the applied traction,  $\sigma_0$ ) on points 1 and 2 at  $t=0$ . After a period of time,  $t$ , the stresses between points 1 and 2 will be redistributed so that the equivalent von Mises stress acting at point 1 is  $\sigma_1$  and that acting at point 2 is  $\sigma_2$ . Normally,  $\sigma_0$  (the applied traction) is known. The objective of the proposed model is to develop expressions to predict  $\sigma_1$  and  $\sigma_2$  and their variations with time. Note that the applied equivalent von Mises stress is  $\sigma_0$  for both the uniaxial and biaxial loading. Now, because of equilibrium the total forces acting on points 1 and 2 at  $t=0$  must be equal to those at time  $t$  such that in terms of the stresses depicted in Fig. 2, we have:

$$dx dz \sigma_1 + dx dz \sigma_2 = (dx dz + dx dz) \sigma_0$$

or,

$$\sigma_1 + \sigma_2 = 2\sigma_0 \quad (1)$$

Due to compatibility:

$$\frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt} \quad (2)$$

where  $\varepsilon$  is the equivalent von Mises total strain at time  $t$  and subscripts 1 and 2 refer to points 1 and 2. In the absence of plastic deformation, the total strain rate is assumed to be the sum of the elastic (superscript  $e$ ) and creep (superscript  $c$ ) strain rates, viz.:

$$\frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_1^e}{dt} + \frac{d\varepsilon_1^c}{dt} \quad (3)$$

$$\frac{d\varepsilon_2}{dt} = \frac{d\varepsilon_2^e}{dt} + \frac{d\varepsilon_2^c}{dt} \quad (4)$$

$$\text{with: } \frac{d\varepsilon_1^e}{dt} = \frac{1}{E_1} \frac{d\sigma_1}{dt} \quad (5)$$

$$\text{and } \frac{d\varepsilon_2^e}{dt} = \frac{1}{E_2} \frac{d\sigma_2}{dt} \quad (6)$$

$$\frac{d\varepsilon_1^c}{dt} = f_1(\sigma_1, t, T) \quad (7)$$

$$\frac{d\varepsilon_2^c}{dt} = f_2(\sigma_2, t, T) \quad (8)$$

where  $E_1$  is the modulus of elasticity of material 1,  $E_2$  is the modulus of elasticity of material 2,  $f_1$  and  $f_2$  are the functions that describe the creep constitutive relationships for materials 1 and 2 respectively.

Combining and integrating equations (1) to (8) and eliminating  $\sigma_1$ , gives:

$$F(\sigma_2, t, \sigma_0) = 0 \quad (9)$$

Eliminating  $\sigma_2$  between equations (1) and (9) will give  $\sigma_1$ . Note that the specific expression for  $F$  depends on the specific expressions for  $f_1$  and  $f_2$ . For example, for the creep power law where  $f_1 = B_1 \sigma_1^{n_1}$  and  $f_2 = B_2 \sigma_2^{n_2}$  with  $B_1$  and  $n_1$  are the creep material constant and the creep stress index of material 1 respectively and  $B_2$  and  $n_2$  are the creep material constant and the creep stress index of material 2 respectively, equations (4) and (5) will change to:

$$\frac{d\varepsilon_1}{dt} = \frac{1}{E_1} \frac{d\sigma_1}{dt} + B_1 \sigma_1^{n_1} \quad (10)$$

$$\frac{d\varepsilon_2}{dt} = \frac{1}{E_2} \frac{d\sigma_2}{dt} + B_2 \sigma_2^{n_2} \quad (11)$$

Now, by eliminating  $\sigma_1$  between equations (1) and (10) and then combining the results with equations (2) and (11), one will obtain:

$$\frac{1}{E_1} \frac{d(2\sigma_0 - \sigma_2)}{dt} + B_1 (2\sigma_0 - \sigma_2)^{n_1} = \frac{1}{E_2} \frac{d\sigma_2}{dt} + B_2 \sigma_2^{n_2}$$

which may be rearranged to:

$$dt = \frac{d\sigma_2}{\frac{E_1 E_2}{(E_1 + E_2)} [B_1 (2\sigma_0 - \sigma_2)^{n_1} - B_2 \sigma_2^{n_2}]} \quad (12)$$

Integrating equation (12) gives:

$$t = \int_{\sigma_0}^{\sigma_2} \frac{d\sigma'}{\frac{E_1 E_2}{(E_1 + E_2)} [B_1 (2\sigma_0 - \sigma')^{n_1} - B_2 \sigma'^{n_2}]} \quad \text{for both the uniaxial and biaxial loading} \quad (13)$$

where  $\sigma'$  is the integral variable. Having determined  $\sigma_2$  using equation (13), then the variation of  $\sigma_1$  with time is followed from equation (1). Note that equation (13) holds for  $t > 0$ . It is obvious that at  $t = 0$ ,  $\sigma_1 = \sigma_2 = \sigma_0$  for all load cases (a) to (c).

## VALIDATION AND RESULTS

The following finite element modelling was carried out to verify equations (1) and (13). Finite element models were generated using the butt-welded plate shown in Fig. 1. The plate was square with a dimension of 150 mm by 150 mm and it had a constant thickness of 20 mm. The weld width was 30 mm and the width of the HAZ was 10 mm. Because of symmetry only  $\frac{1}{4}$  of the plate was meshed using 792 isoparametric 4-node elements as shown in Fig. 3. Three loading cases were modelled, viz., (a) a uniform traction  $\sigma_0 = 130 \text{ MPa}$  applied in the  $x$  - direction, (b) a uniform traction  $\sigma_0 = 130 \text{ MPa}$  applied in the  $y$  - direction, (c) a uniform traction  $\sigma_0 = 130 \text{ MPa}$  applied in both the  $x$  and  $y$  - directions simultaneously. The plate was made of 18%Cr 11%Ni steel and it was subjected to a uniform temperature of 550°C. The material properties used for the finite element analyses were [4]:  $E_1 = E_2 = E_{PM} = 157,000 \text{ MPa}$ ,  $B_{PM} = 1.1 \times 10^{-32} \text{ 1/HR}$ ,  $B_1 = 2.3 \times 10^{-37} \text{ 1/HR}$ ,  $B_2 = 1.7 \times 10^{-41} \text{ 1/HR}$ ,  $n_{PM} = 12$ ,  $n_1 = 14.74$  and  $n_2 = 15.53$ . The finite element analyses were carried out using MSC/NASTRAN [5] and a personal computer Pentium III. The creep finite element analyses were performed assuming the strain-hardening model. Also, to be consistent with the proposed model, the small displacement and strain conditions were used in the finite element analyses.

To verify the mesh, first a creep finite element analysis was performed assuming that the entire plate was made of PM only and subjected to the uniaxial stress  $\sigma_0 = 130 \text{ MPa}$ . The computed stresses were everywhere  $130 \text{ MPa}$  indicating that the mesh was sufficiently fine. Next the creep finite element analyses of the butt-welded plate were carried out for load cases (a) to (c). As might be expected, the equivalent von Mises stresses at points 1 and 2 were the same for all load cases (a), (b) and (c). In all load cases, the maximum von Mises stress was at interface between the weld and HAZ and just in the HAZ, i.e., at point 2 (Fig. 4). Fig. 4 shows the comparison of the equivalent von Mises stresses at points 1 and 2 for load cases (a), (b) and (c) predicted using equations (1) and (13) with those computed using the finite element analyses. The differences were negligible and the results depicted in Fig. 4 show that the proposed model can accurately predict the equivalent von Mises stresses at the interface of two different materials such as the weld and HAZ. Also, Fig. 4 shows that the stresses redistribute from the creep soft material (in this case the weld) to creep hard material (in this case the HAZ).

In above, it was shown that equations (1) and (13) are valid. Next these equations will be used to ascertain the effects of the pertinent materials properties on  $\sigma_1$  (the equivalent von Mises stress at point 1, e.g., at the interface between the weld and HAZ and just inside the weld) and  $\sigma_2$  (the equivalent von Mises stress at point 2, e.g., at the interface between the weld and HAZ and just inside HAZ). To do so, we need to know the explicit creep constitutive equations. For the creep power-law relationships, from equation (13) it is clear that the pertinent material properties are:  $n_1, n_2, B_1, B_2, E_1, E_2$ . Considering the denominator of the right-hand side of equation (13), it is apparent that the first term is positive and the second term is negative noting that  $0 \leq \sigma_1, \sigma_2 \leq 2\sigma_0$ . Therefore, as  $n_1$  or  $B_1$  (e.g., the creep material parameters for the weld) increases,  $\sigma_2$  increases and hence  $\sigma_1$  decreases (see equation (1)). The reverse is true when  $n_1$  or  $B_1$  decreases. This is demonstrated in Figs 5 and 7. Also, as  $n_2$  or  $B_2$  increases,  $\sigma_2$  decreases and hence  $\sigma_1$  increases. The reverse is true when  $n_2$  or  $B_2$  decreases. This is demonstrated in Figs 6 and 8.  $E_1$  and  $E_2$  are multipliers for the first term in the denominator of equation (13), therefore, their increases cause  $\sigma_2$  to increase and hence  $\sigma_1$  to decrease. The reverse is true when  $E_1$  or  $E_2$  decreases. Finally, since the applied traction ( $\sigma_0$ ) is constant and  $\sigma_1 + \sigma_2 = 2\sigma_0$  (see equation (1)), then an increase in  $\sigma_1$  causes a reduction in  $\sigma_2$  and vice versa.

## CONCLUSIONS

A semi-analytical model was developed for predicting the equivalent von Mises stresses at the interface of two materials (e.g., the weld and HAZ or two grades of HAZ, or HAZ and parent material, etc) in a plate that contains a butt-weld and subject to a uniform and constant temperature and traction. It was shown that the explicit relationships between the stresses, time and material properties depend on the creep law employed. For power-law creep, the pertinent material properties were:  $n_1, n_2, B_1, B_2, E_1, E_2$ . It was shown that  $\sigma_1$  (the equivalent von Mises stress at point 1; this could be the weld stress if the interface between the weld and HAZ is considered for which subscript '1' refers to the weld and subscript '2' refers to HAZ) decreases with increases in  $n_1$  or  $B_1$  or  $E_1$  or  $E_2$  but it increases with increases in  $n_2$  or  $B_2$  and vice versa. On the other hand,  $\sigma_2$  (the equivalent von Mises stress at point 2 or in HAZ) increases with increases in  $n_1$  or  $B_1$  or  $E_1$  or  $E_2$  but it decrease with increases in  $n_2$  or  $B_2$  and vice versa. Also, it was shown that as  $\sigma_1$  increases,  $\sigma_2$  decreases and vice versa, i.e., the stresses are transferred from one grade of material to the adjacent grade material.

## **ACKNOWLEDGMENT**

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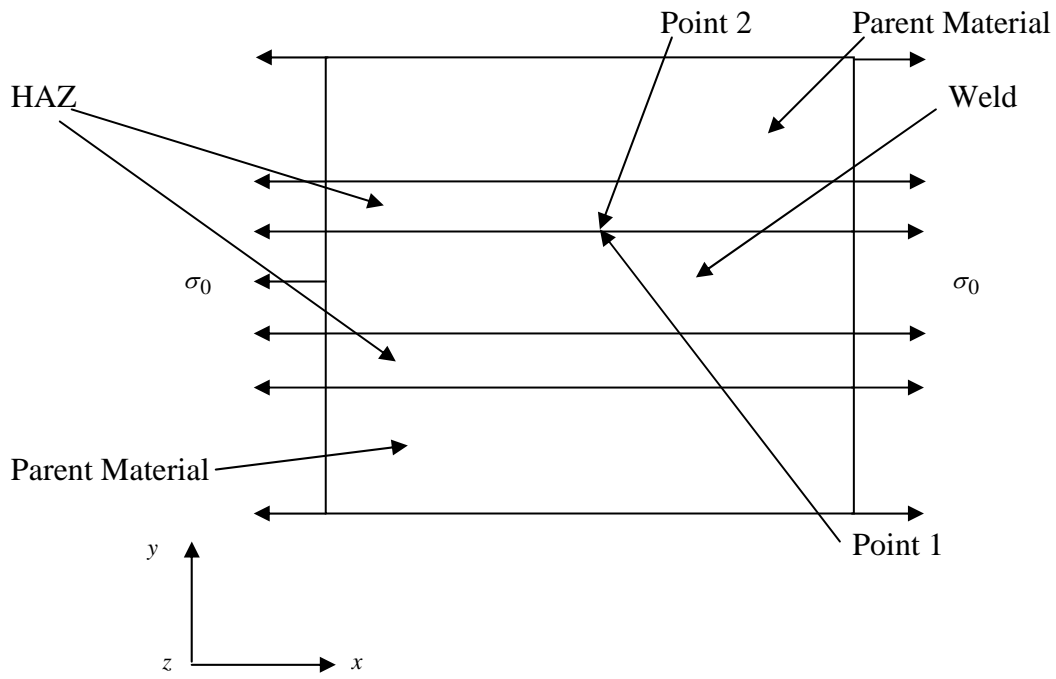


Fig. 1 - Uniformly loaded welded plate.

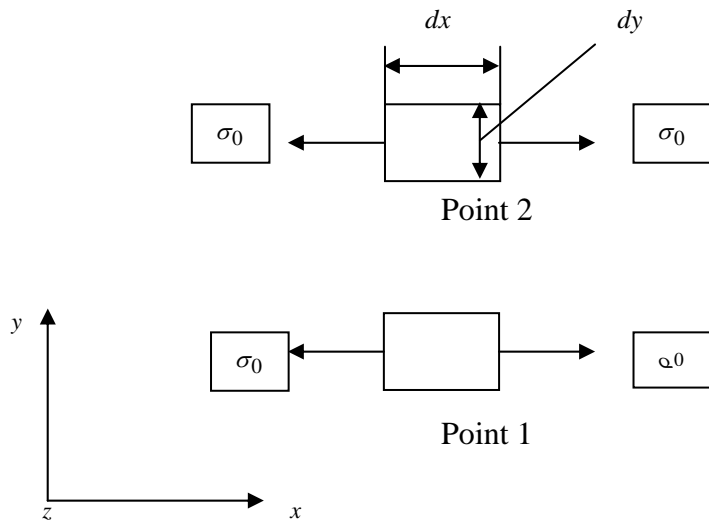


Fig. 2 - Differential elements corresponding to points 1 and 2 at  $t = 0$ .

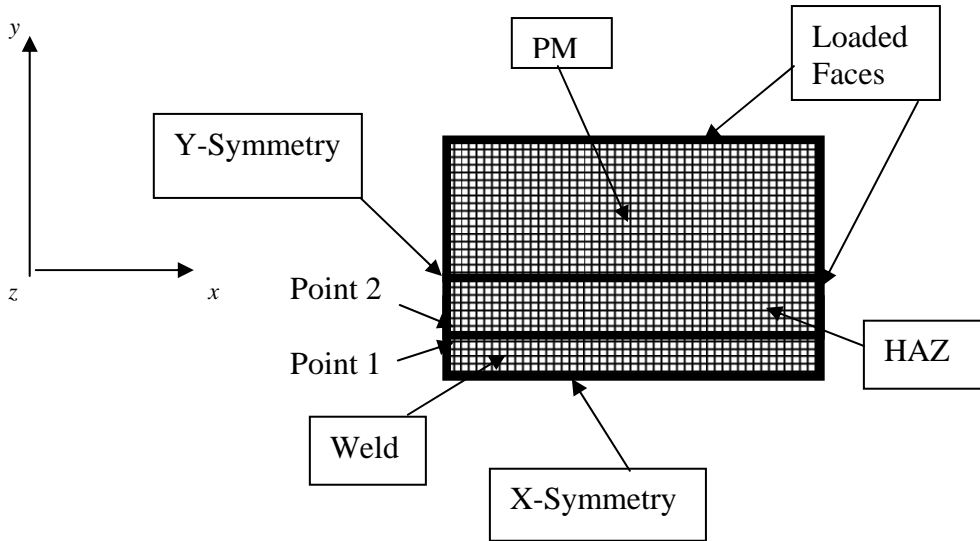


Fig. 3 - Finite element mesh of welded plate.

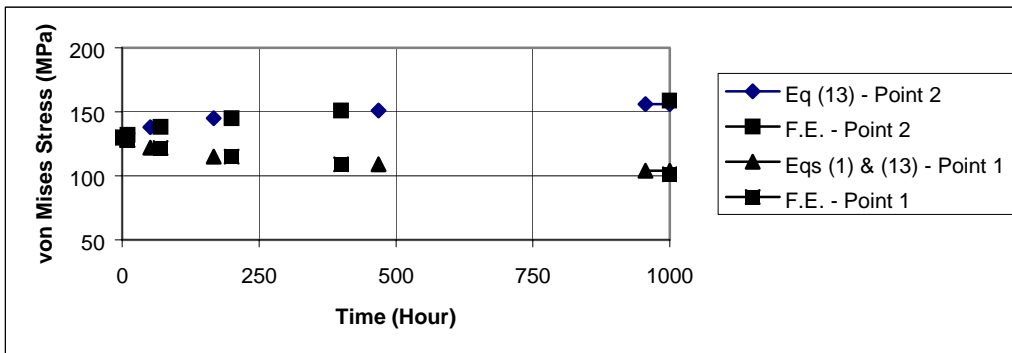


Fig. 4 - Comparison of stresses at Points 1 and 2 predicted by the proposed model and F.E.

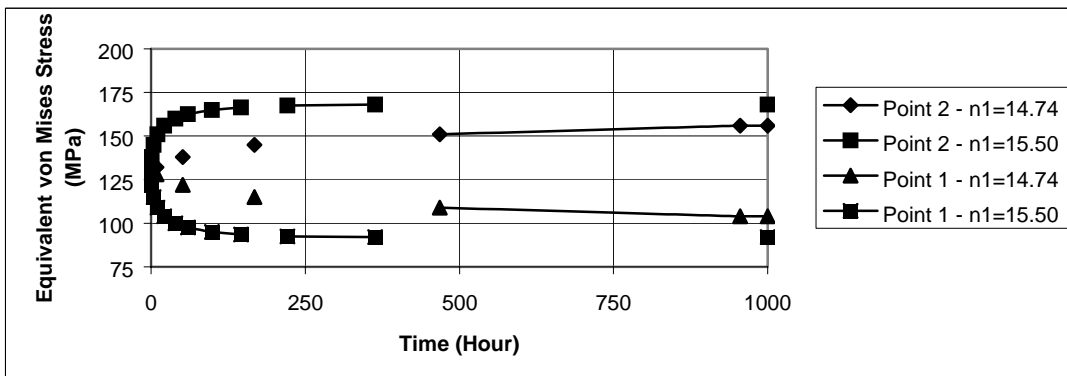


Fig.5 - Variation of stresses at Points 1 and 2 with  $n_1$ .

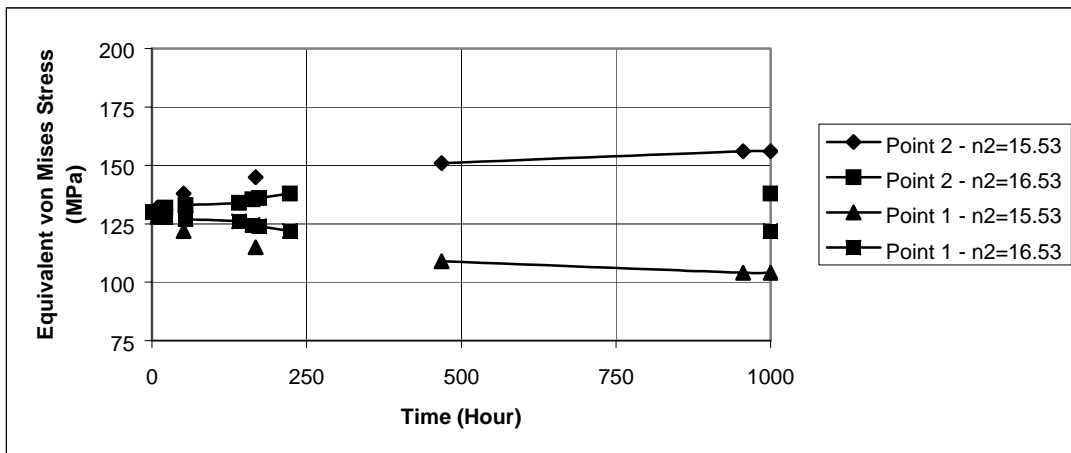


Fig. 6 - Variation of stresses at Points 1 and 2 with  $n_2$ .

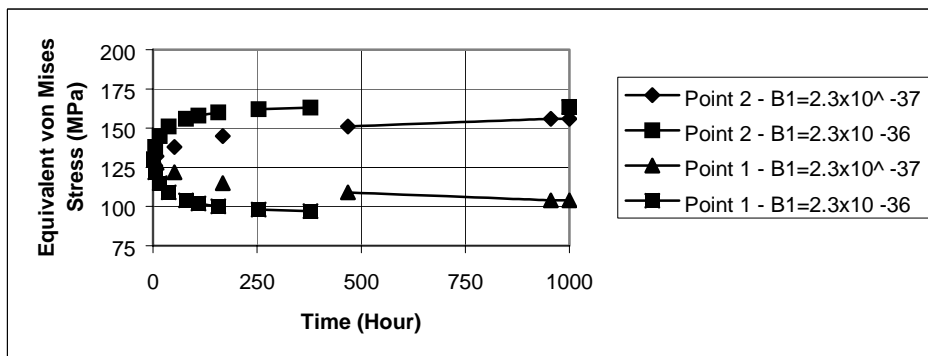


Fig. 7 - Variation of stresses at Points 1 and 2 with  $B_1$ .

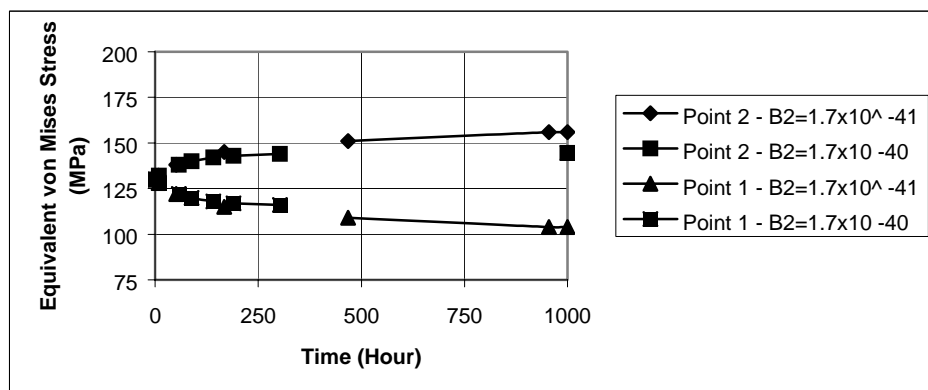


Fig. 8 - Variation of stresses at Points 1 and 2 with  $B_2$ .