SIF2004 Structural Integrity and Fracture. http://eprint.uq.edu.au/archive/00000836

Mechanisms Of Fracturing In Structures Built From Topologically Interlocked Blocks

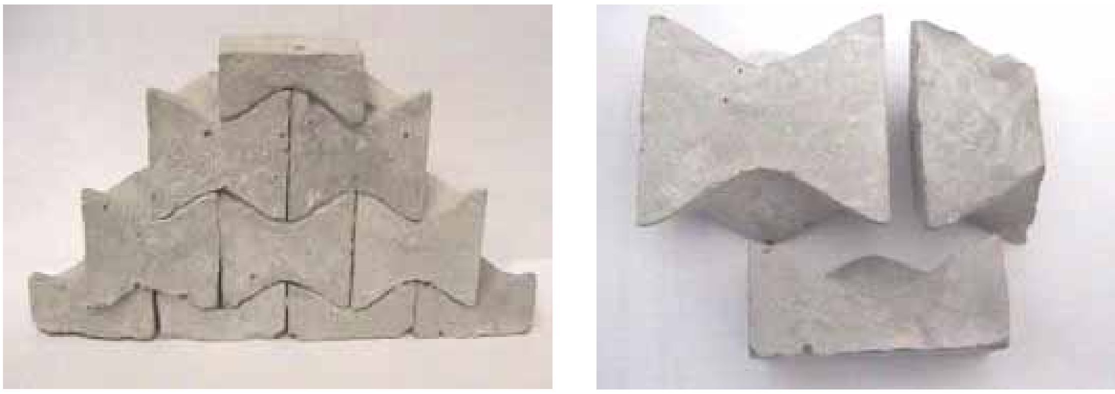
H.C. Khor(1), A.V. Dyskin(1), Y. Estrin(1,2), E. Pasternak(1)
(1) School of Civil and Resource Engineering, The University of Western Australia, 35 Stirling Highway, Crawley WA 6009, Australia
(2) Institut für Werkstoffkunde und Werkstofftechnik, Technische Universität Clausthal, D-38678 Clausthal-Zellerfeld, Germany

ABSTRACT: Failure of materials is in many cases associated with initiation and subsequent propagation of macroscopic fractures. Consequently, in order to increase the strength, one needs to inhibit either crack initiation or propagation. The principle of topological interlocking provides a unique opportunity to construct materials and structures in which both routes of the strength increase can be realised. Materials and structures built on the basis of this principle consist of many elements which are hold together by the special geometry of their shape, together with an external constrain. The absence of the binder phase between the elements allows the interfaces to arrest macroscopic crack propagation. In addition, with sufficiently small size of the elements an increase in local strength and, possibly, in the stress for crack initiation can be achieved by capitalising on the size effect. Furthermore, the ability of some interlocking structures to tolerate missing elements can serve to prevent the avalanche-type failure initiated by failure of one of the elements. In this paper, experimental results and a theoretical analysis with regard to this possibility are presented.

1. INTRODUCTION

Failure of brittle materials and structures is in many cases associated with initiation of a macrocrack and its propagation to failure. Correspondingly, an increase in structural stability can be achieved either by preventing the formation of macrocracks or by inhibiting their propagation. An efficient method of inhibiting macrocrack propagation and thus increasing the macroscopic fracture toughness of the material is to introduce internal interfaces with loosely connected opposite faces capable of arresting a macrocrack. The role of such interfaces can be played by cracks in a heavily cracked body or by internal boundaries in a fragmented body with unbonded fragments, however such a body will be either very weak or not in one piece. The principle of topological interlocking provides a unique opportunity to construct crack arresting fragmented materials and structures which keep their integrity. Materials and structures built according to this principle consist of many elements which are held together by their special geometry and an external constraint. A great benefit of such design is that the absence of the binder phase between the elements allows the interfaces to arrest macroscopic crack propagation. (Note that when a binder phase is introduced to join the elements, it offers a medium where the crack can form and propagate.) Furthermore, one can capitalise on the size effect by employing elements of sufficiently small dimensions, leading to an increased local strength and, possibly, the crack initiation stress [Ashby and Bréchet, 2003]. Both recipes for enhancing strength - preventing the formation of the macrocrack or by inhibiting its propagation - can thus be implemented using topological interlocking. Furthermore, the ability of some interlocking structures to tolerate defects, such as missing elements, can serve to prevent avalanche-type failure initiated by failure of an individual element.

Topological interlocking is achieved either by special arrangements of convex blocks of simple habitus, such as platonic shapes [Dyskin et al., 2003a], or by blocks with specially designed curved surfaces [Dyskin et al., 2003b] such that if a plate-like assembly is constrained at the periphery, no block can be removed from the assembly because its movement is obstructed by the neighbouring blocks. An example of the first type of interlocking is given by plate-like assemblies of tetrahedra [eg, Glickman, 1984; Dyskin et al., 2001]. The second type of interlocking is exemplified by a set of so-called osteomorphic bricks [Robson, 1978; Khor et al., 2002; Dyskin et al., 2003b] Figure 1, permitting production of both plate-like and corner-like structures, as well as columns. In the osteomorphic block structures, each curved surface of a block obstructs its removal in both directions normal to the assembly.



(a) (b)

Figure 1: Osteomorphic blocks: (a) plate-like structure; (b) full osteomorphic block and two types of half-block necessary to furnish the edges.

Point loading applied at the middle of a plate-like structure made of osteomorphic blocks clumped at the edges showed that failure is localised within the blocks on which load is exerted, while the assembly as a whole still keeps its integrity [Khor et al., 2002]. This presents a striking contrast to the failure mode of a solid reference plate under the same type of loading in which fractures transverse the entire plate. The load bearing capacity of the fragmented plate was, however, much lower than the one of the solid plate. The present paper analyses the reasons for the low load bearing capacity and presents results of a different test where the peak load was similar or in one instance greater than the one for a solid plate.

2. MECHANISM OF LOW LOAD BEARING CAPACITY IN LARGE DEFLECTION BENDING TESTS

In bending tests [Khor et al., 2002] the plate-like structure of interlocking blocks is clamped and fixed at the edges, and subjected to in-plane compression q from the edges and the displacement-controlled indentation producing a force F in the middle of the structure. Two main features were observed. Firstly, the bearing capacity of the interlocking plate was much lower than the one of the reference solid plate of the same thickness made of the same material. Secondly, the deflections (indentor displacements) achieved were rather large, close to the thickness of the structure as opposite to small deflections of the reference solid plate.

Our analysis of the mechanism of low bearing capacity of the interlocking plate will be based on the fact that in both cases the fractures were initiated in the indentation area (In the case of interlocking structure the fractures were confined within the loaded block.) These fractures are obviously caused by the indentor-generated local tensile stresses. The tensile stresses produce fractures when their magnitude exceeds the value $\sigma_t + q$, where σ_t is the tensile strength of the material and q is the constraining pressure, both being the same for the interlocking structure and the reference solid plate. Since the only difference between these two cases is the presence of large (comparable with the plate thickness) deflections in the interlocking plate, it is natural to look for the mechanism in the non-linear effects related to the large deflections.

We consider large deflections of a plate with fixed boundaries under point loading. It is known [e.g. Timoshenko and Gere, 1961; Landau and Lifshitz, 1959] that in this case the neutral plane is no longer stress-free, but is rather subjected to tensile (membrane) stress, $\Delta\sigma$, Figure 2, as opposed to the cases of cylindrical bending of a plate or bending of a beam. This is a pure geometrical feature: cylindrical bending of a plate does not change its perimeter, while bending of a circular plate with fixed perimeter under a central load causes the plate diameter to stretch and thus produces tensile stresses. When deflection is small, this stress is negligible, however, it becomes noticeable when the deflection, $\zeta(r)$, is no longer small as compared to the plate thickness.

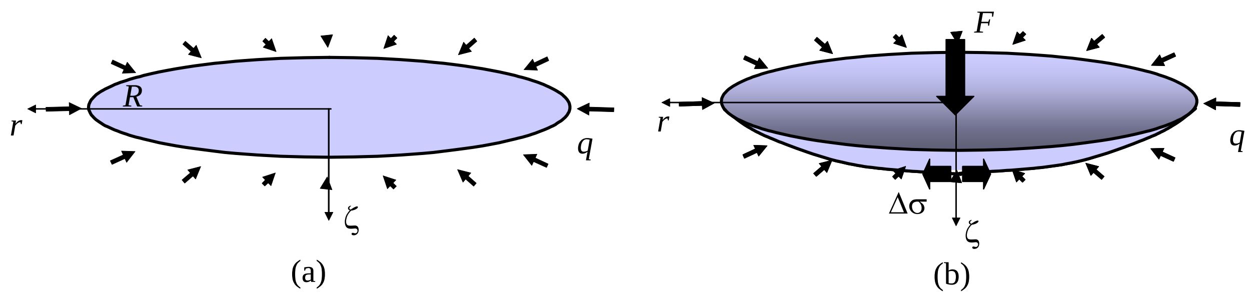


Figure 2: Mechanism of membrane stresses in a plate with clamped perimeter: (a) circular plate of radius R under confining pressure q before indentation loading; (b) circular plate under indentation force F. Large plate deflection $\zeta(r)$ induces membrane stresses $\Delta\sigma$, which reduce the confining pressure and, hence, the resistance to tensile fractures.

In order to estimate this stress, we consider a circular plate of thickness h clamped and fixed at the edges. Deflection of such a plate in a polar co-ordinate frame with the origin at the plate centre, Figure 2b, in the approximation of small deflections has the form [e.g. Landau and Lifshitz, 1959]:

$$\zeta(r) = \frac{F}{8\pi D}\left(\frac{R^2 - r^2}{2} - r^2 \ln\frac{R}{r}\right), \quad D = \frac{Eh^3}{12(1-\nu^2)}, \tag{1}$$

where E, ν are the (effective) Young's modulus and Poisson's ratio of the structure.

In the first approximation, the stress $\Delta\sigma$ can be obtained by substituting the small deflection approximation (1) into a balance equation of the plate, which in a corresponding Cartesian co-ordinate frame with x and y axes in the plate plane has the form [e.g. Landau and Lifshitz, 1959]

$$\Delta^2\chi + E\left[\frac{\partial^2\zeta}{\partial x^2}\frac{\partial^2\zeta}{\partial y^2} - \left(\frac{\partial^2\zeta}{\partial x\partial y}\right)^2\right] = 0, \quad \Delta\sigma_x = \frac{\partial^2\chi}{\partial y^2}, \quad \Delta\sigma_y = \frac{\partial^2\chi}{\partial x^2}, \quad \Delta\sigma_{xy} = -\frac{\partial^2\chi}{\partial x\partial y}, \tag{2}$$

where χ denotes the Airy stress function. Transferring (2) to polar co-ordinates, substituting (1) in the result and taking into account that due to central symmetry, χ is independent of the polar angle, one has

$$\Delta^2\chi + \frac{EF^2}{16\pi^2 D^2} r \ln\frac{R}{r}\left[1 - \ln\frac{R}{r}\right] = 0, \quad \Delta\chi = \frac{1}{r}\left(r\chi'_r\right)'_r. \tag{3}$$

Using the boundary condition, $\Delta\sigma_r$=0, and the condition of boundness of stresses at the origin, one obtains the circumferential stress component $\Delta\sigma_\theta$:

$$\Delta\sigma_\theta = -\frac{EF^2}{64\pi^2 D^2}\left[\frac{3}{4}\left(r\ln\frac{R}{r}\right)^2 - \frac{5}{8}r^2\ln\frac{R}{r} + \frac{51}{64}r^2 - \frac{13}{64}R^2\right]. \tag{4}$$

The following condition of fracture generation can be identified near the plate centre (r=0):

$$\sigma_F + \Delta\sigma_\theta(0) = \sigma_t + q, \quad \sigma_F = kF, \quad \Delta\sigma_\theta(0) = \kappa\frac{F^2R^2}{Eh^6}, \quad \kappa = \frac{117(1-\nu^2)^2}{256\pi^2}. \tag{5}$$

Here it is assumed that the tensile stress generated by the indentation, σ_F, is proportional to the force, F, the proportionality constant being denoted by k. From this, the critical failure force, F_{cr}, is given by

$$F_{cr} = \frac{\sigma_t + q}{k\xi}\left[\sqrt{1+2\xi} - 1\right], \quad \xi = 2\kappa\frac{R^2(\sigma_t + q)}{k^2Eh^6}. \tag{6}$$

Figure 3 shows the dependence of the normalised failure force, $kF_{cr}(\sigma_t+q)^{-1}$, on ξ. It is seen that for low ξ, which corresponds to large values of Young's modulus, E, and, hence, low deflections, $F_{cr}\approx(\sigma_t+q)\,k^{-1}$ holds. As E is reduced, the failure force monotonically decreases tending to zero. That is why in the interlocking structures, which are characterised by low values of Young's modulus, the force of fracture initiation is considerably lower than in solid plates. Fortunately, in contrast with solid plates, in the interlocking structures the fracture initiation does not result in complete failure.

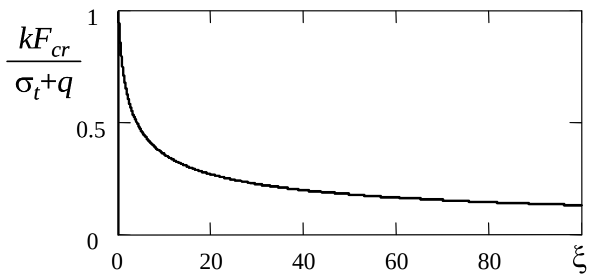


Figure 3: Normalised failure force vs. $\xi=2\kappa R^2(\sigma_t+q)/(k^2Eh^6)$.

The above analysis suggests that if deflection is prevented by an external constraint, the force of fracture initiation will be similar in both interlocking and solid plates. This is investigated experimentally in the following section.

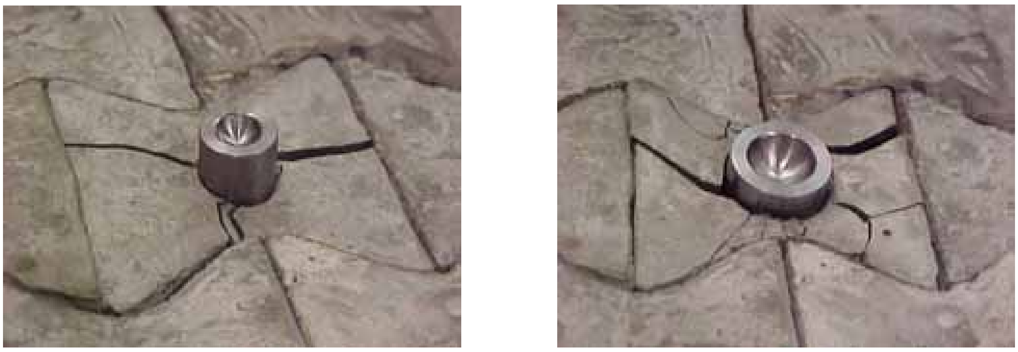
3. COMPRESSION TESTS

Solid plates and plate-like interlocking structures of rectangular shape, of size 280 x 260 x 20mm were loaded by an indentor. These structures rested on a flat surface such that no deflection was possible. Both the solid plate and the interlocking blocks were cast from cement paste and cured for 21 days in a humidity room. Two indentor sizes were used: 10 mm diameter (thin cylinder) and 15 mm diameter (thick cylinder). The results of the tests are shown in Table 1.

Table 1. Results of the indentation tests on plate-like structures rested on a flat surface.

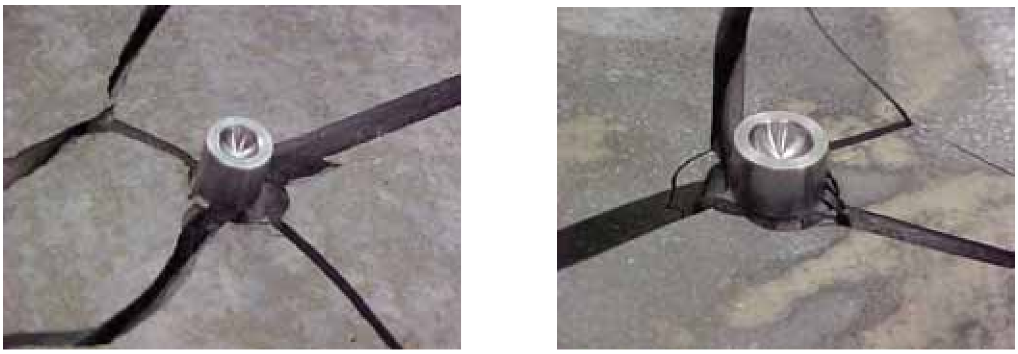
	Peak Strength (kN) Test 1	Peak Strength (kN) Test 2	Peak Strength (kN) Average
Osteomorphic Blocks with Thin Cylinder	6.9	6.7	6.8
Solid Plate with Thin Cylinder	7.0	-	7.0
Osteomorphic Blocks with Thick Cylinder	12.3	11.56	11.93
Solid Plate with Thick Cylinder	9.9	-	9.9

As evident from Table 1, in these tests the peak loads for the solid plate and the interlocking assembly were indeed similar. Furthermore, in the case of the thick cylinder indentor the peak load for the interlocking structure was 20% higher that for the solid plate. At the peak load fractures were generated, which in the case of the interlocking assembly were contained within the block immediately subjected to loading, while other blocks remained intact, Figure 4. By contrast, the solid plate was completely fractured, Figure 5. This result displays the superior properties of the interlocking assembly where only local fracture at the site of loading was observed, while the overall structure still retained its integrity. Furthermore, the interlocking structure could withstand multiple fractures before the whole structure ultimately failed.



(a) (b)

Figure 4: Assembly of osteomorphic blocks loaded by (a) thin and (b) thick cylindrical indentors.



(a) (b)

Figure 5: Solid plate loaded by (a) thin and (b) thick cylindrical indentors.

4. CONCLUSION

In indentation tests the reduced failure load of interlocking structures is associated with their low bending rigidity leading to deflections which are comparable with the thickness of the plate. Such deflections create appreciable tensile stresses within the plate which counteract the stabilising effect of the applied constraining pressure. This suggests that the most efficient use of the interlocking structures is in protective coatings and covers when the effect of enhanced fracture toughness and tolerance to missing blocks appears to be beneficial, especially when there is a risk of multiple damages, for example in a war situation. Essentially, the principle of topological interlocking permits manufacturing of fragmented bodies kept in one piece without connectors with their tendency to act as stress-concentrators. Then, as soon as the fragmented body is created, one can capitalise on two favourable properties, *viz.* the enhanced fracture toughness and the scale effect, when by making the blocks smaller one can increase their strength and consequently the load at which fracture initiates.

ACKNOWLEDGEMENTS: Support from the Australian Research Council through Discovery Grant DP0210574 (2002-2004) and Linkage International Grant LX0347195 (2002-2004) is acknowledged.

REFERENCES

Ashby M.F. and Bréchet, Y.J.M. Designing hybrid materials *Acta Materialia* 51 (2003) pp. 5801–5821.

Dyskin, A.V., Estrin, Y., Kanel-Belov, A.J. and Pasternak, E. A new concept in design of materials and structures: Assemblies of interlocked tetrahedron-shaped elements. *Scripta Materialia*, 44 (2001) pp. 2689-2694.

Dyskin, A.V., Estrin, Y., Kanel-Belov, A.J. and Pasternak, E. Topological interlocking of platonic solids: A way to new materials and structures. *Phil. Mag. Letters* 83(3) (2003a) pp. 197-203.

Dyskin, A.V., Estrin, Y., Pasternak, E., Khor, H.C. and Kanel-Belov, A.J. Fracture resistant structures based on topological interlocking with non-planar contacts. *Advanced Engineering Materials*, 5(3) (2003b) pp. 116-119.

Estrin, Y., Dyskin, A.V., Kanel-Belov, A.J. and Pasternak, E. Materials with novel architectonics: Assemblies of interlocked elements. In B. Karihaloo (ed.) *IUTAM Symposium on Analytical and Computational Fracture Mechanics of Non-homogeneous Materials*, Kluwer Academic Press (2002) pp. 51-56.

Glickman, M. The G-block system of vertically interlocking paving, In: *Second International Conference on Concrete Block Paving*, Delft, April 10-12 (1984) pp. 345-348.

Khor, C., Dyskin, A.V., Pasternak, E. Estrin, Y. and Kanel-Belov, A.J. Integrity and fracture of plate-like assemblies of topologically interlocked elements. In: *Structural Integrity and Fracture*, SIF2002 (Dyskin, A.V., Hu, X.Z. and Sahouryeh, E., Eds), Swets & Zeitlinger, Lisse (2002) pp. 449-456.

Landau, L.D., Lifshitz, E.M. *Theory of Elasticity*, Pergamon Press, London, Addison-Wesley Publ. Co., Reading, Mass (1959).

Robson, D.A. Deutsches Patent DE-AS 25 54 516 (1978).

Timoshenko, S.P. and Gere, J.M. *Theory of Elastic Stability*. McGraw-Hill Book Company. New York, Toronto, London (1961).