

# Numerical Modeling Of Obliquely Incident Guided Wave Scattering By A Crack In A Laminated Composite Plate

W. Karunasena

Civil & Environmental Engineering, School of Engineering, James Cook University,  
Townsville, QLD 4811, Australia

**ABSTRACT:** In this paper, we consider scattering by a guided wave incident obliquely on a surface breaking crack in a laminated composite plate, with a view to ultrasonic nondestructive assessment of cracks. The solution to this problem is the first step towards analyzing the general three-dimensional scattering problem. The method used for modeling is a hybrid method which combines finite element method with wave function expansion procedure. The reciprocity relations governing the reflection and transmission coefficients are established and are used to check the numerical accuracy. Also, the principle of energy conservation is used as another check for the accuracy of the numerical results. Numerical results for reflection and transmission coefficients are presented for an 8-layer cross-ply laminated plate.

## 1. INTRODUCTION

Due to low density, high performance, high corrosion resistance and efficient tailorability of material properties to suit the application, laminated composite materials are receiving wide attention in aerospace, civil and mechanical engineering applications. Defects such as cracks and delaminations in composite structures have become a major problem from the point of view of structural integrity. Guided elastic waves in laminated composites possess characteristics that make them particularly useful for applications in non-destructive evaluation of such defects in plate-like structures. These guided waves can travel far along the plate, thus providing a means of inspection of an otherwise inaccessible area. In order to use guided waves in ultrasonic nondestructive applications, it is necessary to investigate the phenomenon of scattering of these waves by defects.

Dispersive behaviour of guided waves in laminated plates of finite thickness has been studied extensively in recent years. It has been shown that dispersive modal propagation behavior is strongly influenced by the anisotropic properties of each lamina or layer and the stacking sequence of the layers. Several investigators have successfully used comparison of experimental results with modeling predictions to determine the anisotropic elastic constants of individual layers in composite plates. References to these works can be found in the symposium volumes edited by Datta et al. (1990) and Kinra et al. (1994). Both free and fluid-loaded plates have been considered. Chimenti (1997) has published a comprehensive review of guided waves in composite plates and their use for material characterization. Also, Datta (2000) has given a detailed review of the theory of guided waves in composite plates and shells. Although a vast body of work on guided ultrasonic waves in layered plates and shells now exists, relatively few studies have dealt with scattering of these waves by cracks or delaminations. To our knowledge, most of these studies have been confined to the problems of horizontally polarized shear (SH) waves and plane strain (two-dimensional) waves.

In this paper, we present a model analysis of scattering of a guided wave incident obliquely on a long symmetric surface breaking crack. The solution to this problem is the first step towards analyzing the general three-dimensional scattering problem. A conventional finite element modeling approach is not suitable for solving this problem as it involves a very large finite element mesh to get satisfactory solutions due to energy propagation to large distances by propagating wave modes. The method used is a hybrid method in which the plate is divided into two regions: an

interior region bounded by two imaginary vertical boundary planes and two exterior regions outside these vertical boundaries. The interior region is modeled by finite elements while the field outside is represented by wave function expansion (ie modal sums). The geometry of the problem is depicted in Figure 1. The incident wave shown in Figure 1(b) is assumed to have harmonic time dependence. The symmetric surface-breaking crack is taken to be infinite in length in the  $y$ -direction and is in the plane of  $yz$ . As shown in Figure 1(b), the plane of the incident wave makes an angle  $90^\circ - \phi^{\text{in}}$  with the  $yz$ -plane. The analysis is presented for a plate with an arbitrary stacking sequence where each ply can have an arbitrary fibre direction ( $\theta$ ) with respect to the global  $x$ -axis. Reciprocity relations governing the reflection and transmission coefficients are established and are used to check the numerical accuracy. Also, the principle of energy conservation is used as another check for accuracy of the numerical results. Numerical results of the scattering problem are presented for an 8-layer cross-ply laminated graphite-epoxy composite plate.

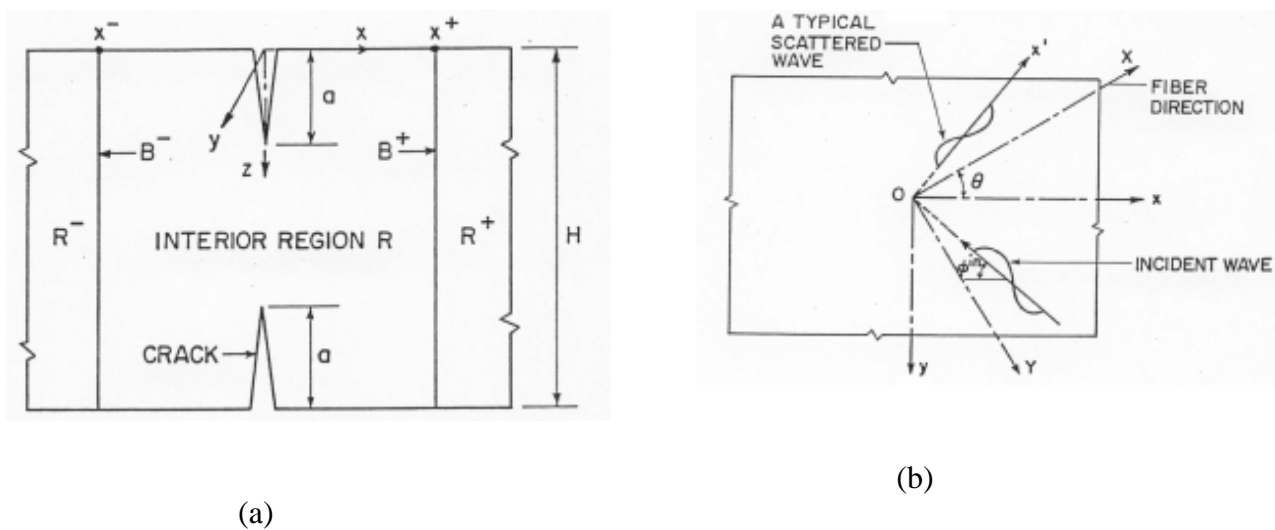


Figure 1: Geometry of the problem

(a) Elevation showing normal edge crack.

(b) Plan view of a typical lamina (layer) showing fibre orientation and wave normals.

## 2. FORMULATION

Time-harmonic waves are considered. The plate is assumed to be composed of perfectly bonded layers of equal thickness and with transversely isotropic elastic properties having the axis of symmetry of each layer lying in the  $xy$  plane. It is assumed that the two faces of the plate  $z = 0$  and  $z = H$  are stress-free. Also, the crack faces are assumed to be open with zero traction. As indicated above, the incident wave is a guided wave mode propagating in a direction  $90^\circ - \phi^{\text{in}}$  with the negative  $y$ -axis.

### 2.1 Governing Equations

Since the incident wave is not coincident with a symmetry axis of a layer, the particle motion associated with the incident wave will have all three displacement components in the  $x$ -,  $y$ -, and  $z$ -directions. These will be denoted by  $u_x(x,y,z,t) \equiv u(x,y,z,t)$ ,  $u_y(x,y,z,t) \equiv v(x,y,z,t)$ , and  $u_z(x,y,z,t) \equiv w(x,y,z,t)$  where  $t$  denotes time. The differential equation governing the wave motion is, in index notation,

$$\sigma_{ij,j} = \rho \partial^2 u_i / \partial t^2 \quad (i,j = 1,2,3) \quad (1)$$

where  $u_i$ ,  $i = 1, 2, 3$ , stand respectively for  $u$ ,  $v$ , and  $w$ , and  $\sigma_{ij}$  the stress components are related to the strain components,  $\varepsilon_{ij}$ , by the relation

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & D_{16} \\ D_{12} & D_{22} & D_{23} & 0 & 0 & D_{26} \\ D_{13} & D_{23} & D_{33} & 0 & 0 & D_{36} \\ 0 & 0 & 0 & D_{44} & D_{45} & 0 \\ 0 & 0 & 0 & D_{45} & D_{55} & 0 \\ D_{16} & D_{26} & D_{36} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} \quad (2)$$

where  $\gamma_{ij} = 2 \varepsilon_{ij} = u_{i,j} + u_{j,i}$  and  $D_{ij}$  are the coefficients of the constitutive matrix with respect to the global  $x,y,z$  coordinate system. It is convenient to use a local cartesian coordinate system  $X, Y, Z$  in each layer (or sublayer discussed later) with origin in the middle of the layer,  $X$ -axis aligned with the axis of elastic symmetry of the layer and  $Z$ -axis normal to plane of the layer. Let  $C_{ij}$  denote the coefficients of the constitutive matrix with respect to the  $X,Y,Z$  coordinate system. Then  $D_{ij}$  are related to  $C_{ij}$  through the expressions given in Karunasena et al. (1991a).

Both incident and scattered displacements must satisfy the equation of motion (1) and the stress free boundary conditions at the plate surfaces and the crack faces. It is extremely difficult if not impractical to find a close form solution to the wave scattering problem described above. Therefore, we resort to a hybrid numerical method to solve this problem.

## 2.2 Hybrid Method

The hybrid method combines finite element formulation in a bounded interior region of the plate with wave functions of the total field in the exterior region. The interior region consists of the flaw and a small region of the plate surrounding the flaw. The regions are connected along vertical boundaries  $B^+$  at  $x = x^+$ , and  $B^-$  at  $x = x^-$  as shown in Figure 1. Note that the axis of flaw coincides with the  $y$ -axis. However, since the orientation of the axis of the flaw is not known *a priori* it is not, in general, possible to excite the incident wave in such a way that  $\phi^{\text{in}} = 0^0$ . Thus, it is necessary to consider the general case where  $\phi^{\text{in}} \neq 0^0$ . This general case is considered here. It is assumed that the incident wave is propagating in a direction making an arbitrary angle with the  $x$ -axis, and in general, the fibre direction in each lamina may be at an arbitrary angle to the  $x$ -axis. The wave motion associated with the incident and scattered waves have all three particle displacement components. Let  $k^{(0)}$  be the wave number of the incident wave in the direction of propagation. Thus,  $k^{(0)}$  should be one of the admissible real roots of the dispersion equation for off-axis propagation. Since the flaw extends to infinity in  $y$ -direction, the scattered field must have the same wave numbers in the  $y$ -direction as the incident field. Thus, each of the scattered wave modes will have a constant wave number  $\zeta_0 (= k^{(0)} \sin \phi^{\text{in}})$  in the negative  $y$  direction. Therefore, for time-harmonic waves,  $y$  and  $t$  variation can be factored out as

$$\begin{Bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{Bmatrix} = \begin{Bmatrix} \hat{u}(x, z) \\ \hat{v}(x, z) \\ \hat{w}(x, z) \end{Bmatrix} \exp[-j(\zeta_0 y + \omega t)] \quad (3)$$

where  $\omega$  is the circular frequency and  $j = \sqrt{-1}$ .

#### *Finite element model of the interior region*

The procedure for finite element formulation for the interior region R for this (general) case is very similar to that for the plane strain case given in Karunasena et al. (1991b). As in that case, the finite element representation of the interior region includes singular elements at the crack tip. The finite element mesh used for this problem consists of six noded quarter point triangular crack tip elements which are surrounded by a layer of transition elements followed by conventional four node elements. The standard discretization process in the finite element method leads to

$$\delta\{\bar{q}\}^T [S]\{q\} - \delta\{\bar{q}_B\}^T \{P_B\} = 0 \quad (4)$$

where

$$[S] = [K] - \omega^2 [M] \quad (5)$$

in which: [K] and [M] are, respectively, the global stiffness and mass matrices of the region; {q} is the nodal displacement vector corresponding to interior nodes; and {q<sub>B</sub>} and {P<sub>B</sub>} are, respectively, the nodal displacement vector and the interaction force vector corresponding to the boundary nodes.  $\delta$  implies first variation and overbar denotes complex conjugate.

#### *Wave functions for the exterior regions*

The wave field in the exterior regions R<sup>+</sup> and R<sup>-</sup> is the superposition of those due to the incident wave and the scattered waves. Using the wave function expansion, the scattered wave field can be expressed in terms of the wave functions (ie wave modes) supported by the free laminated infinite composite plate with no flaws and the unknown reflected and transmitted wave amplitudes. The theoretical details of the methodology adopted to obtain wave functions can be found in our work reported in Karunasena et al. (1991c). The procedure starts with dividing each layer into several sublayers. Let N be the total number of sublayers in the plate. The exact dispersion relation of the infinite plate is developed using the propagator matrices as

$$f(\omega, k) = 0 \quad (6)$$

where k denotes the x-direction wave number of a typical wave mode. It is well known that the wave modes are dispersive and at any given frequency  $\omega$ , there are only a finite number of propagating modes that carry energy away from a source of excitation or upon scattering from an inhomogeneity or crack. However, in order to satisfy the boundary conditions at the source or at a boundary of discontinuity it is necessary to include also the nonpropagating modes in the modal representation of the displacement field. The wavenumbers (k) for the propagating and nonpropagating modes at a given frequency of excitation can be found by solving the dispersion equation (6) for the plate. For each wave number k, the corresponding displacement wave function

(which is basically a vector containing x and z direction displacement at each sublayer level) can be determined using the propagator matrix for each sublayer. This has been discussed in the references cited above.

### Global solution

The global solution is obtained by imposing the continuity of total (incident plus scattered) displacements and tractions on the boundaries. This is achieved by substituting for  $\{q_B\}$  and  $\{P_B\}$  from the wave function expansion into equation (4). This leads to a system of linear equations to solve for the unknown reflected wave amplitudes ( $A_m^+$ ) and transmitted amplitudes ( $A_m^-$ ). These amplitudes are then used to obtain boundary nodal displacements and, in turn, to obtain interior nodal displacements. The reflection coefficient  $R_{pm}$  of the m-th reflected mode and transmission coefficient  $T_{pm}$  of the m-th transmitted mode, due to the p-th incident wave mode, are given by

$$R_{pm} = A_m^+ / A_p^{in}, \quad T_{pm} = \begin{cases} A_m^- / A_p^{in} & \text{for } m \neq p \\ (A_p^{in} + A_m^-) / A_p^{in} & \text{for } m = p \end{cases} \quad (7)$$

in which,  $A_p^{in}$  is the amplitude of the incident wave mode.

### Energy conservation and reciprocity relations

Energy balance and reciprocity of energy among scattered modes are used as checks for accuracy of numerical results. Reflected and transmitted energies are carried only by the propagating modes (which are the modes with real wave numbers only). Let  $I_{pn}^+$  and  $I_{pn}^-$  be, respectively, the time-averaged value of the energy flux associated with the n-th reflected and n-th transmitted wave mode through the plate cross section due to the p-th incident wave mode. Then the percentage error in energy balance,  $\varepsilon$ , can be defined as

$$\varepsilon = \left[ I_{pn}^{in} - \sum_{n=1}^{N_{pr}} (I_{pn}^+ + I_{pn}^-) \right] 100 / I_{pn}^{in} \quad (8)$$

where  $I_{pn}^{in}$  is the energy flux of the p-th incident wave mode and  $N_{pr}$  represents the number of propagating modes in the scattered field. If the modeling results are accurate, we expect  $\varepsilon$  to be close to zero.

Let  $E_{pn}$  be the proportion of energy of the p-th incident mode transferred into the n-th transmitted mode during the scattering process. Details of the derivation of reciprocity relations for energy sharing by propagating modes has been documented in Karunasena et al.(1991b) for plane strain wave scattering. Extending the procedure for this problem, it can be shown that

$$E_{pn} = E_{np}. \quad (9)$$

The reciprocity relation in equation (9) serves as another numerical check of the computations.

## 3. NUMERICAL RESULTS AND DISCUSSION

The hybrid method discussed above has been used to obtain numerical results for scattering by a symmetric normal edge crack in a homogeneous graphite-epoxy plate with fibres in x-direction ( $0^0$ ) and an 8 layer graphite-epoxy cross-ply laminated plate with a configuration  $0^0/90^0/0^0/90^0/90^0/0^0/90^0/0^0$  configuration. In both problems, the  $0^0$  fibres are aligned in the x-

direction. The elastic properties of graphite-epoxy lamina are given in Table 1. Note that graphite-epoxy has transversely isotropic material properties.

Lamina	$C_{11}$	$C_{33}$	$C_{13}$	$C_{55}$
$0^\circ$	160.73	13.92	6.44	7.07
$90^\circ$	13.92	13.92	6.92	3.50

Table 1: Elastic properties of graphite-epoxy lamina (in GPa)

Due to symmetry of the problem with respect to the mid-plane of the plate, the scattered field consists of only symmetric or antisymmetric modes depending on whether the incident mode is a symmetric or antisymmetric one. Thus only half thickness of the plate need to be modeled in the analysis. Due to space limitations the

scattering results for the 8 layer cross-ply laminated plate only are presented here. Numerical results for the magnitudes of reflection and transmission coefficients ( $|R_{pn}|$  and  $|T_{pn}|$ ), and proportions of transmitted energies ( $E_{pn}$ ) are presented in Table 2. The results correspond to a normalized frequency  $\Omega (= \omega H / (2\sqrt{(C_{55} / \rho)_0}))$  of 4.0 and a normalized crack length ( $= a/(H/2)$ ) of 0.5. In this table, p and n denote the incident and the scattered wave mode numbers, respectively, and all incident modes considered are symmetric modes. It can be seen from this table that the energy balance and the reciprocity relations among proportions of energy are satisfied with negligible errors. Numerical results for  $|T_{11}|$  and  $|R_{11}|$  show that mode one transmission and reflection coefficients are quite sensitive to the orientation of the crack in xy plane. Also it is seen that reflection and transmission coefficients are quite sensitive to the incident wave mode number. Although not shown here, our computations showed that coefficients are sensitive to the frequency and crack depth.

p	$k_p$	n	$ R_{pn} $	$ T_{pn} $	$E_{pn}$	$ \varepsilon $
1	3.3624	1	0.8401	0.4240	0.1797	0.04%
		2	0.1432	0.5706	0.1081	
2	0.6622	1	0.0466	0.1893	0.1080	0.25%
		2	0.7370	0.5828	0.3397	

(a)

$\theta$  for  $0^\circ$  lamina =  $0^\circ$ ,  $\phi^{\text{in}} = 45^\circ$ ,  $\zeta_0 = 3.3624$

p	$k_p$	n	$ R_{pn} $	$ T_{pn} $	$E_{pn}$	$ \varepsilon $
1	4.3931	1	0.3686	0.3171	0.1006	0.02%
		2	0.4614	0.0768	0.0394	
		3	0.1963	0.1072	0.0073	
2	1.9389	1	0.1308	0.5111	0.0391	0.08%
		2	0.9173	0.8243	0.6795	
		3	0.0297	1.3467	0.1730	
3	1.4683	1	0.1726	0.0687	0.0074	0.47%
		2	0.6732	0.1283	0.1725	
		3	0.0874	0.2032	0.0413	

(b)

$\theta$  for  $0^\circ$  lamina =  $22.5^\circ$ ,  $\phi^{\text{in}} = 22.5^\circ$ ,  $\zeta_0 = 1.8197$

Table 2: Scattering results for an 8 layer cross-ply laminated plate for a normalized frequency of 4.0 and a normalized crack length of 0.5.

### 3. CONCLUSION

A hybrid method combining the finite element method with the wave function expansion procedure has been used to study time harmonic wave scattering by a guided wave incident obliquely on a surface breaking crack in a laminated composite plate. The method can be easily

applied to other crack shapes and delaminations in composites plates. The accuracy of the results obtained has been verified by checking the energy balance and the satisfaction of the elastodynamic reciprocity relations. It is found that the reflection and transmission coefficients are quite sensitive to the orientation and depth of the crack, the frequency and the incident wave mode number. This suggests that judicious choice of frequency and incident mode can be made to obtain optimum results from ultrasonic non-destructive testing of flaws. The hybrid formulation based finite element code developed here has the capability to investigate scattering of ultrasonic elastic waves by flaws over a wide range of the parameters involved. This enables one to perform a detailed parametric study of the scattering problem and the results of this study can be used to interpret ultrasonic test measurements to characterize flaws.

## ACKNOWLEDGMENTS

The author is grateful to Prof. S.K. Datta, Head, Dept. of Mechanical Engineering, University of Colorado, Boulder, U.S.A and Prof. A.H. Shah, Dept. of Civil Engineering, Univ. of Manitoba, Canada for their advice and suggestions to solve this scattering problem.

## REFERENCES

- Chimenti, D E, Guided waves in plates and their use in material characterization, *Appl. Mech. Rev.* **50**, pp. 247-284 (1997).
- Datta, S K, Achenbach, J D, and Rajapakse, Y S (eds.), *Elastic Waves and Ultrasonic Nondestructive Evaluation*, Elsevier Publishers, North-Holland, Amsterdam (1990).
- Datta, S K, Wave propagation in composite plates and shells, *Comprehensive Composite Materials*, Vol. 1, Chapter 18, (2000).
- Karunasena, W, Bratton, R L, Shah, A H, and Datta, S K, Elastic wave propagation in laminated composite plates, *J. Eng. Mat. Tech., ASME*, **113**, pp. 413-420 (1991a).
- Karunasena, W, Shah, A H, and Datta, S K, Plane strain wave scattering by cracks in laminated composites plates, *J. Eng. Mech., ASCE*, **117**, pp. 1738-1754 (1991b).
- Karunasena, W, Shah, A H, and Datta, S K, Reflection of plane strain waves at the free edge of a laminated composite plate, *Int. J. Solids Structures*, **27(8)**, pp. 949-964 (1991c).
- Kinra, V K, Clifton, R J, and Johnson, G C (eds.), *Wave Propagation and Emerging Technologies*, The American Society of Mechanical Engineers, New York (1994).

