

Cracks in fractal materials

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ABSTRACT: The proposed analysis is based on modelling a material with self-similar structure by a continuum sequence of continua of increasing scales each determined by its own size of the averaging volume element. The scaling is represented by power laws with the exponents determined by the microstructure, but not necessarily by the material fractal dimension. The tensorial quantities must scale isotropically (the same exponent for all non-zero components) with only prefactors accounting for anisotropy. This is prescribed by the linear relationships between the tensor components in different coordinate sets and the fact that the power functions with different exponents are linearly independent.

Stresses are defined in each continuum (and are measured in conventional units of stress) with the scaling law controlling the transition from one continuum to another, i.e. from one stress field to another. Within each continuum the cracks produce conventional stress singularities. However, as the point of singularity is approached, the transition to finer continua is necessary, resulting in apparent non-conventional stress singularity.

A specific case considered is the collective growth of cracks which are already distributed self-similarly. In this regard we consider the crack growth under localized loading. Such a loading models two main situations: (1) tensile cracks growing under the action of heterogeneous stress field generated by material heterogeneities or residual strain and; (2) sliding zones (shear cracks) in a fault system produced by local losses of shear strength over time associated with either the increase in stress or the phenomenon of delayed fracture. If the cracks form self-similar sets then this mechanism of crack growth can either maintain or destroy the self-similarity. It was found that if the cracks are uniformly distributed and isotropically oriented, their self-similar size distribution is maintained. If, however, the cracks are all parallel to each other (transverse isotropic material) the crack growth destroys self-similarity. The situation drastically changes if the parallel cracks are localised in a narrow layer because then the crack growth maintains self-similarity. Thus in order to preserve self-similarity in parallel cracks a mechanism of localisation in multiple crack growth should be at work.

