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Shear Lag And Beam Theories For Structures

B N Cox,[1] M Andrews,[2] R Massabò,[3] A Rosakis,[4] N Sridhar,[1] and Q D Yang[1]

[1] Rockwell Scientific Co. LLC, Thousand Oaks, California, U.S.A.
[2] Department of Civil and Environmental Engineering, Northwestern Univ., Evanston, IL, U.S.A.
[3] Department of Structural and Geotechnical Engineering, University of Genova, Genova, Italy
[4] Department of Aeronautics, California Institute of Technology, Pasadena, California, U.S.A.

ABSTRACT: Dynamic problems are solved using beam theory and shear lag approximations, and also FEM. For a laminated plate incorporating through-thickness fibers, highlights are: 1) Inertia complicates the fiber pullout problem considerably. 2) Disturbances propagate along frictionally coupled fibers at less than the bar wave speed. 3) Unstable regimes appear in interfacial friction. 4) Large scale bridging creates oscillatory, predominately mode II crack profiles and 5) strongly modifies fracture at low to intermediate velocities. These results imply that dynamic delamination damage evolution will be dominated by distributed (not localized) bridging and friction effects. Solutions for single cracks with small process zones are less relevant than those for multiple cracks with large scale bridging, for which some initial solutions are discussed.

1 INTRODUCTION

Figure 1 shows a typical problem of current interest in the dynamic performance of structures. A projectile impacts upon a laminated armored structure, which absorbs energy and limits damage by a series of mechanisms, including comminution of ceramic tiles, viscous flow of the comminuted ceramic, spreading of the load transferred to structural elements, multiple delamination of a structural skin, and detachment of structural stiffeners. Performance characteristics of interest include the maximum dynamic deflection and the residual strength of the skin/stiffener assembly. Such structures are currently designed by intuition and analyzed by fabricating and testing hardware. Predictions of the ballistic performance, which might greatly reduce the cost of design and optimization, are unavailable because the physics and mechanics of the various mechanisms involved are not well known. Here we review recent efforts to understand some aspects of the problem of Fig. 1, which are also relevant to diverse other dynamic damage problems.

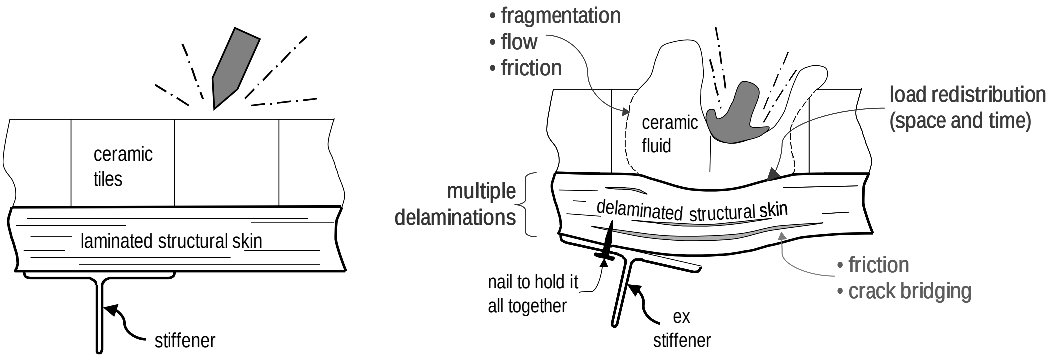

Figure 1. A typical complex problem in dynamic damage evolution.

2 DYNAMIC FIBER PULLOUT AND PUSH-IN

Through-thickness reinforcement, such as pins, stitches, or woven yarns, greatly enhances delamination resistance under both static and dynamic loading. A central problem in this phenomenon is the mechanics of pullout or push-in of a fiber (or pin, stitch, or yarn) embedded in a matrix that is a half-space. For static loading, this problem can be accurately described in mode I conditions using simple shear lag models, which incorporate the debond energy and interfacial friction as scalar parameters. Quite simple models of reasonable accuracy are also now available for mode II and mixed mode conditions (Cox, 2004). In contrast, little attention has been paid to the dynamic case.

Figure 2 compares experimental and theoretical stress fields (σ_1 - σ_2, where x_1 and x_2 are the in-plane coordinates) in a model planar fiber coupled by friction alone (the interface was initially not bonded) to a matrix (not shown) and subjected to a dynamic pulse load at the left end. The experimental data were acquired with dynamic photoelasticity methods. The calculations were per-formed with finite element methods, assuming a con-stant friction stress, apart from a linear variation with shear displacement rate for small rates, which is necessary because the num-erical procedures do not converge if the friction changes sign as a step function. The stress contours show regions, during loading and where the friction stress is saturated (constant), where they are not far from vertical, consistent with the simplified Lamé-like fields of a shear lag model. More complex behaviour is found approaching and during unloading (zones labeled “constant friction” and “unloading zone” in Fig. 2), where both experiment and model suggest, among other characteristics, instability or chaotic behaviour at the interface. The instability is presumably related to that predicted for Coulomb friction laws (Adams, 1998; Cochard and Rice, 2000), but here the distinction arises that the friction is not related to the normal stress. These results suggest that a fundamental difference exists between problems of uniform far-field loading, the case assumed in

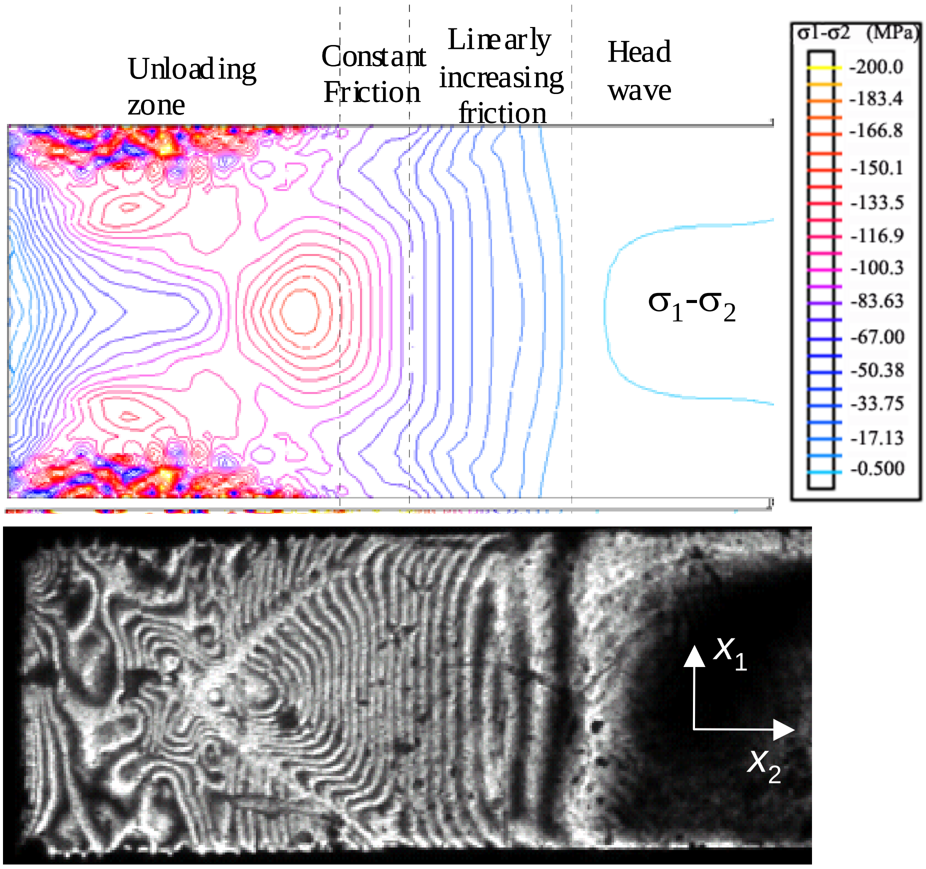

Figure 2. Snap-shot of stress fields: dynamic fiber push-in.

prior studies of instability at frictional interfaces, and time-varying loads; and that loading and unloading show distinct interfacial physics.

Other finite element calculations confirm that many of the characteristics of pullout/push-in problems are well approximated by Lamé-like solutions, at least in the case that friction is assumed to be of uniform magnitude (Sridhar *et al.*, 2003). Such analytical solutions have led to some interesting results that are not easily seen in numerical work, where the strong nonlinearity whenever friction changes sign causes difficulty. Behaviour is much richer than in the corresponding static loading case (Nikitin and Tyurekhodgaev, 1990; Sridhar *et al.*, 2003). Under linearly increasing end loads, zones of interfacial slip, slip-stick, and reverse slip are all possible,

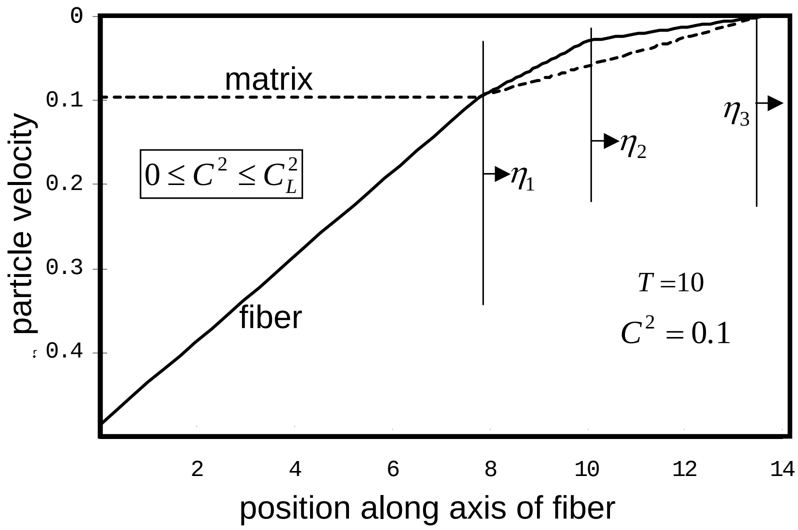

Figure 3. Particle velocity variations in a pullout case: slip and reverse slip zones under linearly increasing end load (Sridhar *et al.*, 2003).

depending on the loading rate and the properties of the fiber and matrix. Figure 3 illustrates the interfacial particle velocity jump for a case that includes reverse slip.

Another characteristic of interest is the stiffness of the response of the end-point displacement to dynamic loading. For linearly increasing loading, inertial effects increase the effective stiffness; but if loading stops increasing and is held constant, the momentum of the fiber continues to displace the fiber end and the final displacement is larger than for static loading. Estimates show that such inertial effects could significantly modify crack bridging laws and therefore the dynamic propagation of cracks bridged by fibers (Cox *et al.*, 2001).

3 LARGE SCALE BRIDGING EFFECTS

Beam theory methods can provide accurate solutions for many aspects of delamination fracture in thin, long specimens (Freund, 1993; Hellan, 1978; Kanninen, 1974; Suo *et al.*, 1992). When long zones of crack bridging are present in static cracks, beam solutions show an oscillatory behaviour, reminiscent of a beam on a Winkler foundation (Massabò and Cox, 2001). The oscillations can result in closure of the crack tip after some crack growth and divergence of the critical load in mode I (Fig. 4a). The same tendency prevails in solutions for mixed mode delamination, so that the crack tip conditions are driven towards mode II (assuming that the crack is not deflected out of the delamination plane) (Massabò and Cox, 2001). Thus friction effects extending over long domains of the crack wake will be central to crack propagation in structures with through-thickness reinforcement. In the dynamic regime, different domains of oscillatory and non-oscillatory solutions arise, depending on the stiffness, *S*, of the bridging mechanism and the crack velocity (Fig. 4b) (Sridhar *et al.*, 2002).

Features of the bridging traction law, including its initial slope, peak, and total area (work of rupture) establish length scales that control fracture (Bao and Suo, 1992; Cox and Marshall, 1994; Massabò and Cox, 1999; Rose, 1987). One length scale is proportional to the length of the bridging zone when the critical condition for bridging ligament failure has been reached. Beam solutions for

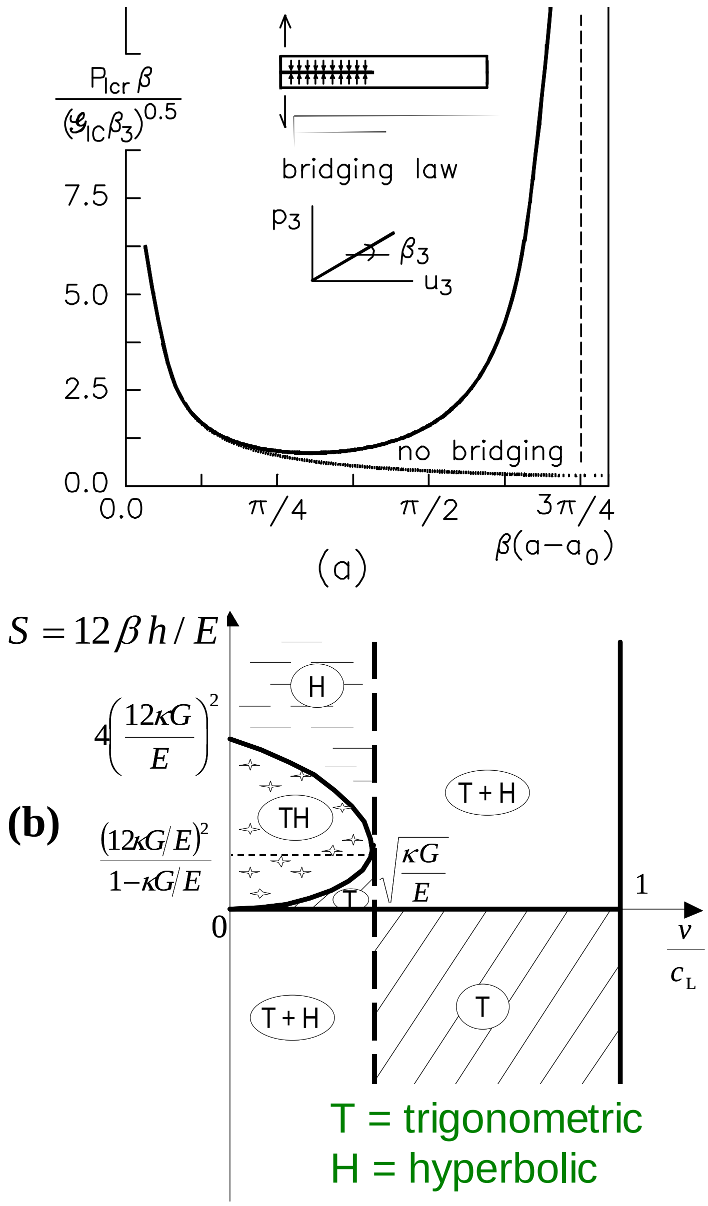

Fig. 4. Large scale bridging effects: (a) closure of crack tip in static mode I loading; and (b) oscillatory nature of dynamic solutions.

a wedge loaded DCB specimen show that, as the crack velocity increases, the zone length falls continuously (Sridhar *et al.*, 2002). Figure 5 shows this effect, with the zone length, A, normalized by its length for zero velocity and vanishing initial bridging stiffness (B in Fig. 5), which is controlled by the static bridging length scale. (The zone length is not zero as $B \rightarrow 0$, but becomes determined by the location of the critical displacement for an unbridged crack profile.) The decrease with velocity is caused by the increasing dominance of kinetic energy, which overcomes the bridging tractions to define the crack profile independently of the bridging effect or the intrinsic fracture energy; and the critical crack displacement is attained closer to the crack tip, as inertial resistance supports closer penetration of the wedge towards the crack tip. Consistently, bridging effects have a significant fractional effect on the critical load for wedge propagation only at moderate crack velocities, less than ~ $0.2c_R$, where c_R is the Rayleigh wave velocity (Sridhar *et al.*, 2002). However, in this regime they are dominant.

4 MULTIPLE DELAMINATION PROBLEMS

Experiment shows that dynamic delamination generally results in multiple cracks, yet, apart from studies of buckling problems, surprisingly few studies exist of the mechanics of multiply cracked beams. Here some highlights of recent work for static loading are summarized. Consider a homogeneous, isotropic beam, with two cracks of arbitrary length and location in the vertical direction, subject to a static force *P* (Fig. 6a). Euler-Bernoulli beam theory shows that one crack can cause either shielding or amplification of the energy release rate, *G*, for the other. Fig. 6b shows how G_L for the lower crack (*G* normalized by its value in the absence of the upper crack) changes when the length of the upper crack varies, for one choice of crack locations. The different curves represent different levels of approximation in dealing with possible interpenetration of sublaminates, which can arise in this problem in the absence of opposing crack surface tractions.

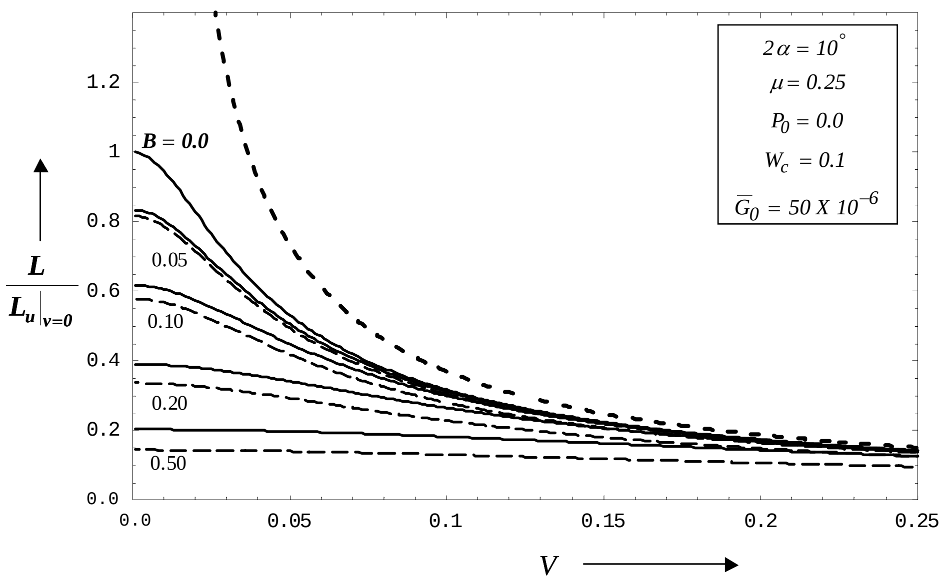

Fig. 5. Reduction of the bridging zone length with crack velocity in a wedge loaded DCB specimen.

Friction is not included. For the case shown, when the upper crack is longer (left side of the diagram), strong amplification occurs; when it is shorter (right side), amplification occurs but is weak. (The coincidence of all curves on the right side implies the absence of interpenetration.) A jump in G_L occurs when the cracks are of equal length.

Figure 6c maps regions of amplification and shielding for different heights of the two cracks in the beam. If the positions fall in the lower left regions, e.g., point (a), then G_L will always be amplified; if they fall in the shaded region, point (b), there will be a mixture of amplification and shielding depending on the relative lengths of the cracks; and if the positions fall in the upper region, point (c), then G_L will always be reduced.

When the cracks have similar lengths, beam segment 1 becomes stocky and the validity of Euler-Bernoulli beam theory is questionable. Fig. 7 compares results of finite element calculations with beam theory results for a typical configuration. Beam theory works remarkably well over the entire range of crack lengths. When the two crack tips are close, the relative error increases, but the qualitative behavior remains similar. The error is mainly due to the assumption of built-in beams, i.e., $\phi_1 = \phi_2 = \phi_0$ at a_U where ϕ_i is the bending rotation, which neglects the different root rotations at the crack tips, which cause different degrees of contact.

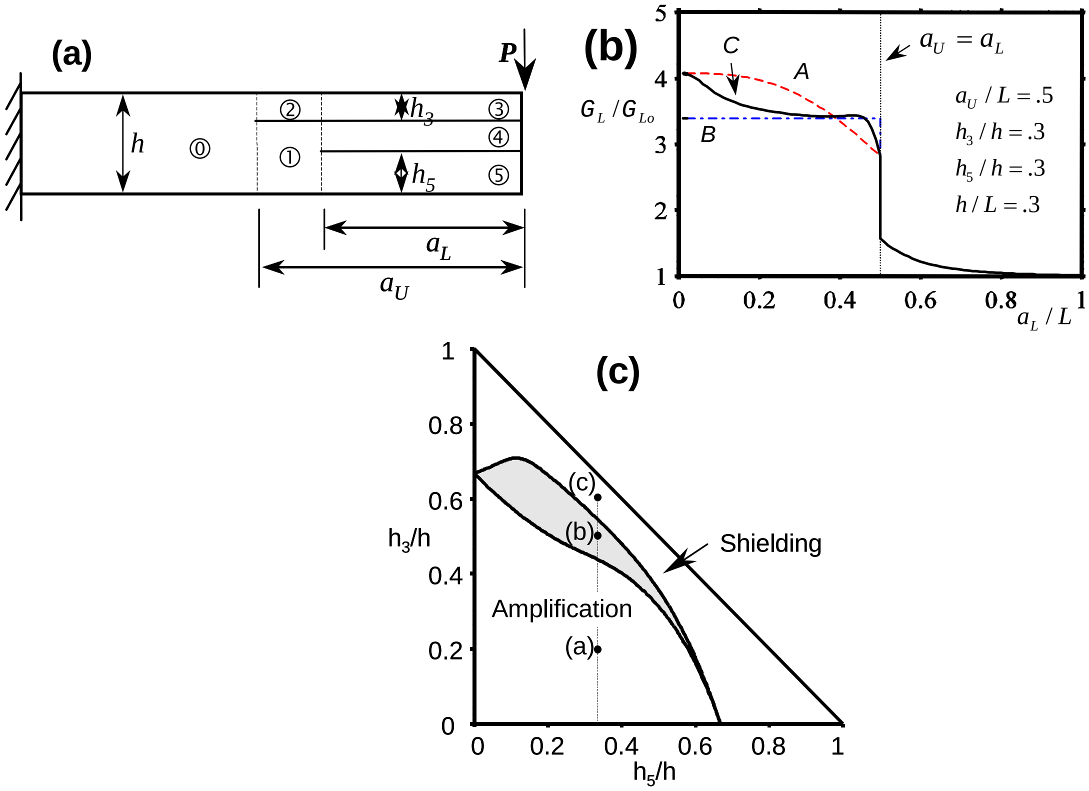

Fig. 6 (a) Cantilever beam with two cracks. (b) *G* for one crack as other varies in length. (c) Map of shielding and amplification.

Study of the macrostructural response of the two-crack system has also revealed that 1) for certain crack geometries, there is a local strain hardening behavior due to the shielding effect that leads to hyper-strength phenomena, while for other geometries, there is crack pull-along, in which one crack will begin to propagate and later draw the second crack along behind; and 2) jumps in the

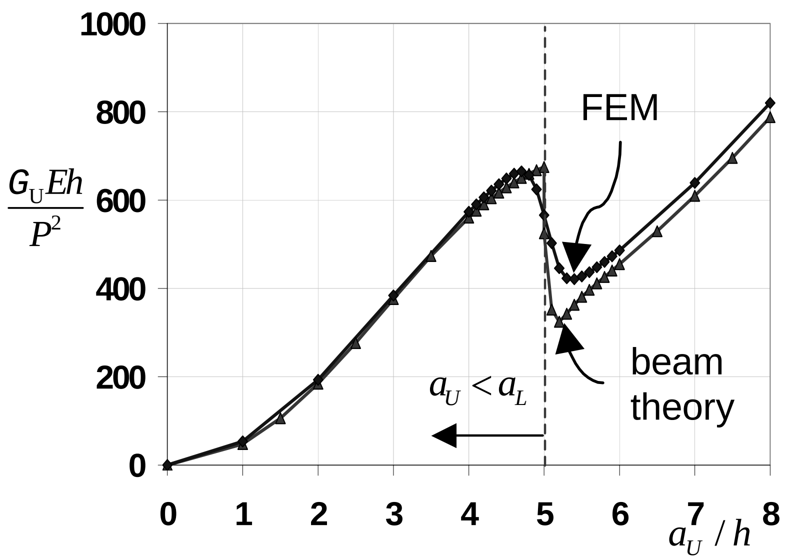

Fig. 7. Comparison of beam theory and FEM.

energy release rate (Fig. 6b) can also be seen in the quasi-static load deflection curve as local snap-back instabilities. Extension of this type of model to systems of more than two cracks and to incorporate long-range friction effects will be reported elsewhere. Both generalizations significantly enrich the nature of the possible solutions. The generalized models are especially relevant to understanding delamination response to impact, among other applications.

5 CONCLUSIONS

Certain problems in dynamic damage evolution can be well aproximated by beam and shear models. Great challenges remain in understanding large scale friction, mixed mode fracture, and multiple delamination effects.

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