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A Non-Destructive Crack Detection Technique Using Vibration Tests

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ABSTRACT: A method of structural damage detection using non-destructive vibration test is presented. It uses the Frequency Response Function (FRF) data and a finite element model of the virgin structure to construct and display Damage Location Vector (DLV). It is shown that DLV can detect, locate and assess the extent of damage. By using the raw FRF data and its wealth of information, DLV can handle experimental noise and the inherent incompleteness of data.

1 INTRODUCTION

It is desirable to be able to detect structural damage invisible to the naked eyes at an early stage, preferably by non-destructive testing (NDT) methods. Current NDT techniques employed such as ultrasound, X ray, dye penetrants, magnetic particle and acoustic emission, are often limited to observation in a limited area and rely on a presumption of the likely area of damage. Structural damage detection using non-destructive vibration test data has received considerable attention since early 1990s [Zimmerman and Smith, 1992]. The underlying principle is that damage will change the dynamic characteristics such as natural frequencies, damping loss factors and mode shapes. For early damage, it is reasonable to assume that the mass is constant hence the mass matrix remains unchanged and only the stiffness matrix is subject to change induced by damage. Most of the works on structural damage detection carried out so far use modal analysis data, [Doebling *et al*, 1996; Ren and De Roeck, 2002] which are processed from FRF data, the most compact form of data obtained from vibration tests of structures. Such methods are based on algorithms involving extensive numerical computing including curve fitting techniques. In recent years, a different approach has been proposed to use measured FRF data directly to detect structural damage [Choudhury, 1996; Hwang and Kim, 2004]. Unlike modal data, FRFs can be retrieved without further numerical processing and hence without the associated errors and loss of information as modal data are. The FRF data also provide abundance of information at all measured degree of freedom (DOF) and at a great number of sampling frequencies, much of these would have been lost in using modal analysis data.

FRFs data can also be obtained from the so called spatial model of the structure comprising of the matrices of stiffness [K], of mass [M] and of damping [C] of the structure. These matrices can be obtained from an analytical model for a discrete system but are mostly assembled by numerical techniques such as finite element method (FEM). Both the modal models (natural frequencies, corresponding mode shapes and damping loss factors) and the FRFs data can then be found. However FEM is practically limited to the virgin structure as the damaged structure is not yet known. In many practical problems requiring structural damage detection, properties of the virgin structure are often not available, there fore either its FRF data have to be determined from an FEM model or they should have been acquired and stored, a foresightedness rarely found in practice. The experimental modal analysis approach [He and Fu, 2001], using vibration tests in which excitation and response signals are recorded, can deal with both the virgin and damaged structure. The FRF data of an experimental model, or grid, can be processed and stored, albeit the grid is very much simplified compared to an FEM model. Inherently, crack detection using vibration tests faces two challenging problems: the effect of noise and coordinate incompatibility. The former covers inaccuracy in signals and data due to many sources, among them random noise of the electronic measuring system. The second is caused by the fact that the experimental model is inadvertently much coarser than the FEM models, leading to what is called coordinate incompatibility between the experimental model and the FEM model. This is also known as incompleteness of experimental data. The method of Damage Location Vector (DLV) using FRFs obtained from non-destructive vibration testing has shown promise in crack detection and tackling these two challenges, not only for simulated damage and noise but for experimental data as well [Choudhury, 1996; Huynh, 2004]. This paper presents the application of DLV in crack detection in a plate structure. First a brief presentation of the theory of DLV.

2 THEORETICAL BASIS OF DLV

A continuous structure can be modelled as a multi-degree of freedom (MDOF) system. Its equation of motion can be written as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

where {x} is the structural nodal displacement vector, also termed DOF; {F} is the structural nodal force vector, and • represents differentiating with respect to time. In the case of negligible damping, the equation becomes for a virgin structure without damage and the damaged structure,respectively:

$$[M]_{UD}\{\ddot{x}\} + [K]_{UD}\{x\} = \{0\} \tag{1}$$

$$[M]_{D}\{\ddot{x}\} + [K]_{D}\{x\} = \{0\} \tag{2}$$

The subscripts UD, D are used to indicate the undamaged and damaged structure respectively. Damage will be considered to result in a reduction in stiffness only, hence $[M]_{UD} = [M]_D$.

Let $[Z(\Omega)]$ be the dynamic stiffness matrix, $[\alpha(\Omega)]$ be the receptance frequency response function (RFRF) matrix at a particular frequency Ω and the **k**th column of this matrix be denoted by $\{\alpha(\Omega)\}_k$, where:

$[Z(\Omega)] \equiv ([K]-\Omega^2[M])$ and $[\alpha(\Omega)] \equiv ([K]-\Omega^2[M])^{-1}$

then it follows from the orthogonality of the dynamic stiffness matrix :

$$[Z(\Omega)]_{UD}\{\alpha_{UD}(\Omega)\}_k = \{\delta\}_k = [Z(\Omega)]_D\{\alpha_D(\Omega)\}_k \tag{3}$$

where $\{\delta\}_k$ is a vector, the elements of which are zero except the **k**th element which is equal to 1. The dynamic stiffness matrix before and after damage can be related as follows:

$$[Z(\Omega)]_D = [Z(\Omega)]_{UD} - [\Delta Z(\Omega)] \tag{4}$$

where $[\Delta Z(\Omega)]$ represents the difference in dynamic stiffness matrix between the damaged and undamaged structure. Equation (3) can be rewritten as:

$$[Z(\Omega)]_{UD}\{\{\alpha_D(\Omega)\}_k - \{\alpha_{UD}(\Omega)\}_k\} = [\Delta Z(\Omega)]\{\alpha_D(\Omega)\}_k \tag{5}$$

Define a damage location vector (DLV) as:

$$\{d(\Omega)\} \equiv [\Delta Z(\Omega)]\{\alpha_D(\Omega)\}_k \tag{6}$$

and let $\{\Delta\alpha(\Omega)\}_k \equiv (\{\alpha_D(\Omega)\}_k - \{\alpha_{UD}(\Omega)\}_k)$ be the difference of the **k**th column vectors of the response frequency function matrix of the damaged structure and undamaged or virgin structure, then Equation (5) can be rewritten as:

$$\{d(\Omega)\} = [Z(\Omega)]_{UD}\{\Delta\alpha(\Omega)\}_k \tag{7}$$

It can be seen from equation (6) that the **j**th element of $\{d(\Omega)\}$ would be nonzero if the **j**th row of $[\Delta Z(\Omega)]$ is nonzero as a consequence caused by structural damage. In another word, a damage to an element would affect [K] and in turns, [Z] and $[\alpha]$, hence a non-zero element of DLV points to the DOF associated with damage and a zero value would indicate no associated damage. Thus the DLV reflects the change in the response brought about by structural damage, it also points to the DOFs that have been affected by the damage hence the name *damage location vector*. These affected DOFs would then point to the damaged element. DLV can be evaluated at any particular frequency by equation (7). In summary DLV requires:

- A spatial model in the form of mass matrix [M] and stiffness matrix [K] that describes the structure when it is free of damage. These matrices are usually provided by an FEM model of the undamaged structure.
- A column of measured FRF data from vibration test denoted as $\{\alpha_D(\Omega]\}^R$.

Here Ω is an arbitrary frequency and "R" stands for reduced since the measured FRF data in the vector $\{\alpha_D(\Omega]\}^R$ would come from a limited number of DOFs which are merely a subset of the DOFs used by the FEM model. There are two main reasons for this incompleteness of data: the first is the difference in DOFs used in the FEM model and the experimental grid, namely FEM model usually has far more elements and DOFs; the second is not only the experimental model is of lesser number of nodes, at each node out of six possible DOFs that should be measured, only one DOF, usually of linear acceleration, is available. That is not to mention that due to limitation in physical resources and testing time, it is very often that, at best, the measured DOFs constitute only one column of the response function matrix. Furthermore the frequency range covered in measurement is limited, any attempt, even a judicious one, to widen the range is usually made at the expense of frequency resolution. In short, the experimental model is far much coarser compared to the FEM models, leading to what is called coordinate incompatibility between the experimental model and the FEM model, also known as incompleteness of experimental data. To overcome this incompatibility, some mathematical techniques have been used to expand the DOF of the experimental model to reconcile with the DOF of the FEM model to obtain the full set of DOF. The technique adopted here is called Dynamic Expansion Method A [Choudhury, 1996], which is briefly described here: first the full set of DOFs is partitioned into the measured ones or master DOFs and the unmeasured ones or slave DOFs. The structural stiffness and mass matrices [K] and [M] are then partitioned accordingly to Kmm, Kms, Ksm, Kss and Mmm, Mms, Msm, Mss, in a similar fashion as carried out in Guyan reduction technique of structural dynamics, where subscript m stands for master DOF and s for slave DOF. The unmeasured set of FRF is obtained from the measured set by:

$$\{\Delta\alpha_s\}_k = -[\{[Kss]-\omega^2[Mss]\}^{-1}\{[K_{sm}]-\omega^2[Msm]\}]\{\Delta\alpha_m\}_k \qquad (8)$$

This set is then adjoined to the measured set to obtain a complete set of DOFs, hence the term "Dynamic Expansion". Equation (8) involves considerable computing and is best handled by programming or by using software like MATLAB.

To facilitate the identification of the affected DOF, the evaluation of DLV is carried out over a chosen range of frequency, followed by plotting DLV in a three-dimensional plot, in which the first axis represents frequency values corresponds to the frequency range over which the damage location algorithm is applied to, the second axis is the numbered DOFs of the structure and the third axis represents the absolute value of the of elements of DLV, each corresponds to one DOF. This three-dimensional plot is called Damage Location Vector Plot (DLVP). Another way to display damage is by adding data corresponding to a particular DOF at all frequencies together, i.e. the second axis shows the values of DLV obtained at all frequency sampling points added together to indicate cumulative damage. This two-dimensional plot is called cumulative damage location plot (CDLP). This method to detect structural damage is called Damage Location Vector (DLV). Frame and truss structures have been studied [Huynh et al, 2003, Tung and Tran, 2003], showing that DLV can detect, locate and assess the extent of damage. In this paper DLV is applied to real physical damage and experimental noise in a plate structure.

3 DAMAGE DETECTION IN A PLATE STRUCTURE

The structure is a rectangular steel plate of 0.3 by 0.6 m, of 3 mm thick, density of 8179 Kg/m^3 and Young's modulus of 200 GPa. It is fixed at one of the short edge.

In this study, [K] and [M] are found from the Discrete Kirchoff's Theory (DKT). For a single rectangular thin plate element, [K] and [M] were derived from first principles by [Przemieniecki, 1967]. It has been shown that these matrices perform as well as a more complex plate element (SHELL63) of six DOFs per node provided by ANSYS software. A meshing of 18 elements, each of 0.1 m square was used [Huynh, 2004], Figure 1. The assembling of the structural mass and stiffness matrices of the plate structure was carried out using an algorithm written in MATLAB. Three cases of physical damage in the form of saw cuts were studied: case A with damage in element 11, an internal element; case B in element 16, a boundary element; and case C, a multiple site damage with damages in both elements 11 and 16.

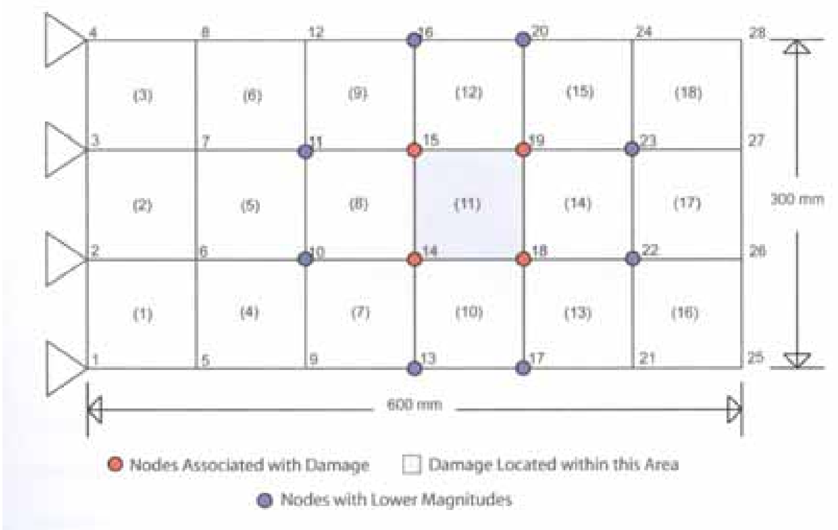

Figure 1: FEM model, also showing damaged element found by DLV for Damage Case A

The plates were tested in vibration tests using impact hammer to obtain FRF curves through a Bruel & Kjaer Frequency Spectrum Analyser and ICATS, a modal analysis software developed by Imperial College, London. In these tests, an accelerometer was attached to the plate at a fixed node to measure the response of lateral linear acceleration and the hammer was moved around the plates, hitting the plate at all nodes in turn, each time an average of five readings was recorded. These experimental FRF data inherently contained both real noise and coordinate incompatibility, as only 24 lateral accelerations were measured out of 72 DOFs of the FEM model using DKT element. The experimental FRF data were expanded using the Dynamic Expansion A method to obtain the full set of one column of the FRF matrix as previously presented in Section 2.

3.1 Effect of experimental noise

In order to see the effect of real noise on the DLV algorithm FRFs were also obtained for the virgin structure and were fed into the DLV algorithm which should show a zero DLV if there were no noise nor coordinate incompatibility. The corresponding CDLP plot is shown in Figure 2, clearly indicating that the effect of noise is randomly distributed over the frequency range and DOFs.

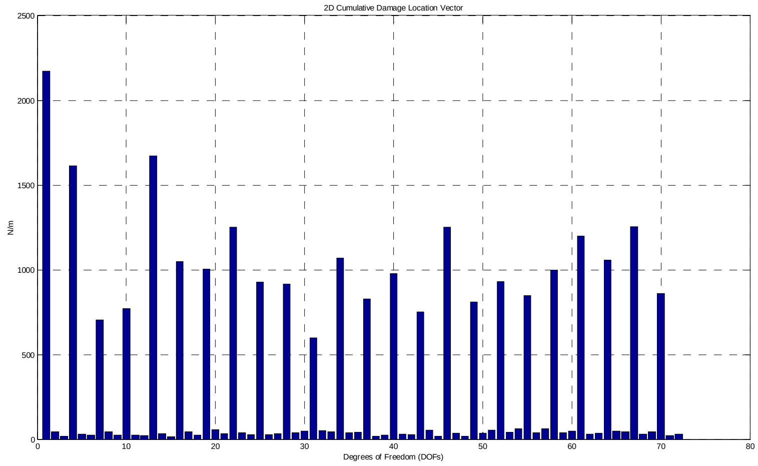

Figure 2: CDLP due to experimental noise in undamaged plate (vertical scale magnified)

From Figure 2, the maximum magnitude of elements of DLV was found to be of order 2000 units out of initial values of order of 2 millions units, i.e. the effect of noise is negligible, of order 0.1%. DLV was tested with simulated noise level of 5% and was found to perform well [Huynh, 2004].

3.2 Experimental Study of Damage Case A

The damaged plate of Case A is shown in Figure 3. DLV was applied to experimental FRF data, yielding the DLVP and CDLP shown in Figures 4 and 5 respectively. The CDLP shown in Figure 5 clearly shows four prominent peaks associated with DOFs 28, 31, 40 and 43 pointing to nodes 14, 15, 19, and 18 (shown as red dots in Figure 1), which identify correctly element 11 as the damage site. There are also eight less dominant peaks pointing to nodes 10, 11, 16, 20, 23, 22, 21, 13, 17 shown as blue dots. These nodes, in the immediate neighbourhood of element 11, are also affected by damage, albeit the effect is of second order (20% of that inflicted on nodes of the damaged element): the effect decays very quickly with distance from the damage.

Figure 3: Damage introduced at element 11

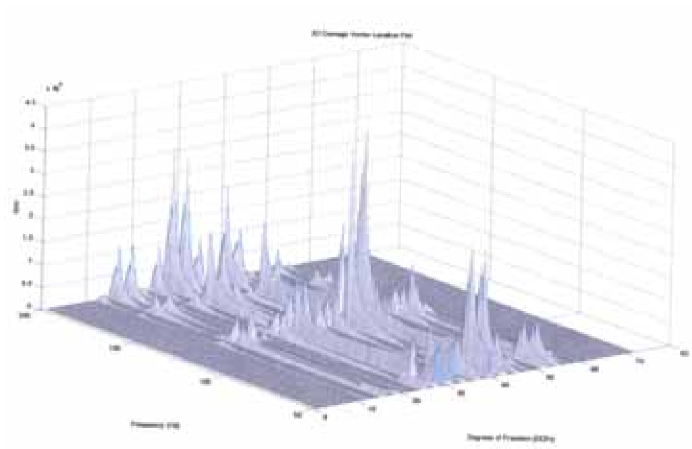

Figure 4: DLVP for Damage Case A

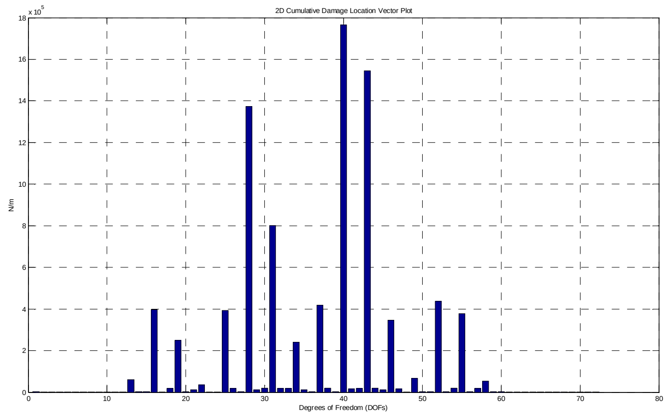

Figure 5: CDLP for Damage Case A

3.3 Experimental Study of Damage Case B

The resulting CDLP is shown in Figure 6. Similarly by inspecting the CDLP and referring to the element and node numbering scheme, it was found that DOFs 49, 52, 61 and 64 were most affected, pointing to nodes 21, 22, 25 and 26, shown as red dots, which identify element 16 as the damaged location, as shown in Figure 7. It should be noted that at DOFs 37, 40, 55 and 67 have magnitudes of order 20% or less of the above dominant DOFs. These DOFs point to nodes 17, 18, 23, 27, which are neighbouring nodes of element 16 leading to similar conclusion as found in Section 3.1.

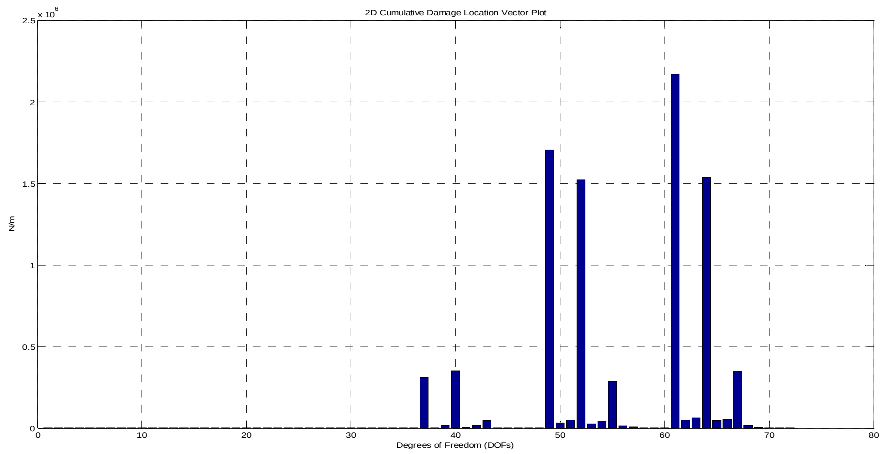

Figure 66: CDLP for Damage Case B

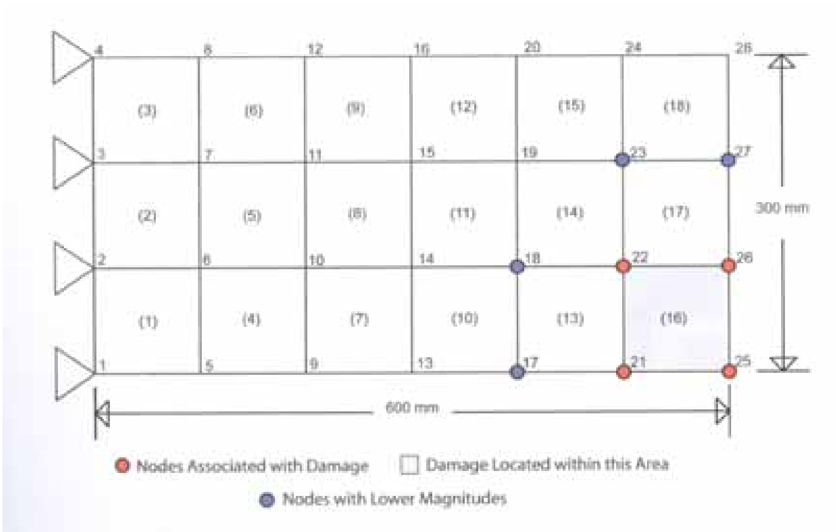

Figure 7: Damaged element found by DLV for Damage Case B

3.4 Experimental Study of Damage Case C

The CDLP of the multi-site damage case C is shown in Figure 8. It can be seen that the effect of noise is again random and negligible but there are now peaks of various magnitudes. The first seven highest values can be identified as in the immediate area of possible damage location, they are then mapped in red dots to their locations (element 16) on the plate as shown in Figure 9, the remaining DOFs with relatively smaller magnitude have been mapped in blue dots. Damage also exists at element no. 11, as three of its nodes are red and one, node 15, is blue. However, it may not be argued that element no. 14 might also contain damage: had it indeed contained damage then the magnitude of its DOF at node 23 would have been much larger. Thus it can be seen that DOFs in the immediate neighbourhood of an element containing damage are also slightly affected, but the effect decays quickly for nodes further away as evidenced by the fact that no damage was detected for element 1, 2 and 3 as expected.

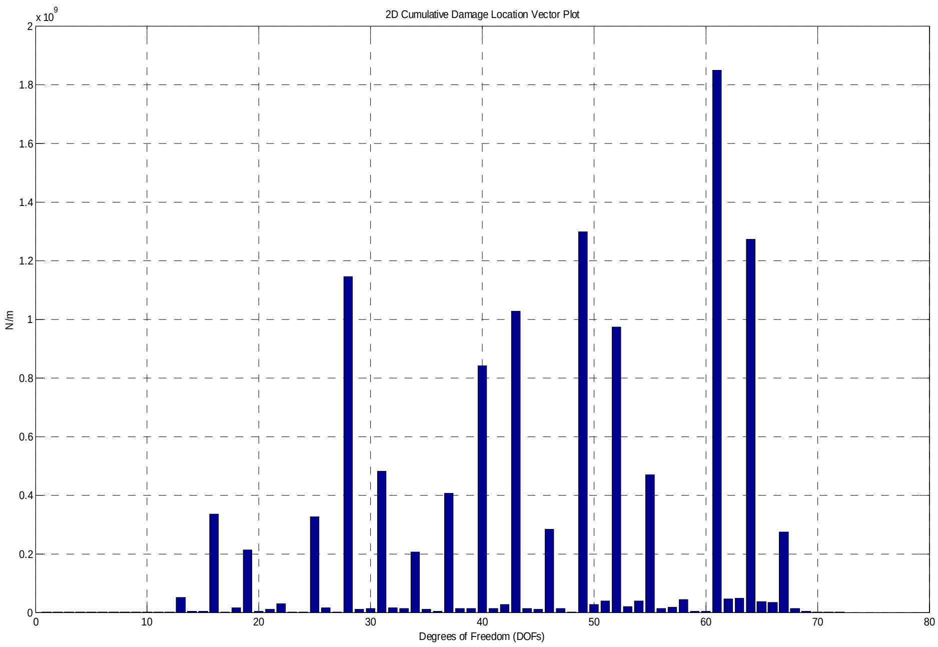

Figure 8: CDLP for Damage Case C

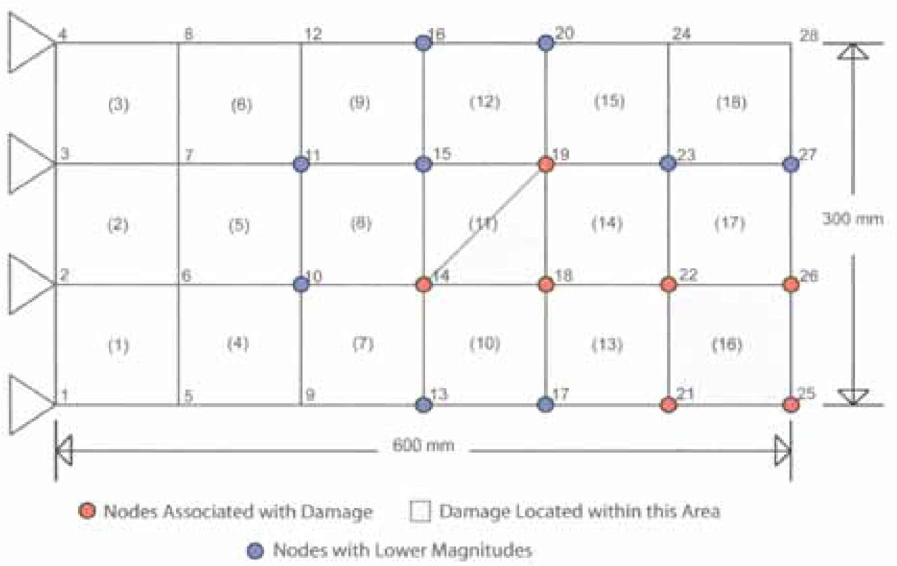

Figure 9: Damaged elements found by DLV for Damage Case C

4 CONCLUSION

By using the abundance of FRF data, DLV can overcome the problem of noise and coordinate incompatibility between the FEM model and the experimental model. It was found that the effect of experimental noise on damage detection by DLV was negligible and randomly distributed, and DLV is capable even when only one third of the FRFs could be obtained by experiments. It was also found that in the case of a plate structure, the effect of damage in an element is felt primarily by its nodes and to some lesser extent by the neighbouring nodes. Further investigations are needed to see how sharp a crack that DLV can detect and to provide a more automatic scheme of identifying damaged elements in the case of multi-site damage.

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