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The Influence Of Elasticity, Temperature And Fracture On Large Scale Geological Flow

Hans-Bernd Muhlhaus[(1,2)], Matt Davies[(1)], Lutz Gross[(1)], Louis Moresi[(3)]
[(1)] Earth Systems Science Computational Centre, The University of Queensland, St Lucia, QLD 4072
[(2)] CSIRO Division of Exploration and Mining, 26 Dick Perry Ave., Kensington WA 6051
[(3)] School of Mathematical Sciences, Monash University, PO Box 28M, Clayton, VIC 3800

ABSTRACT: Realistic simulations of earth processes such as faulting, shearing, magma flow, subduction and convection often require the consideration of non-Newtonian Effects such as elasticity and power law creep. As the deformations involved in geological deformation are often large the constitutive relationships must maintain certain geometric terms to ensure that the tensor properties of the model are conserved. A model with such properties is termed as objective. There are a wide range of objective, visco-elasto-plastic models to choose from. The main structural difference between these models consists in the choice of the objective stress rate, e.g. Jaumann, Oldroyd, Truesdell - rates (see Kolymbas and Herle, 2003, for a recent discussion).

In this paper we give an outline of a thermo-visco-elastic-plastic model including a discussion of numerical aspects such as the derivation of a consistent incremental form. The viscous part of the deformation involves a combination of both Newtonian and power law creep. Plastic deformations are described by means of a standard Prandtl-Reuss flow rule combined with a von Mises yield criterion. In planetary scale flow modeling the yield criterion is required as a stress limiter during episodic events e.g. in connection with the initiation of subduction. The temperature sensitivity of the viscous deformation is considered by means of an Arrhenius relation involving a pressure dependent reference (melting) temperature.

The salient features of the model are first explored by means of analytical and numerical solutions of a simple shear problem for an infinite strip with fixed and prescribed shear velocities on the bottom and top of the layer respectively.

The model has been implemented into the finite element code FINLEY consisting of two main parts: (i) a Python-powered scripting facility (EScript) for the formulation of the governing equations, and
(ii) a parallelized matrix assembly and solution engine.

The relative role and importance of elasticity, Newtonian creep, power law creep and temperature dependence of the rheological parameters are explored by 2D finite element simulations of natural convection. We consider a quadratic domain with zero normal velocities on the boundaries, fixed temperatures on top and bottom and zero heat flux on the sides. The different cases are compared in terms of the time histories of the Nusselt number and the histories of fractions of the elastic- and viscous mechanical powers. For instance

$$\frac{1}{V}\int_V \left(\frac{\tau^2}{\eta_N}\right) dV \;/(Nu-1)$$

is the average power fraction of the Newtonian part of the deformation; τ is the second deviatoric invariant of the stress tensor. In the case considered here *Nu-1* is the total mechanical power.

In the simple shear study we compare the shear stress- shear strain curves for a constant applied shear strain rate, assuming infinitesimal theory (no co-rotational stress terms), Jaumann and Naghdi models . It turns out that for a Jaumann model combined with Newtonian creep (Maxwell model) at relatively high values of the Weissenberg number (*Wei*=applied strain rate times viscosiy/shear modulus) the stress strain curve has a peak followed by strain softening until under increasing strain a steady state is reached. This behavior is observed at *Wei*> 1. This kind of geometric softening is also present if Truesdell's definition of the co-rotational terms is used. This softening behavior is the reason why Maxwell models are still considered a challenge in the CFG community in connection with polymer flow simulations. Geometric softening is not present in the Naghdi and the infinitesimal deformation model. An important conclusion of this study is that geometric softening disappears in all models if a stress limiter in the form of power law creep or a yield criterion is included in the model. Provided, of course, that the transition stress or yield stress is low enough

to prevent the stress entering the softening regime. The latter is the case in rocks and metals but not necessarily the case in polymers.

In natural convection with temperature and pressure insensitive parameters, including elasticity and combined Newtonian and power law creep it turns out that elasticity is unimportant for realistic values of the Weissenberg number. The Weissenberg number is defined as $Wei = <\eta>/(t_{thermal}\mu)$, $t_{thermal}$ is the time scale of thermal diffusion used in convection simulations, $<\eta>$ is the average effective viscosity at steady state and μ is shear modulus. For mantle convection conditions, $Wei<10^{-4}$. However elasticity does have an effect if the creep parameters are strongly temperature dependent.

The speed and efficiency of the solution scheme is determined crucially by the way in which the constitutive nonlinearities are handled. The presence of nonlinearities requires sub-iterations within each time step. There are two possible formulations: the secant and the tangent method. The tangent method is more complex and requires the derivation of a consistent incremental form of the constitutive relationships. We have compared three cases for an effective Rayleigh number of $Ra= 10^6$: (1) secant method with iterations, (2) tangent method with iteration, (3) tangent method without iterations. The secant method requires about 4-5 sub-iterations per step; the tangent iteration require 1-4 iterations whereby more then one iteration was only necessary around the first few time steps. Thereafter there was virtually no difference between the tangent methods with or without iterations. The CPU time for the tangent method without iterations was about one-third that of the secant method with iterations.

The role of elasticity is explored in connection with natural convection simulation including elasticity, combined Newtonian and power law creep as well as rate independent plasticity. It turns out that elasticity as well as rate independent yielding and strong temperature sensitivity of the viscosity are key ingredients for persistent transient convection at realistic (earth-like) material parameters.

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