

An Analysis Of Fatigue Crack Growth Of A Notched Aircraft Component Under Compression-Dominated Spectrum Loading

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ABSTRACT: In engineering structures, fatigue cracks often emanate from geometrical discontinuities such as holes and notches. Experimentally it has been observed that there exists a notch-affected zone, in which the crack growth exhibits a transitional behaviour. Depending on the loading level and the geometry of the notch, the crack growth rate may initially decrease with crack length to reach a minimum. It will then either grow at an accelerated rate, or stop growing. This report details ongoing work in modelling crack growth in the presence of notch plasticity. The local stress-strain distribution ahead of a notch root is determined based on an empirical distribution of the equivalent stress, and the evolution of the notch root stress and strain is calculated using Neuber's rule and an Armstrong-Chaboche type nonlinear kinematic hardening model. The stress intensity factor is then calculated using a Green's function approach. A crack growth analysis program has been developed, implementing the above procedures.

1 INTRODUCTION

Engineering structures subjected to cyclic load often fail by fatigue at geometrical discontinuities such as holes, notches and other sudden shape changes. To analyse the durability and damage tolerance of such components it is important to know the stress distribution near the notch. This is different from the traditional static strength analysis where only the notch root stress is required, and where the component is deemed to have failed if that stress exceeds the material yield stress. For components managed according to the philosophy of damage tolerance such as aero-structures, the yielding of material at the notch root is permitted, as long as the stress does not cause unstable crack growth and any stable crack propagation can be accurately predicted. If the notch root plastic zone size is negligible compared with the crack size, the normal engineering practice is to apply linear elastic fracture mechanics (LEFM), and the crack growth correlating parameter, *e.g.*, ΔK_{eff} is calculated using remote stress and various approximate formulas. However, there are cases when the plastic zone size is large compared with the crack size. One example is the cold proof load test of the wing pivot fitting of F111 aircraft, in which significant plastic deformation takes place near geometric discontinuities such as the fuel flow vent holes.

It is, therefore, important to determine the stress distribution as well as the evolution of the stress field near a notch root. A considerable amount of work has been carried out to determine the notch root stress, but relatively fewer studies have been performed on the elastic-plastic stress distribution near a notch root. Ball [1990] proposed a method of using the elastic stress distribution and a generalized Neuber's rule to determine the notch root stress field, but the method suffers from the fact that Neuber's rule was used to calculate the elastic-plastic response stress at a distance from the notch root. Based on the results of extensive finite element studies, Wang et al [1999] developed a procedure for determining the elastic-plastic stress distribution for static loading.

In this paper, a general procedure for the determination of notch root stress field is presented. The notch root stress and its evolution under cyclic loading are determined using Neuber's rule and a nonlinear kinematic constitutive model. The method developed by Wang et al [1999] is extended to cyclic loading, and modified to ensure the continuity of stress and load constancy. A detailed numerical procedure is presented, and preliminary numerical results are used to demonstrate the potential of the procedure developed.

2 NEAR-NOTCH STRESS DISTRIBUTION

2.1 The material constitutive model

The Armstrong-Chaboche[1986] type constitutive model is chosen to model the notch plasticity, because of its capability to model kinematic and isotropic hardening, and capturing transient behaviour such as strain ratchetting and mean stress relaxation[Chaboche 1986]. Although mathematically complicated, its implementation has proved to be rather robust [Hu et al. 2001; Hu and Wang 2003]. In the case of uniaxial loading, the model may be summarised by the following set of equations, representing the yield surface, the evolution rule for the back stress and the isotropic hardening,

$$\nu(\sigma - X) - \sigma_y - R \leq 0 \quad (1)$$

$$\dot{X} = C\dot{\varepsilon}^p - \gamma X\dot{p} \quad (2)$$

$$R = R_s(1 - e^{-bp}) \quad (3)$$

where σ is the response stress, X the back stress, σ_y the uniaxial yield stress. C and γ are two material constants to be determined from the cyclic stress-strain curve of the material, ε^p and p the plastic strain and the equivalent plastic strain, respectively. In Eqn (1) R represents the effect of isotropic hardening, and it is described by the function defined in Eqn (3), in which R_s and b are two material constants. The symbol ν takes the value of +1 or -1 depending on the load direction. It should be pointed out that isotropic hardening can be turned off by setting $R_s = 0$, but for some materials it may be necessary to keep isotropic hardening to represent the experimental data accurately. Assuming proportional loading, the constitutive model can be readily applied for multi-axial loading.

2.2 Notch root stress and strain

The notch stress-strain response can be determined by using Neuber's rule and the constitutive model outlined above. Denoting the Young's modulus of the material by E and the far field strain by Δe , Neuber's rule [Neuber 1961] states that the response stress range $\Delta\sigma$ and strain range $\Delta\varepsilon$ satisfies

$$\Delta\sigma\Delta\varepsilon = K_t^2 E(\Delta e)^2 \quad (4)$$

$$\sigma = E(\varepsilon - \varepsilon^p) \quad (5)$$

Therefore, for a given increment in remote load, the local stress and strain response can be determined from the constitutive equations and Neuber's rule. In fact, the above two equations can be combined to give the following implicit function of plastic strain ε^p , which can be solved numerically using Newton-Raphson's method,

$$f(\varepsilon^p) = \nu(\sigma(\varepsilon^p) - X(\varepsilon^p)) - R(\varepsilon^p) - \sigma_y = 0 \quad (6)$$

The stress and strain ranges can then be calculated from Eqn (4) and (5) above.

2.3 Near-notch stress distribution

To be specific, let us consider a specimen with a central hole in a plate. In this case, the elastic stress distribution is known [Wang et al. 1999], with a stress concentration factor of about 3 at the hole edge (the theoretical value of 3 is achieved when the width of the plate is very large). If the

applied load is high enough to cause yielding at the edge, then elastic-plastic stress will be bounded by the yield criterion. To determine the distribution quantitatively, we need to know the plastic zone size, the distribution of stress within the plastic zone and that outside the plastic zone.

2.4 Elastic-plastic response

Based on finite element analysis, it has been proposed that the equivalent stress range would follow a relation described by the following function,

$$\frac{\Delta\sigma_{\text{eq}}}{\sigma_0} = \frac{A}{x + \alpha} \quad (7)$$

where A and α are constants to be determined from the consideration of stress continuity at the elastic-plastic boundary, and the load balance, and σ_0 is either the flow stress for the first half cycle, or twice the proportional limit for subsequent yielding. The two constants can be determined as follows.

First, at the elastic-plastic boundary $x = x_p$, we have (ignoring isotropic hardening),

$$\Delta\sigma_{\text{eq}} = \sigma_0 \quad (8)$$

Therefore we have

$$A = x_p + \alpha \quad (9)$$

where x_p is the plastic zone size. Substituting this into (8), we have

$$\frac{\Delta\sigma_{\text{eq}}}{\sigma_0} = \frac{x_p + \alpha}{x + \alpha} \quad (10)$$

At the notch root, the elastic-plastic response stress $\Delta\sigma_{\text{eq}}$ can be determined using Neuber's rule. Therefore, we have the following relation at the notch root

$$\frac{\Delta\sigma_{\text{eq}}(0)}{\sigma_0} = \frac{x_p + \alpha}{\alpha} \quad (11)$$

Solving for α , we obtain the following relationship between α and x_p

$$\alpha = \frac{x_p \sigma_0}{\Delta\sigma_{\text{eq}}(0) - \sigma_0} \quad (12)$$

Substituting this back to Eqn (8), we have a distribution of equivalent stress that is solely determined by the plastic zone size x_p . Once α is determined, the equivalent stress at any point x can be calculated from

$$\Delta\sigma_{\text{eq}} = \frac{x_p + \alpha}{x + \alpha} \sigma_0 \quad x \leq x_p \quad (13)$$

The parameter x_p can be determined by considering the fact that whether yielding occurs or not, the load carried by the specimen is the same. Therefore,

$$\int_0^W \Delta \sigma_{yy}^E(x) dx = \int_0^{x_p} \Delta \sigma_{yy}(x) dx + \int_{x_p}^W (\Delta \sigma_{yy}^E(x) + \Delta \sigma) dx \quad (14)$$

where $\Delta \sigma$ is the difference in the assumed elastic-plastic stress distribution in the y-direction and the elastic stress distribution at x_p ,

$$\Delta \sigma = \sigma_{yy}(x_p) - \sigma_{yy}^E(x_p) \quad (15)$$

Once the distribution of the equivalent stress is thus determined, the stress σ_{yy} at any position x may be calculated in a similar fashion as detailed in [Wang et al. 1999].

3 THE CRACK GROWTH MODEL

The fatigue crack growth model implemented in FASTRAN [Newman 1992] is based on the crack closure model extensively studied by several researchers [Fühling and Seeger 1979; Newman 1981; Wang and Gustavsson 1987]. This model attempts to simulate the experimental observation made by Elber [1971] that due to the residual plastic deformation left in the wake of a growing crack, the crack remains closed during part of the subsequent tensile loading, and the crack only grows when it is fully open.

In FASTRAN, the crack tip plastic zone and the crack wake are discretized into strip elements. Each element undergoes elastic-perfect plastic deformation under cyclic loading, and based on the amount of crack extension calculated, some crack tip elements are 'broken up' to facilitate crack growth. Due to the permanent plastic deformation in these elements they may come into contact upon subsequent unloading, thus causing crack closure. It should be noted that closure may occur when the remote stress is still tensile. According to the crack closure model, further crack growth can only happen if the external load is such that the crack is fully open, and this external stress is known as the crack opening stress. Hence the effective crack growth driving force is determined by the stress range between the maximum stress and the crack opening stress.

For crack growth with severe notch plasticity, two distinct stages can be considered for the crack growth: (a) the crack tip is engulfed in the notch plastic zone, and (b) the crack tip has grown out of the notch plastic zone. As a first approximation, it is proposed that the effect of notch plasticity is solely reflected in the calculation of K_{\max} in $\Delta K_{\text{eff}} = K_{\max} - K_{\text{open}}$, while K_{open} may be approximated by the existing FASTRAN solution, *i.e.* by using a scaled remote stress (taking into consideration of the stress concentration caused by the notch) to calculate the crack opening stress.

4 CALCULATION OF THE STRESS INTENSITY FACTOR USING A GREENS FUNCTION APPROACH

For the analysis of crack growth, the stress intensity factor needs to be computed for each loading cycle. When the stress distribution ahead of the notch root has been determined by the procedure detailed above, the stress intensity factor at maximum stress can be determined using Green's function approach and Bueckner's principle: the crack tip stress intensity factor for a traction free crack in an externally loaded body is equivalent to that for a crack with an applied pressure distribution in a body with no externally applied loads when the applied pressure is equivalent to the stress field that would exist if the externally loaded body were crack-free. Hence, the basic formula for K is given by

$$K = \int_0^c G(x) \sigma(x) dx \quad (16)$$

where c is the current crack length, $G(x)$ the Green's function and $\sigma(x)$ the stress distribution. It should be pointed out that the stress distribution is calculated disregarding the existence of the crack. For a given remote load S , a different notch configuration leads to a different stress distribution $\sigma(x)$, and a different crack geometry gives different Green's functions. When these functions are established, the above integration may be performed numerically using a globally adaptive scheme based on Gauss-Kronrod rules, as shown, for instance by Press et al [1997].

4.1 Two symmetric through cracks from a hole

The Green's function for two radially symmetric through cracks at a circular hole in an infinite plate was developed by Wu and Carlsson [1991]. For a symmetric but otherwise arbitrarily distributed load system on the crack faces, the stress intensity factor can be found by the Green's function method, Eqn (16). As there is no closed formula for the Green's function, numerical results in terms of a function $F_{1,2}$ over the full range of the two independent, normalized parameters c/R and x/c were developed by Wu and Carlsson [1991], with

$$G = \frac{F_{1,2}}{\sqrt{\pi c}}, \text{ where } F_{1,2} = A_1 + \frac{A_2}{[1 - (x/c)^{A_3}]^{1/2}}$$

and the values of A_1 , A_2 and A_3 are given in tabulated format.

4.2 Numerical example

The above algorithms have been implemented in a crack growth analysis program, CGAP, which is based on the crack growth model of FASTRAN, with an added graphical user interface, a residual stress module and a probabilistic crack growth module, as well as the capabilities detailed in this report. As part of the verification process, the case of two symmetric through-cracks emanating from a central hole in a plate subjected to remote spectrum loading, as shown in Figure 1, has been analysed. This is a simplified representation of a structural component on F111 military aircraft, and the material is D6ac steel. Experiments have shown that crack growth in this component is very slow, if at all, but analytical results using existing software do not correlate well with this observation. One possible cause of the inaccuracy is the calculation of the stress intensity factor using the remote stress, and the current approach attempts to make improvements in this calculation. Preliminary results, as shown in Figure 2, appear promising; where the solid and dashed lines represent, respectively, the crack growth using the current approach and the LEM approach. The current approach predicts significantly slower crack growth, as expected.

5 CONCLUSION

A numerical model has been developed to predict the near-notch elastic-plastic stress distribution, which is then implemented in a crack growth analysis program to model the crack growth in the notch-plasticity affected zone. Preliminary results show an encouraging trend in terms of crack growth rate, in comparison with the remote stress approach.

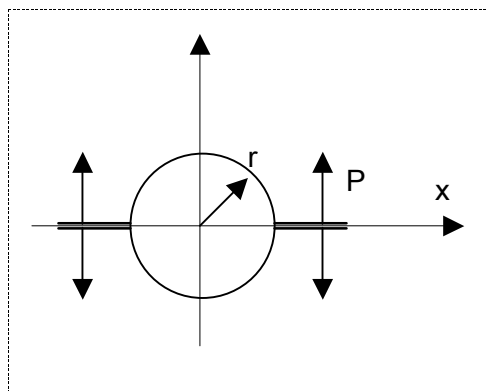


Figure 1 Two symmetric through-cracks emanating from a hole under remote spectrum loading.

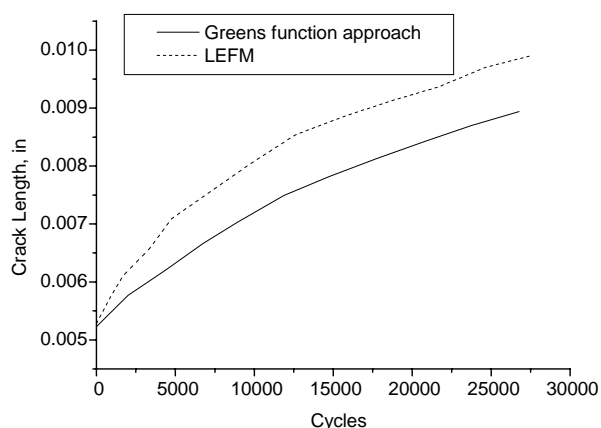


Figure 2 Crack growth curves predicted using (1) remote stress approach and (2) local stress approach.

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