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Extremely asymmetrical scattering of electromagnetic waves in gradually varying periodic arrays

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This paper analyses theoretically and numerically the effect of varying grating amplitude on the extremely asymmetrical scattering (EAS) of bulk and guided optical modes in non-uniform strip-like periodic Bragg arrays with stepwise and gradual variations in the grating amplitude across the array. A recently developed new approach based on allowance for the diffractive divergence of the scattered wave is used for this analysis. It is demonstrated that gradual variations in magnitude of the grating amplitude may change the pattern of EAS noticeably but not radically. On the other hand, phase variations in the grating may result in a radically new type of Bragg scattering—double-resonant EAS (DEAS). In this case, a combination of two strong simultaneous resonances (one with respect to frequency, and another with respect to the phase variation) is predicted to take place in non-uniform arrays with a step-like phase and gradual magnitude variations of the grating amplitude. The tolerances of EAS and DEAS to small gradual variations in the grating amplitude are determined. The main features of these types of scattering in non-uniform arrays are explained by the diffractive divergence of the scattered wave inside and outside the array.

I. INTRODUCTION

Extremely asymmetrical scattering (EAS) of waves in periodic arrays is a new type of Bragg scattering that is realized when the scattered wave propagates parallel or almost parallel to the boundary(ies) of a strip-like periodic array [1–11]. This type of scattering is radically different from the conventional Bragg scattering in periodic arrays. For example, EAS is characterized by a strong resonant increase of the scattered wave amplitudes inside and outside the array; the smaller the grating amplitude, the larger the amplitudes of the scattered waves [2, 6–11]. In addition, the incident and scattered waves inside the array each split into three waves [6–11]. Two of these scattered waves and two of the incident waves inside the array are evanescent waves which are localized near the array boundaries [6–8]. The third scattered wave is a plane wave propagating at a grazing angle into the array [6–8].

Our recent publications [6–8] have demonstrated that the physical reason for EAS is related to the diffractive divergence of the scattered wave inside and outside the array. A new powerful approach for simple analytical analysis of EAS, based on allowance for this diffractive divergence, was introduced and justified [6–11]. In the case of bulk electromagnetic waves, this approach was shown to give the same coupled mode equations as the dynamic theory of scattering that was previously used for the theoretical analysis of EAS [1–5, 8]. However, the dynamic theory of scattering is not suitable for analysis of EAS of guided and surface waves on account of extremely

awkward calculations involved, while the new approach is readily applicable for all types of waves, including surface and guided optical and acoustic modes [6, 7, 9, 10].

EAS has enormous potential for new important practical applications in the development of novel optical communication devices (e.g. narrow-band optical filters, resonators, couplers, switches, lasers, etc), optical sensors and measurement techniques. For example, the strong resonant increase of the scattered wave amplitude during EAS may result in high-quality EAS-based resonators and high sensitivity of sensors and measurement techniques. New non-collinear geometry of EAS can lead to the development of highly tuneable optical and ultrasonic devices, and may also result in an improved side-lobe structure of filtered signals. The possibility of concentration of the scattered wave energy in narrow channels can be used for amplification and lateral compression of waves, as well as for effective coupling of a planar waveguide and a fibre.

However, manufacturing EAS-based structures and devices will inevitably be related with various imperfections of periodic arrays. Because of the resonant character of EAS, it is possible to expect that at least some of these imperfections may be crucial for the experimental observation and practical use of this type of scattering. Thus, a very important practical problem is to investigate theoretically the effect of array imperfections on EAS, and determine the tolerance of the scattering. In addition, non-uniform arrays are often manufactured on purpose. For example, non-uniform chirped gratings can effectively compress pulses broadened due to dis-

persion of optical modes in a slab or optical fibre [12] (i.e. the dispersion can be compensated). Launching of solitons into non-uniform nonlinear Bragg gratings can be significantly easier than for uniform nonlinear Bragg gratings [13, 14]. The side-lobe structure of filtered signals can be noticeably improved (suppressed) by using non-uniform gratings with slowly varying grating amplitude [15].

There are also two other special reasons for using nonuniform arrays in the case of EAS. Firstly, edge effects at the array boundaries may result in noticeable undesirable energy losses in the scattered wave, caused by an additional re-scattering of the scattered wave. These losses may be noticeable because of the large amplitude of the scattered wave propagating along the boundaries. Non-uniform arrays with gradually increasing grating amplitude can be used for a substantial reduction of this effect. Secondly, nonuniform arrays with varying grating amplitude may result in a radically new type of EAS [10], as well as cause concentration of the wave energy within narrow channels inside the array [11]. In addition, non-uniform arrays may result in a significant reduction of the relaxation time to steady-state EAS.

Therefore, EAS of bulk or guided optical waves has been analysed for non-uniform arrays with step-like variations of the grating amplitude [10, 11]. It was demonstrated that a step-like variation in magnitude of the grating amplitude may result in a noticeable rearrangement of the intensity distribution of the scattered wave inside the array [11]. The sensitivity of EAS to small uniform and non-uniform (step-like) variations in the grating amplitude was also determined [11]. Step-like phase variations in the grating inside the array were shown to have a much stronger effect on EAS than the magnitude variations [10]. In thin arrays, they resulted in a radically new type of EAS double-resonant extremely asymmetrical scattering (DEAS) that is characterized by a unique combination of two sharp simultaneous resonances with respect to frequency and the phase variation in the grating [10].

However, non-uniform arrays with step-like variations of the grating amplitude will hardly reduce edge effects at the array boundaries. Moreover, the presence of additional interfaces at which the step-like variations take place must increase undesirable edge effects and the resultant energy losses. This is especially the case for DEAS, where the scattered wave amplitude is exceptionally large [10]. Step-like variations in the grating amplitude will also hardly improve the side-lobe structure of processed (filtered) signals. Finally, manufacturing periodic arrays usually results in imperfections characterized by slow (but not step-like) variations in the grating amplitude (e.g. due to non-uniform etching, photolithography, etc). However, EAS and DEAS in non-uniform arrays with slow variations of the grating amplitude have not been analysed previously, though such analysis is crucial for successful development of new practical applications of these types of scattering.

Therefore, the aim of this paper is to analyse theoretically and numerically the steady-state EAS and DEAS of bulk and guided electromagnetic waves in non-uniform periodic arrays with slowly varying grating amplitude. Coupled wave equations describing EAS and DEAS in arrays with slowly varying grating amplitude are derived. The field structure in the scattered waves is determined inside and outside the array by means of numerical solution of the coupled wave equations. The tolerance of EAS and DEAS to small slow variations in the grating amplitude (array imperfections) is also investigated theoretically. The results are compared with those obtained for EAS and DEAS in uniform and non-uniform arrays with step-like variations of the grating amplitude.

II. COUPLED WAVE EQUATIONS

The structure under investigation and the geometry of scattering are presented in figure 1. An incident wave is scattered in a strip-like periodic array of width L . The period of the grating is assumed to be constant in the whole array, while the grating amplitude can vary in phase and/or in magnitude across the array, i.e. in the x -direction (figure 1). We will consider only stepwise and/or slow (i.e. small at distances of the order of the grating period) variations in the grating amplitude. In this case, the grating amplitude can be assumed to be locally constant at each point of the array, except for the values of the x -coordinate at which the steplike variations take place (at $x = L_1$ in figure 1). The array is assumed to be uniform along the y -axis, i.e. in the directions parallel to the array boundaries.

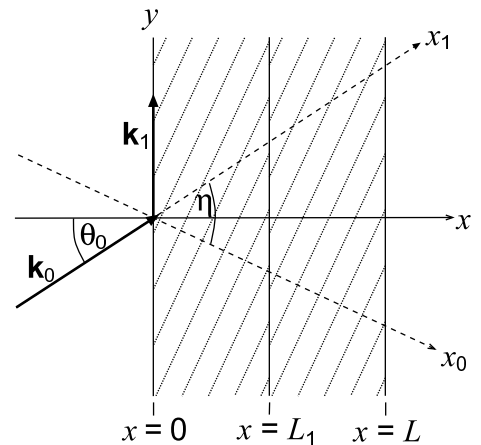


FIG. 1: Scheme for EAS in non-uniform strip-like periodic Bragg arrays with varying grating amplitude and the total array width equal to L .

In sections 2–5 we analyse EAS and DEAS of bulk TE optical waves in periodic Bragg arrays represented by a

periodic variation of the dielectric permittivity:

$$\begin{aligned} \epsilon_s &= \epsilon + \epsilon_1(x) \exp(i\mathbf{q} \cdot \mathbf{r}) + \epsilon_1^*(x) \exp(-i\mathbf{q} \cdot \mathbf{r}) \\ &\quad \text{if } 0 < x < L, \\ \epsilon_s &= \epsilon \quad \text{if } x < 0, \text{ or } x > L, \end{aligned} \quad (1)$$

where the mean dielectric permittivity ϵ is the same in all parts of the structure (inside and outside the array), the amplitude of the grating $\epsilon_1(x)$ is small:

$$|\epsilon_1(x)|/\epsilon \ll 1, \quad (2)$$

and \mathbf{q} is the reciprocal lattice vector ($|\mathbf{q}| = 2\pi/\Lambda$, where Λ is the grating period). There is no dissipation of electromagnetic waves inside or outside the array, i.e. ϵ is real and positive. The complex grating amplitude $\epsilon_1(x)$ varies in magnitude and/or phase within the array.

A plane TE electromagnetic wave (with the electric field parallel to the z -axis) is incident on the array at an angle θ_0 (measured counterclockwise from the x -axis—figure 1). We assume that the Bragg condition is satisfied precisely:

$$\mathbf{k}_1 - \mathbf{k}_0 = -\mathbf{q} \quad (3)$$

where \mathbf{k}_0 is the wavevector of the incident wave, \mathbf{k}_1 is parallel to the array boundaries (figure 1), $|\mathbf{k}_1| = |\mathbf{k}_0| = k_0 = \omega\epsilon^{1/2}/c$, ω is the angular frequency, and c is the speed of light in vacuum.

If condition (2) is satisfied for all values of x , then the amplitudes of the incident and scattered waves vary slowly inside the array, i.e. their variations at distances of about one wavelength are small as compared with the values of these amplitudes. In this case the approximation of slowly varying amplitudes is valid and only two waves—incident and scattered—need to be taken into account inside and outside the array:

$$\begin{aligned} E(x) &= E_0(x) \exp\{ik_{0x}x + ik_{0y}y - i\omega t\} + \\ &\quad E_1(x) \exp\{ik_{0y}y - i\omega t\}, \end{aligned} \quad (4)$$

where $E_0(x)$ and $E_1(x)$ are the slowly varying amplitudes of the electric field in the incident and scattered waves, respectively, $k_{0x} = k_0 \cos \theta_0$, and $k_{0y} = k_0 \sin \theta_0$.

It was shown previously [6–11] that in the geometry of EAS, there are two opposing mechanisms determining the behaviour of the scattered wave amplitude. On the one hand, the scattered wave amplitude must increase along the direction of its propagation (i.e. along the y -axis) due to scattering of the incident wave inside the array. On the other hand, the scattered wave amplitude must decrease along the y -axis due to a significant diffractive divergence of this wave [6–11]. The steady-state EAS occurs when the increment in the scattered wave amplitude caused by the scattering is exactly compensated by the decrement caused by the diffractive divergence [6–11]. The new approach for the theoretical analysis of EAS is based on the separate determination of each of the two contributions to the scattered wave

amplitude. The coupled wave equations describing the steady-state EAS are then derived from the comparison of these contributions [6–11].

Here, we apply this approach to the considered case of EAS and DEAS in non-uniform periodic arrays with slowly varying grating amplitude. During the first step, we determine the contribution due to scattering, disregarding the diffractive divergence. In this case the amplitude of the scattered wave must increase along the y -axis. If the amplitudes of the incident and scattered waves vary slowly inside the array, then at any point in the array, the incident wave amplitude and the grating amplitude can be considered to be locally constant. Thus the scattering-induced increments in the amplitude of the scattered wave along the direction of its propagation at any point of the array are determined only by the local values of the grating amplitude and the amplitude of the incident wave, regardless of the type of Bragg scattering. Therefore, we can find these increments by means of the conventional dynamic theory of scattering in uniform arrays.

The coupled wave equations in the conventional dynamic theory of scattering are well known [8, 12–15]:

$$dE_0/dx_0 = i\Gamma_1 E_1, \quad (5)$$

$$dE_1/dx_0 = i\Gamma_0 E_0, \quad (6)$$

where Γ_0 and Γ_1 are the coupling coefficients, and the x_0 -axis is parallel to the reciprocal vector of the grating (see figure 1). Equation (6) gives the scattering-induced rate of changing the scattered wave amplitude along the x_0 -axis. The rate of changing amplitude of the scattered wave along the direction of its propagation can then be obtained from equation (6) by means of the simple substitution of $dx_0 = -\cos(\pi/2 - \eta + \theta_0)dy = -\sin(\eta - \theta_0)dy$:

$$\left(\frac{\partial E_1}{\partial y}\right)_{\text{scattering}} = -i\Gamma_0 E_0(x) \sin(\eta - \theta_0). \quad (7)$$

where η is the angle (measured counterclockwise) between the positive x_0 -direction and the wavevector of the incident wave (figure 1).

During the second step, we disregard the scattering and consider only the diffractive divergence of the scattered wave (beam). In this case, the rate of decreasing amplitude of the scattered wave can be derived by substituting the entire scattered field $E_{sc} = E_1 \exp(ik_1 y - i\omega t)$ into the Helmholtz equation $\nabla^2 E_{sc} + k_1^2 E_{sc} = 0$:

$$\frac{\partial^2}{\partial x^2} E_1 + \frac{\partial^2}{\partial y^2} E_1 + 2ik \frac{\partial}{\partial y} E_1 = 0. \quad (8)$$

Here, we can neglect the second-order derivative of the slowly varying amplitude E_1 with respect to y as compared with the first-order derivative. This gives us the parabolic equation of diffraction

$$\left(\frac{\partial E_1}{\partial y}\right)_{\text{divergence}} = \frac{i}{2k_1} \frac{\partial^2 E_1}{\partial x^2} \quad (9)$$

that determines the rate of decreasing scattered wave amplitude along the direction of its propagation due to the diffractive divergence.

As has already been mentioned, in the steady-state EAS, the contributions to the scattered wave amplitude, caused by the scattering and diffractive divergence, must exactly compensate each other. Therefore, the sum of rates (7) and (9) must give zero. This condition results in the following equation:

$$\frac{d^2 E_1(x)}{dx^2} + K_0 E_0(x) = 0, \quad (10)$$

where

$$K_0 = -2k_1 \Gamma_0 \sin(\eta - \theta_0). \quad (11)$$

Note that in equations (8)–(11) we have deliberately used k_1 instead of k_0 , even though $k_1 = k_0$ for bulk electromagnetic waves. This was done to make the presented derivation valid for all types of waves, including surface and guided modes, for which k_1 may be not equal to k_0 (e.g. for scattering of TE modes guided by a slab into TM modes of the same slab).

Equation (10) is one of the coupled wave equations describing EAS (or DEAS) in a non-uniform periodic array. The second equation is obtained from equation (5). Recall that in the considered geometry of scattering (see figure 1), the incident wave amplitude must depend only on the x -coordinate. Therefore, substituting $dx_0 = dx_1 \cos \eta = dx \cos \eta / \cos \theta_0$ into equation (5) gives:

$$\frac{dE_0(x)}{dx} = iK_1 E_1(x), \quad (12)$$

where

$$K_1 = \Gamma_1 \cos \eta / \cos \theta_0. \quad (13)$$

Equations (10) and (12) are the complete set of coupled wave equations for EAS (or DEAS) of electromagnetic waves in a non-uniform periodic array with varying grating amplitude. As has already been mentioned above, these coupled wave equations are valid for the description of EAS and DEAS of all types of waves, including surface and guided optical modes. The difference between, for example, bulk and guided optical waves is only in different values of the coupling coefficients Γ_0 and Γ_1 . For bulk electromagnetic waves [8]

$$\Gamma_0 = -\Gamma_1^* = -\epsilon_1^* \omega^2 / [2c^2 k_0 \cos \eta] \quad (14)$$

while for guided modes, these coefficients are determined using one of the conventional dynamic theories of scattering for optical slab modes [12–15] (see section 6 for more detail).

There is no coupling between incident and scattered waves outside the array, i.e. at $x < 0$ and $x > L$. Therefore, the coupling coefficients $K_{0,1}$ are equal to zero outside the array, and the coupled wave equations (10) and (12) take the form:

$$d^2 E_1(x)/dx^2 = 0; \quad dE_0(x)/dx = 0. \quad (15)$$

One can easily see that the coupled wave equations (10), (12) and (15) in the case of non-uniform arrays with slowly varying grating amplitude appear to have exactly the same form as for uniform arrays [6–9]. However, for non-uniform arrays, the coupling coefficients K_1 and K_0 in equations (10) and (12) are dependent on the x -coordinate because the grating amplitude ϵ_1 depends on x . This results in substantial difficulties with analytical solution of these equations. Therefore, in sections 3 and 4, we present results of numerical analysis of the coupled wave equations (10) and (12) for periodic arrays with several typical dependences of the grating amplitude on the x -coordinate. The numerical method and boundary conditions used are described in the appendix.

III. EAS IN NON-UNIFORM ARRAYS

In this section, we analyse numerically the effect of slowly varying grating amplitude on EAS of bulk TE electromagnetic waves in periodic Bragg arrays. The results obtained for the non-uniform arrays are compared with those for EAS in uniform arrays [8]. We also investigate numerically the tolerance of EAS to small gradual variations (imperfections) in the grating amplitude.

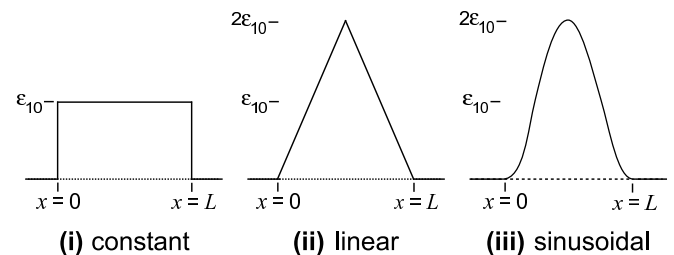


FIG. 2: Three different profiles of the grating amplitude inside a periodic array: (i) constant grating amplitude (uniform array), (ii) linearly varying grating amplitude (non-uniform array), and (iii) sinusoidally varying grating amplitude (non-uniform array). The mean grating amplitudes are the same.

For example, consider three different types of dependences of the grating amplitude on the x -coordinate inside the array—figures 2(i)–(iii). In figure 2(i), the grating amplitude is constant throughout the array, i.e. we consider a uniform array of width L and the grating amplitude ϵ_{10} . EAS in such arrays was investigated previously for bulk [8] and guided [6, 9] optical waves. Mathematically, the profile of the grating amplitude in such an array is given by the equations:

$$\epsilon_1(x) = \begin{cases} \epsilon_{10} & \text{if } 0 < x < L, \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

The second of the considered arrays is a non-uniform array with a linear dependence of magnitude of the grating amplitude on the x -coordinate in such a way that the

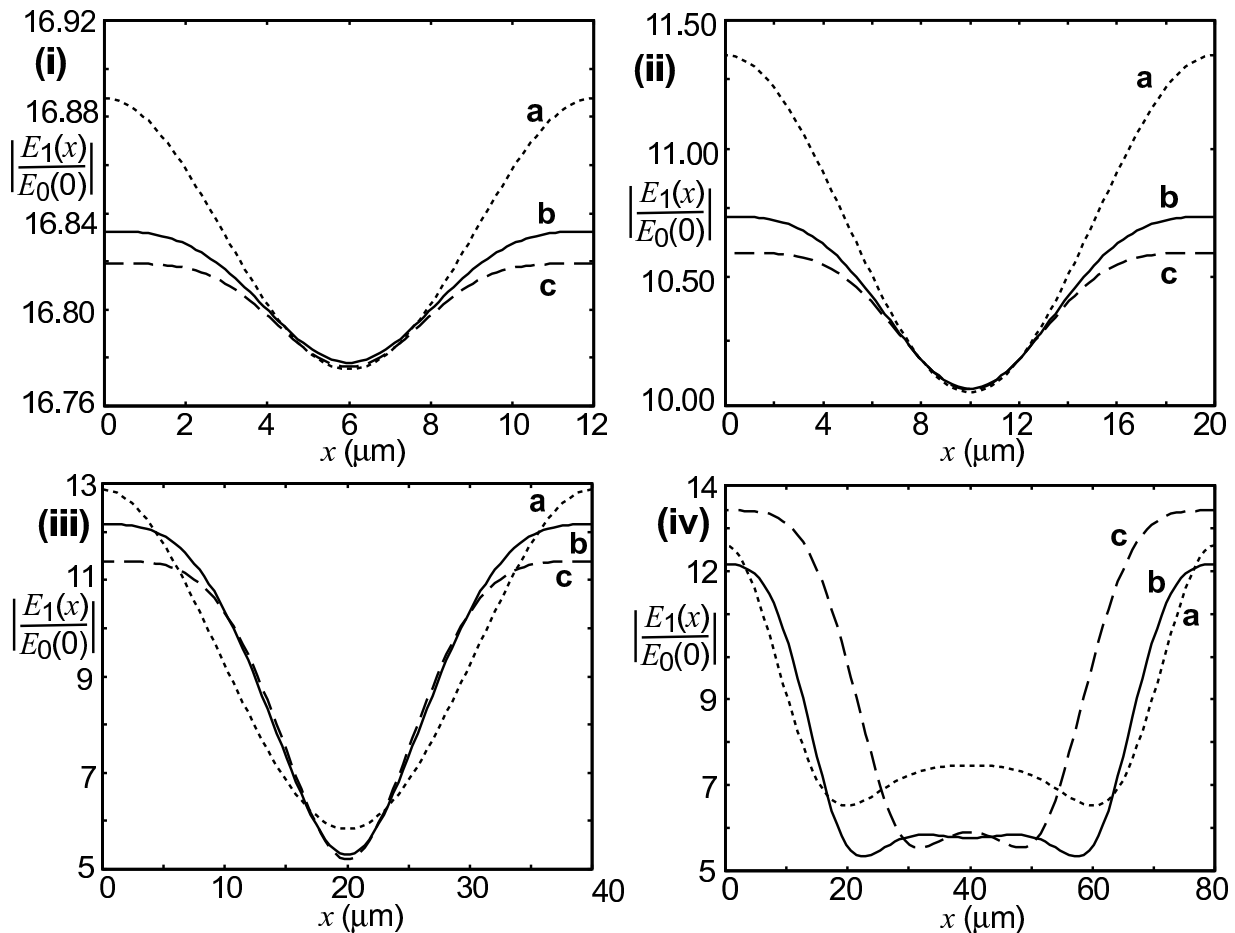


FIG. 3: The dependences of the relative scattered wave amplitudes on distance from the front array boundary inside the arrays with $\epsilon_{10} = 5 \times 10^{-3}$, $\epsilon = 5$, $\theta_0 = \pi/4$, the wavelength in vacuum $\lambda = 1 \mu\text{m}$ (the grating has a period of $0.58 \mu\text{m}$ and is inclined at the angle of $\pi/8$ to the front array boundary); (i) $L = 12 \mu\text{m}$, (ii) $L = 20 \mu\text{m}$, (iii) $L = 40 \mu\text{m}$, (iv) $L = 80 \mu\text{m}$. Curves (a): uniform array with constant grating amplitude ϵ_{10} —figure 2(i). Curves (b): non-uniform array with the linear dependence of $\epsilon_1(x)$ —figure 2(ii). Curves (c): non-uniform array with the sinusoidal dependence of $\epsilon_1(x)$ —figure 2(iii).

grating amplitude is zero at the front and rear boundaries of the array (figure 2(ii)):

$$\epsilon_1(x) = \begin{cases} 4\epsilon_{10}x/L & \text{if } 0 < x < L/2, \\ 4\epsilon_{10}(L-x)/L & \text{if } L/2 < x < L, \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

The gradient of changing grating amplitude in (17) is chosen such that the mean grating amplitude in the non-uniform array is the same as in the uniform array described by equations (16). Thus the integral

$$\int_0^L \epsilon_1(x) dx \quad (18)$$

must be the same for both the dependences presented by equations (16) and (17). This condition is necessary, because EAS is strongly dependent on the grating amplitude [6–9]. If, for example, $\epsilon_1(x)$ increased gradually from zero to ϵ_{10} , the mean grating amplitude would be

noticeably smaller than that of the uniform array, resulting in a significant increase in the scattered wave amplitude. If we assume that the mean grating amplitudes are the same for the uniform and nonuniform arrays of the same widths (i.e. integral (18) is the same for both the arrays), then we will be able to investigate only the effects of non-uniformity of the grating on EAS.

The third of the considered arrays is characterized by sinusoidal variations of the grating amplitude so that the grating amplitude is again zero at the front and rear array boundaries (figure 2(iii)):

$$\epsilon_1(x) = \begin{cases} \epsilon_{10}[1 - \cos(2\pi x/L)] & \text{if } 0 < x < L, \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

Here, the mean grating amplitude is again the same as for arrays (16) and (17), i.e. integral (18) is the same for all three dependences (16), (17) and (19). The relationship between the maxima of the grating amplitudes in arrays

(16), (17) and (19) can be written as

$$\begin{aligned} \max\{\epsilon_1(x)\}_{\text{linear}} &= \max\{\epsilon_1(x)\}_{\text{sinusoidal}} \\ &= 2 \max\{\epsilon_1(x)\}_{\text{stepwise}} \equiv 2\epsilon_{10}. \end{aligned} \quad (20)$$

The selection of non-uniform arrays in the form of (17) and (19) is not arbitrary. Analysis of arrays with zero grating amplitude at the front and rear boundaries (as for arrays in figures 2(ii) and 2(iii)) is practically important because such arrays may significantly improve the side-lobe structure of filtered signals [15] and substantially reduce unwanted edge effects at the array boundaries.

Figure 3 presents the dependences of the scattered wave amplitudes on the x -coordinate inside periodic arrays (16), (17) and (19). It can be seen that for all array widths, slowly varying grating amplitude does not introduce radical changes in the pattern of scattering. It can also be seen that the effect of slowly varying grating amplitude on amplitudes of the scattered waves is noticeably smaller for narrow arrays. For example, for arrays of $12\ \mu\text{m}$ thickness (figure 3(i)), the difference between the scattered wave amplitude E_1 in the non-uniform array with sinusoidal variation of the grating amplitude (19) and the scattered wave amplitude in the uniform array (16) is less than 0.4%. For linear dependence (17) this difference is even smaller (figure 3(i)). At the same time, it can reach 10–15% in arrays of $40\ \mu\text{m}$ width (figure 3(iii)), and over 25% in arrays of $80\ \mu\text{m}$ width (figure 3(iv)).

This can be explained by the diffractive divergence of the scattered wave. In narrow arrays, the diffractive divergence effectively smooths out variations in the amplitude of the scattered wave, caused by different values of the grating amplitude. For such arrays, the scattered wave amplitudes are determined (to a very good accuracy) by an average value of the grating amplitude (figures 3(i) and (ii)). However, the diffractive divergence is effective only within a limited distance. This is because when the scattered wave spreads along the x -axis (due to the diffractive divergence) from one part of the array to another, it experiences rescattering in the grating. As a result, the wave can diverge along the x -axis only until it is re-scattered by the grating, i.e. it can spread only within a finite distance (see also [10]). This distance must increase with decreasing mean grating amplitude, because the efficiency of re-scattering decreases with decreasing grating amplitude. It should also increase with increasing gradient $d\epsilon_1(x)/dx$, because in this case variations of the amplitude of the scattered wave along the wavefront will increase, resulting in stronger diffractive divergence. Typically, the distance through which the scattered wave can spread inside the array before being re-scattered is about several γ^{-1} [10], where $\gamma = (K_0 K_1)^{1/3}$.

In our examples (see equations (16), (17), (19) and figures 2 and 3), the critical array width is about $\sim 3\gamma_{\text{av}}^{-1}$, where γ_{av} is the average value of γ in the considered arrays. Therefore, if $L < 3\gamma_{\text{av}}^{-1}$, then the scattered wave amplitude is very well determined by the average value

of the grating amplitude inside the array (figures 3(i) and 3(ii)). If the array width $L > 3\gamma_{\text{av}}^{-1}$, then the scattered wave amplitudes inside a non-uniform array are significantly affected by local values of the grating amplitude, and the pattern of scattering differs from that for the uniform array with the average grating amplitude (figures 3(iii) and 3(iv)).

So far, we have analysed non-uniform arrays with significant variations of the grating amplitude. Another practically important problem is the effect on EAS of small grating imperfections, e.g. variations in the grating amplitude, which are inevitable during manufacture of periodic arrays. Therefore, in this paper, we analyse tolerance of EAS to small variations of the grating amplitude across the array (i.e. in the x -direction).

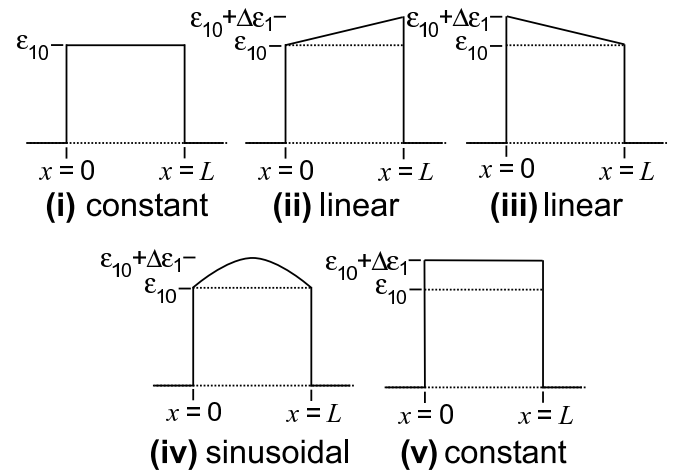


FIG. 4: Five different profiles of the grating amplitude with small variations inside a periodic array: (i) constant grating amplitude ϵ_{10} (uniform array), (ii) linearly increasing grating amplitude (non-uniform array), (iii) linearly decreasing grating amplitude (non-uniform array), (iv) sinusoidally varying grating amplitude (non-uniform array), and (v) constant grating amplitude $\epsilon_{10} + \Delta\epsilon_1$ (uniform array).

Figure 4 presents three different types of small and slow variations of the grating amplitude on the x -coordinate in an initially uniform periodic array. These are linearly increasing (figure 4(ii)), decreasing (figure 4(iii)) and sinusoidal (figure 4(iv)) dependences of the grating amplitude with the minimum and maximum values of the grating amplitude equal to ϵ_{10} and $\epsilon_{10} + \Delta\epsilon_1$, respectively, where $\Delta\epsilon_1 \ll \epsilon_{10}$.

The results of the numerical analysis of the effect of small and slow variations of the grating amplitude on the scattered wave amplitude inside the array are presented in figures 5(i) and 5(ii) for $\Delta\epsilon_1 = \epsilon_{10}/10 = 5 \times 10^{-4}$ for two different array widths: (i) $L = 12\ \mu\text{m}$, and (ii) $L = 20\ \mu\text{m}$. It can be seen that curves (b) and (c) in figure 5(i) coincide with each other. This again reflects the fact that for narrow arrays, the specific dependence of small gradual variations of the grating amplitude does not affect the scattered wave amplitudes that are depen-

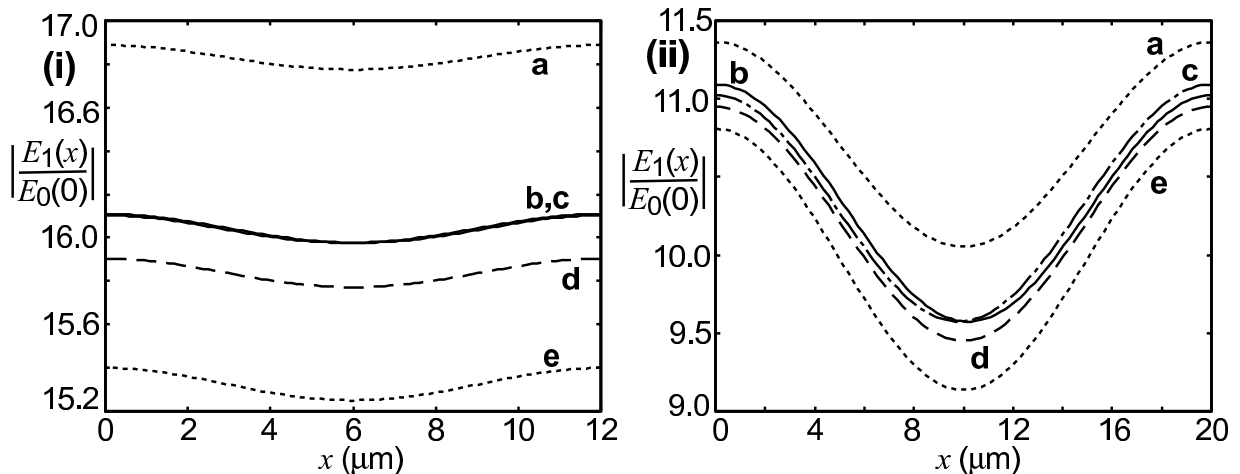


FIG. 5: The dependences of the relative scattered wave amplitudes on distance from the front array boundary inside the arrays (figure 4) with $\epsilon_{10} = 5 \times 10^{-3}$, $\Delta\epsilon_1 = \epsilon_{10}/10$, $\epsilon = 5$, $\theta_0 = \pi/4$, the wavelength in vacuum $\lambda = 1 \mu\text{m}$ (the grating has a period of $0.58 \mu\text{m}$ and is inclined at an angle of $\pi/8$ to the front array boundary); (i) $L = 12 \mu\text{m}$, (ii) $L = 20 \mu\text{m}$. Curves (a): uniform arrays with constant grating amplitude ϵ_{10} —figure 4(i). Curves (b): non-uniform arrays with linearly increasing $\epsilon_1(x)$ —figure 4(ii). Curves (c): non-uniform arrays with linearly decreasing $\epsilon_1(x)$ —figure 4(iii). Curves (d): non-uniform arrays with sinusoidal dependence of $\epsilon_1(x)$ —figure 4(iv). Curves (e): uniform arrays with constant grating amplitude $\epsilon_{10} + \Delta\epsilon_1$ —figure 4(v).

dent only on the average values of the grating amplitude in the array. Curve (d) in figure 5(i) differs from curves (b) and (c). However, this difference is related not with the sinusoidal profile of the dependence of the grating amplitude on the x -coordinate (figure 4(iv)), but with the slightly different value for the mean grating amplitude in this array. If the mean grating amplitudes were the same for all three arrays shown in figures 4(ii)–(iv), then curves (b)–(d) in figure 5(i) would have been indistinguishable. Curves (a) and (e) correspond to the uniform arrays with the grating amplitudes ϵ_{10} (figure 4(i)) and $\epsilon_{10} + \Delta\epsilon_1$ (figure 4(v)), respectively.

Figure 5(ii) shows that as the array width increases, the shape of the dependence of the grating amplitude on the x -coordinate becomes significant. For example, for the array width of $20 \mu\text{m}$, the scattered wave amplitudes are slightly different for arrays with increasing and decreasing grating amplitude (figures 4(ii) and (iii))—see curves (b) and (c) in figure 5(ii). If the array width is increased to 40 or more μm (i.e. $L > 3\gamma_{av}^{-1}$), then the scattered wave amplitude at any point inside the array is approximately the same as it would have been in the uniform array of the same width and with the grating amplitude taken from the point of observation in the non-uniform array. That is, the scattered wave amplitudes are determined by local values of the grating amplitude. Note that this statement is correct only for slow and small variations (imperfections) of the grating amplitude (figure 4). It is not valid for arrays where the grating amplitude tends to zero somewhere in the array or at its boundaries (see figures 2 and 3). If the above statement was valid for such arrays, this would have meant that the scattered wave amplitude in parts of the array with $\epsilon_1 \rightarrow 0$ must

increase to infinity, and due to the diffractive divergence, it must then be infinite in all other parts of the array, which is not possible.

Increase in $\Delta\epsilon_1$ results in variations of the scattered wave amplitudes that are to a very good approximation directly proportional to $\Delta\epsilon_1$. Therefore, the graphs in figure 5 can be used for evaluation of the scattered wave amplitudes for any other value of $\Delta\epsilon_1 \ll \epsilon_1$.

IV. DEAS IN ARRAYS WITH VARYING GRATING AMPLITUDE

As has been seen from the analysis of the previous section, magnitude variations of the grating amplitude may result in noticeable, but not dramatic, changes in the field distribution inside the array. For narrow arrays, such changes can even be neglected altogether. However, if we consider phase variations in the grating, the situation may change very radically, and especially for narrow arrays.

For example, consider three different types of dependences of the grating amplitude on the x -coordinate inside a non-uniform array—figures 6(i)–(iii). In figure 6(i), the magnitude of the grating amplitude is constant throughout the array, but the phase of the grating experiences a stepwise variation $\phi \approx 180^\circ$ in the middle of the array, i.e. the sign of the grating amplitude $\epsilon_1(x)$ changes when crossing the interface between the two sections of the array at $x = L_1 = L/2$ —see figure 6(i) and figure 1. In all other parts of the array the phase of the grating is not changing (figure 6(i)). Thus we have a non-uniform array consisting of two joint uniform arrays with different

phases of the gratings. In the general case with arbitrary stepwise phase shift ϕ between the gratings in the joint arrays, the dependence of the grating amplitude on the x -coordinate is given by the equations:

$$\epsilon_1(x) = \begin{cases} \epsilon_{10} & \text{if } 0 < x < L/2, \\ \epsilon_{10} \exp(i\phi) & \text{if } L/2 < x < L, \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

If $\phi \approx 180^\circ$, then equations (21) give the dependence of the grating amplitude presented in figure 6(i).

The second considered type of non-uniform arrays is characterized by a linear dependence of magnitude of the grating amplitude on the x -coordinate in such a way, that the amplitude of the grating is zero at the front and rear array boundaries, as well as at the interface $x = L_1 = L/2$ (figure 6(ii)). There is also a stepwise variation in the phase of the non-uniform grating at $x = L_1 = L/2$, which is equal to $\phi \approx 180^\circ$. Thus we have two joint non-uniform arrays with linearly varying magnitude of the grating amplitude and constant phase of the grating in each of the joint arrays (figure 6(ii)). If the stepwise variation of the phase between the two joint arrays is arbitrary, then the dependence of the grating amplitude on the x -coordinate is given by

$$\epsilon_1(x) = \begin{cases} 4\epsilon_{10}x/L & \text{if } 0 < x \leq L/4, \\ 4\epsilon_{10}(L/2 - x)/L & \text{if } L/4 < x \leq L/2, \\ 4\epsilon_{10} \exp(i\phi)(x - L/2)/L & \text{if } L/2 < x \leq 3L/4, \\ 4\epsilon_{10} \exp(i\phi)(L - x)/L & \text{if } 3L/4 < x < L, \\ 0 & \text{otherwise,} \end{cases} \quad (22)$$

where the gradient of changing magnitude of the grating amplitude is chosen so that the mean magnitude of the grating amplitude in the array described by equations (22) is the same as for the array described by equations

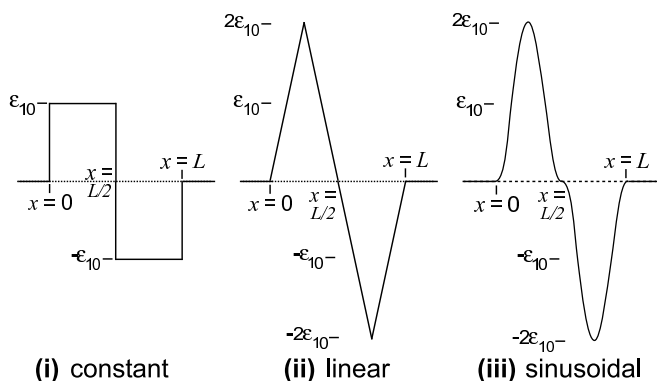


FIG. 6: Three different profiles of the grating amplitude with a stepwise phase variation $\phi \approx 180^\circ$ at $x = L/2$: (i) constant magnitude of the grating amplitude, (ii) linearly varying grating amplitude, and (iii) sinusoidally varying grating amplitude.

(21). That is,

$$\int_0^{L/2} |\epsilon_1(x)| dx = \int_{L/2}^L |\epsilon_1(x)| dx = G, \quad (23)$$

where the constant G is the same for both the dependences (21) and (22). The reasons for using this condition are the same as the reasons for integral (18) being the same for the arrays with the grating amplitude given by equations (16), (17), and (19).

The third of the considered arrays with a phase variation in the grating is presented in figure 6(iii). It is characterized by sinusoidal variations of magnitude of the grating amplitude inside the joint arrays in such a way that this magnitude is again zero at the boundaries of the periodic array and at the interfaces $x = L_1 = L/2$. The phase of the grating is again constant inside each of the joint arrays, but experiences a stepwise variation $\phi \approx 180^\circ$ at $x = L_1 = L/2$ (where the magnitude of the grating amplitude is zero)—figure 6(iii). If the phase variation ϕ is arbitrary, then such non-uniform arrays can be described by the equations:

$$\epsilon_1(x) = \begin{cases} \epsilon_{10}[1 - \cos(4\pi x/L)] & \text{if } 0 < x < L/2, \\ \epsilon_{10}e^{i\phi}\{1 - \cos[2\pi(2x/L)]\} & \text{if } L/2 < x < L, \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

where condition (23) is again satisfied.

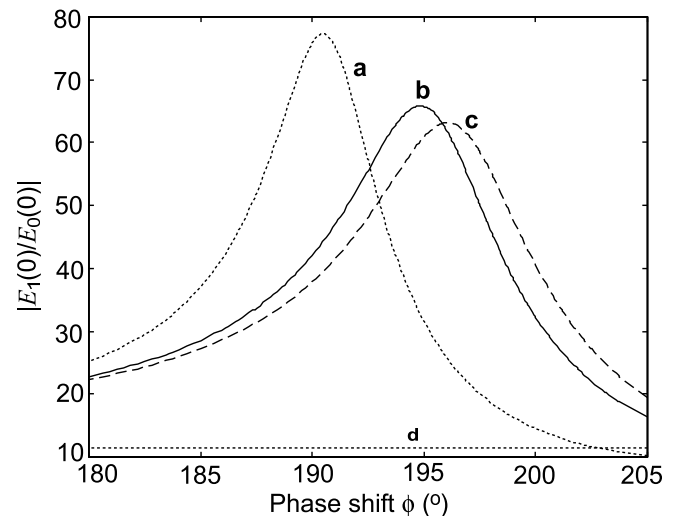


FIG. 7: The dependences of the relative scattered wave amplitudes at the front boundary on the phase shift ϕ for the non-uniform arrays described by equations (21), (22), (24) with $\epsilon_{10} = 5 \times 10^{-3}$, $\epsilon = 5$, $\theta_0 = \pi/4$, $L = 20 \mu\text{m}$, and the wavelength in vacuum $\lambda = 1 \mu\text{m}$. Curve (a): non-uniform array with constant magnitude of the grating amplitude. Curve (b): non-uniform array with the linear dependence of $\epsilon_1(x)$. Curve (c): non-uniform array with the sinusoidal dependence of $\epsilon_1(x)$. Curve (d): uniform array with $\phi = 0$.

The dependences of the ratio of the amplitudes of the scattered and incident waves at the front array boundary

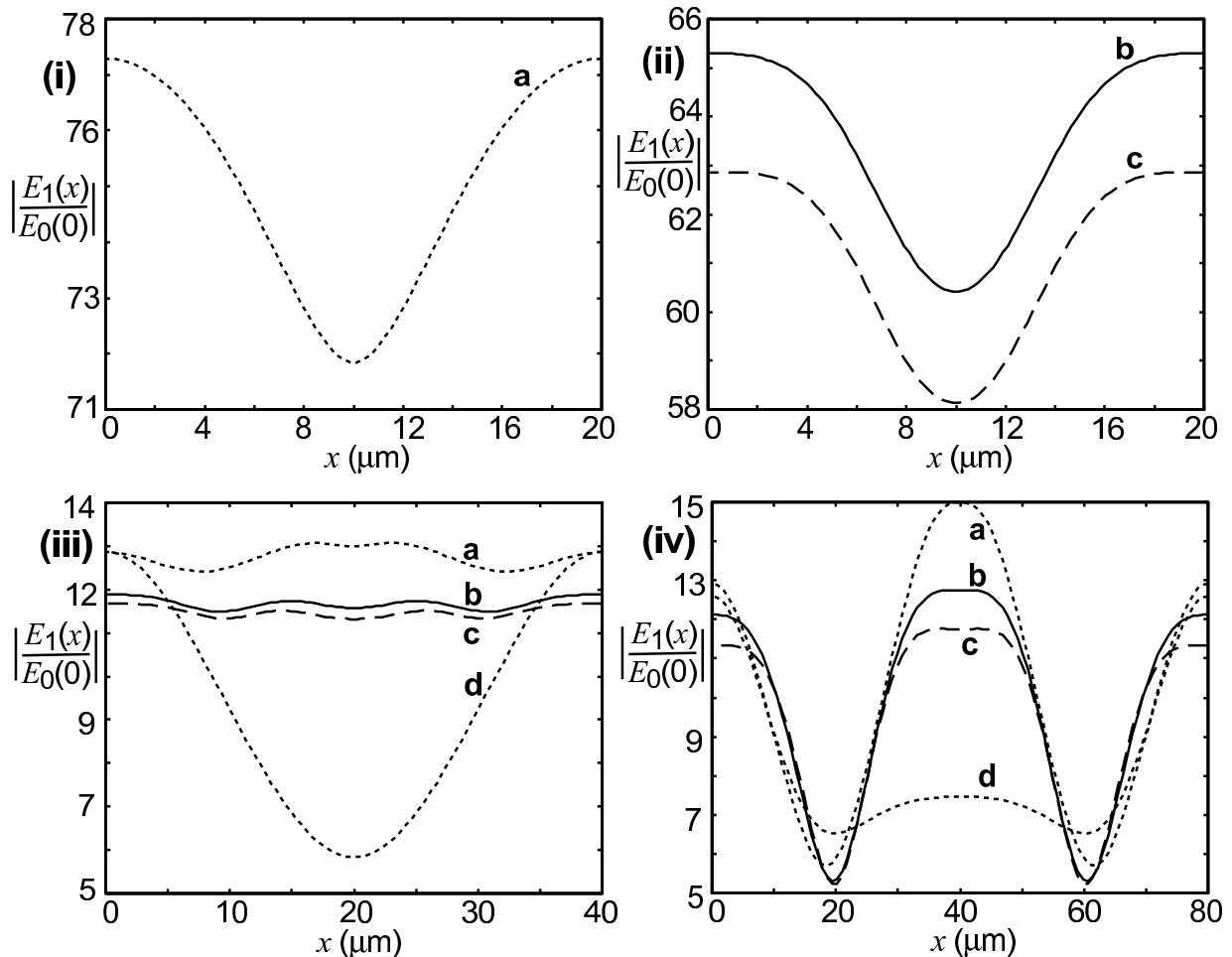


FIG. 8: The dependences of the relative scattered wave amplitudes on distance from the front array boundary inside the non-uniform arrays described by equations (21), (22), (24) with $\epsilon_{10} = 5 \times 10^{-3}$, $\epsilon = 5$, $\theta_0 = \pi/4$, the wavelength in vacuum $\lambda = 1 \mu\text{m}$; (i) and (ii) $L = 20 \mu\text{m}$, (iii) $L = 40 \mu\text{m}$, (iv) $L = 80 \mu\text{m}$. Curves (a): non-uniform arrays with constant magnitude of the grating amplitude (see equations (21)); (i) $\phi = 190.4^\circ$, (iii) $\phi = 178.6^\circ$, (iv) $\phi = 157.6^\circ$. Curves (b): non-uniform arrays with the linear dependence of $\epsilon_1(x)$ (see equations (22)); (ii) $\phi = 194.7^\circ$, (iii) $\phi = 182.5^\circ$, (iv) $\phi = 133.8^\circ$. Curves (c): non-uniform arrays with the sinusoidal dependence of $\epsilon_1(x)$ (see equation (24)); (ii) $\phi = 195.9^\circ$, (iii) $\phi = 184.6^\circ$, (iv) $\phi = 129.4^\circ$. Curves (d): uniform arrays with $\phi = 0$ and constant grating amplitude ϵ_1 .

on the phase shift ϕ in non-uniform arrays (21), (22) and (24) with $L = 20 \mu\text{m}$ and $\epsilon_{10} = 5 \times 10^{-3}$ are presented in figure 7 for bulk TE electromagnetic waves that are incident onto the arrays at the angle $\theta_0 = \pi/4$. This figure demonstrates that when the value of the phase shift is relatively close to π , then the amplitude of the scattered wave at the front array boundary very strongly (resonantly) increases as compared with the conventional EAS (cf curves (a)–(c) with curve (d) in figure 7). Thus we have two simultaneous resonances in the structure. One of these resonances occurs at a resonant frequency (wavelength) that is determined by the Bragg condition (3). This resonant is obviously common for all types of Bragg scattering. In the case of EAS, it results in a strong increase of the scattered wave amplitude see curve (d) in figure 7.

The second resonance takes place at an optimal (reso-

nant) phase shift in a non-uniform array. This resonance occurs on the background of an already resonantly large scattered wave amplitude that is typical for the conventional EAS. As a result, the scattered wave amplitude increases many times further as compared with the incident wave amplitude (see curves (a)–(c) in figure 7). This effect was called DEAS [10].

Curve (a) in figure 7 corresponds to the non-uniform array with the grating amplitude given by equations (21). This is the case of DEAS that was analysed previously in [10] for bulk electromagnetic waves. Curves (b) and (c) demonstrate that similar strong DEAS also occurs in non-uniform arrays with varying magnitude of the grating amplitude, e.g. in arrays with the grating amplitude given by equations (22) and (24). It can be seen that the maxima of the scattered wave amplitude at the front array boundary are about 15% lower than for curve (a), and

resonant values of the phase variation are shifted to larger values, i.e. to about 195° as compared with 190.4° for the array with constant magnitude of the grating—curve (a). This demonstrates that slowly increasing grating amplitude may result in noticeable changes of resonant values of ϕ as compared with DEAS in arrays with constant magnitude of the grating amplitude. On the other hand, curves (b) and (c) are very similar, and this indicates that there is no significant difference in DEAS in non-uniform arrays with linearly and sinusoidally varying grating amplitude. Thus DEAS proves to be fairly insensitive to the particular profile of the slowly varying grating amplitude inside the array.

Physically, the differences between curves (a)–(c) in figure 7 are related to the fact that if the grating amplitude has the profile described by equations (22) and (24) (see also figures 6(ii) and 6(iii)), then the amplitude of the grating near the middle of the array (where the phase variation takes place) is very small, and it is smaller for the sinusoidal dependence (24). Thus, the efficiency of re-scattering in this area is low. However, this is the region near the interface with the phase variation, where the diffractive divergence of the scattered wave and its re-scattering in the array result in increasing the incident wave amplitude, which in turn gives rise to larger scattered wave amplitudes [10]. Therefore, the amplitude of the scattered wave must be smaller in arrays with the grating amplitude that decreases in the middle of the array (i.e. near the interface with the phase variation in the grating). Thus the resonance with respect to phase variation is broader and weaker for curves (b) and (c), than for curve (a) in figure 7. Similarly, the resonance for curve (c), corresponding to sinusoidal variations of the grating amplitude, is slightly weaker and broader than that for the linear dependence (curve (b)).

The dependences of the scattered wave amplitude on the x -coordinate inside the non-uniform arrays (21), (22) and (24) of different widths are presented in figure 8. Here, the values of the phase shift ϕ are chosen so that to give maximal scattered wave amplitude at the interface $x = L/2$ where the step-like phase variation occurs. It can be seen that as the array width decreases from $40\text{--}20\ \mu\text{m}$, the scattered wave amplitude increases dramatically (more than five times). This feature is typical of DEAS in arrays with constant and varying magnitude of the grating amplitude (see also [10]). As can be seen from figure 8, $L = L_c \approx 40\ \mu\text{m}$ is a critical array width, because strong DEAS takes place only if $L < 40\ \mu\text{m}$. This critical array width is the same for all the arrays (21), (22) and (24), regardless of whether the magnitude of the grating amplitude is varying or not. As has already been mentioned, the critical array width is determined by the distance within which the scattered wave may spread along the x -axis before being re-scattered by the grating [10]. The average grating amplitudes in arrays (22) and (24) are the same as in array (21) (see condition (23)), and therefore this distance is the same for the analysed arrays.

If $L > L_c$ (figure 8(iv)), then the amplitude of the scattered wave at the front and rear boundaries of the periodic arrays are approximately the same as in the case of EAS in the uniform array, because the diffractive divergence of the scattered wave from the opposite side of the array makes only negligible contribution to the scattering. However, in the middle of the array near the interface with the phase variation, we observe noticeable maxima of the scattered wave amplitudes for all three considered arrays. The width of these maxima is the same and equal (as expected) to the critical array width. The maximum corresponding to array (24) is the smallest, while the maximum corresponding to the array with constant magnitude of the grating amplitude is the biggest figure 8(iv). This is again due to the fact that the scattering near the interface $x = L_1$ in arrays (22) and (24) is impeded by the small grating amplitude.

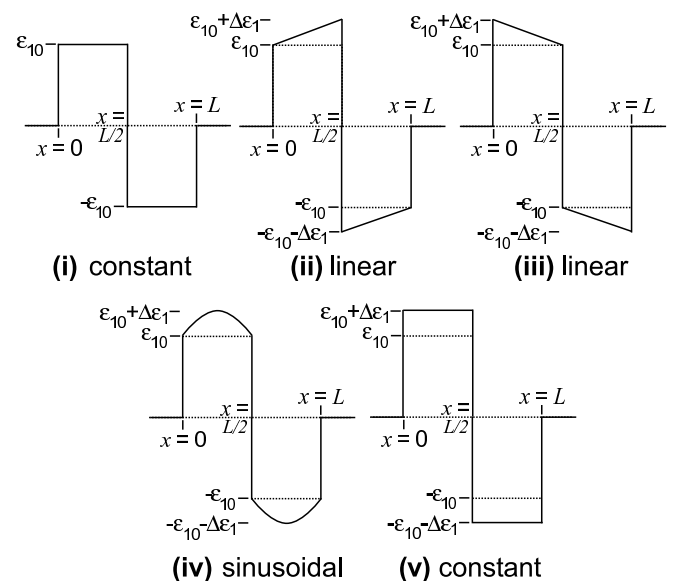


FIG. 9: Five different profiles of the grating amplitude inside non-uniform periodic arrays. The grating amplitude has small gradual magnitude variations, and a stepwise phase variation $\phi \approx 180^\circ$ at $x = L/2$.

The effect of small and gradual linear and sinusoidal variations of magnitude of the grating amplitude (figure 9) on DEAS is presented in figure 10 for $\Delta\epsilon_1 = \epsilon_{10}/10 = 5 \times 10^{-4}$ and $L = 20\ \mu\text{m}$. Note that unlike EAS in non-uniform arrays with small variations of the grating amplitude (figure 5), DEAS appears to be more sensitive to the particular shape of the dependence of $\epsilon_1(x)$ on x —cf curves (b) and (c) in figure 10. The sensitivity of DEAS to small imperfections of the array is also noticeably stronger than for EAS (the difference between curves (e) and (a) in figure 10 reaches up to $\sim 50\%$). This is understandable because the resonance during DEAS is noticeably sharper (due to the combination of the two simultaneous resonances) than the resonance during EAS.

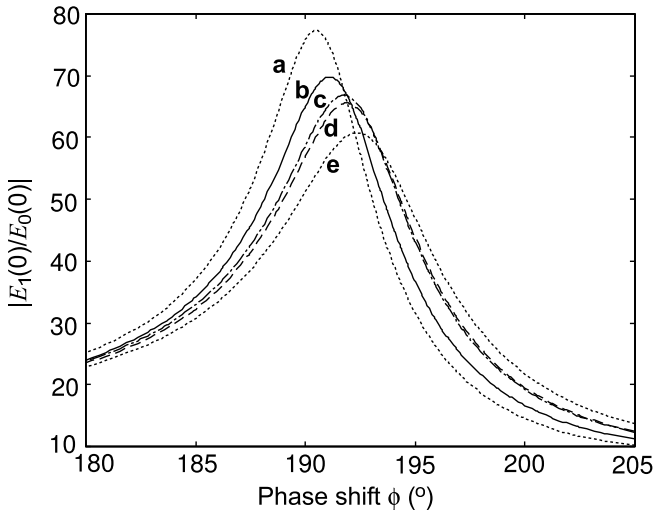


FIG. 10: The dependences of the relative scattered wave amplitudes at the front array boundary on the phase shift ϕ for the non-uniform arrays presented in figure 9 with $\epsilon_{10} = 5 \times 10^{-3}$, $\Delta\epsilon_1 = \epsilon_{10}/10$, $\epsilon = 5$, $\theta_0 = \pi/4$, $L = 20 \mu\text{m}$, and the wavelength in vacuum $\lambda = 1 \mu\text{m}$. Curves (a)–(e) correspond to the non-uniform arrays represented by figures 9(i)–(v), respectively.

V. GUIDED ELECTROMAGNETIC MODES

It was mentioned in section 2 that equations (5)–(13) and (15) are valid for all types of waves, including guided optical modes. This is because the approach based on allowance for the diffractive divergence of the scattered wave, which led to the coupled wave equations (10) and (12), is readily applicable for analysis of EAS of guided modes in a slab with a periodically corrugated boundary. However, in this case the coupling coefficients Γ_0 and Γ_1 are no longer given by equations (14), but are determined in one of the following modern theories of mode coupling in corrugated optical waveguides: boundary perturbation theory [16], mode matching theory [17], local-normal-mode theory [18], or direct approximate solution of the wave equation [19]. These theories are correct for any type of polarization (TE and TM) of slab modes. Therefore, the results of this paper are also correct for any type of guided slab modes.

The geometry of EAS and DEAS of slab modes is again presented by figure 1. In this case the plane of the figure is the plane of a slab, and the periodic structure is represented by a periodic corrugation of a slab boundary. If we consider a non-uniform periodic groove array with varying grating amplitude, then the corrugation is given by the equations

$$\xi = \begin{cases} d + \xi_1(x)f(x_0) & \text{for } 0 < x < L; \\ d & \text{for } x < 0 \text{ or } x > L, \end{cases} \quad (25)$$

where d is the thickness of the guiding slab, $\xi_1(x)$ is the varying grating amplitude (corrugation amplitude),

$f(x_0)$ is an arbitrary periodic function with a period of $2\pi/q$, with $\max\{|f(x_0)|\} = 1$, and a mean value of zero. Dissipation is neglected and all media in contact are isotropic.

For the approximation of slowly varying amplitudes to be valid, the corrugation must be small as compared with the grating period:

$$|\xi_1|q/(2\pi) \ll 1. \quad (26)$$

This inequality is similar to condition (2) for bulk waves.

If the corrugation is non-sinusoidal (the function $f(x_0)$ is not a sine or cosine), it can be expanded into the Fourier series

$$f(x_0) = \sum_{p=-\infty}^{+\infty} f_p \exp(ipq_x x + ipq_y y) = \sum_{p=-\infty}^{+\infty} f_p \exp(ipq x_0). \quad (27)$$

In this case, only two complex conjugate terms (similar to equations (1)) satisfying the Bragg condition

$$\mathbf{k}_1 - \mathbf{k}_0 = -p\mathbf{q} \quad (28)$$

must be taken into account. Here, \mathbf{q} is the reciprocal lattice vector of the grating, which is parallel to the x_0 -axis—see figure 1.

The vectors \mathbf{k}_0 and \mathbf{k}_1 are the wavevectors of the incident and scattered guided modes. Note that in the case of guided modes, $|\mathbf{k}_0| = |\mathbf{k}_1|$ only for $\text{TE}_n\text{--TE}_n$ or $\text{TM}_n\text{--TM}_n$ scattering (n is the order of the mode). If EAS is related with polarization change (e.g. $\text{TE}_n\text{--TM}_n$ scattering), or with change of the mode order (e.g. $\text{TE}_n\text{--TE}_m$ scattering with $n \neq m$), then $|\mathbf{k}_0| \neq |\mathbf{k}_1|$. That is why in equations (8)–(11) we used k_1 even though for bulk waves k_1 is equal to k_0 .

Therefore, all the speculations, derivations and results obtained in sections 3–5 are applicable for EAS and DEAS of guided optical modes in periodic groove arrays with varying corrugation amplitude if $\epsilon_1(x)$, ϵ_{10} , and $\Delta\epsilon_1$ are replaced by $\xi_{1p}(x) = \xi_1(x)f_p$, $\xi_{10p} = \xi_{10}f_p$, and $\Delta\xi_{1p} = \Delta\xi_1 f_p$, respectively. For example, consider EAS or DEAS of an incident zeroth order TE slab mode into a scattered zeroth order TE slab mode in the structure: vacuum—GaAs slab (with permittivity 12.25)—AlGaAs substrate (with permittivity 10.24); a slab thickness of $d = 0.6 \mu\text{m}$, angle of incidence $\theta_0 = \pi/4$, the wavelength in vacuum $\lambda = 1.5 \mu\text{m}$, and the corrugation is assumed to be sinusoidal, i.e. $f(x_0) = \sin(ax_0)$. If the corrugation amplitude in this structure ξ_1 is given by the equation $\xi_1 = 3.8\epsilon_1$ (μm), where ϵ_1 is the grating amplitude used in sections 3–5, then the coupling coefficients K_0 and K_1 are the same as for the bulk TE waves in sections 3–5. Therefore, all the graphs from these sections are valid for the scattering of the guided slab modes in the above structure with the same (as for bulk waves) array widths see sections 3–5.

VI. CONCLUSIONS

In this paper, we have analysed theoretically and numerically the effect of the grating amplitude, slowly varying across a non-uniform periodic array, on EAS and DEAS of bulk and guided optical waves. The main features of EAS in DEAS in such non-uniform arrays were explained by the diffractive divergence of the scattered wave.

In particular, it was shown that the pattern of EAS in narrow arrays with gradually varying magnitude of the grating amplitude is almost exactly the same as for the uniform array with the same width and grating amplitude equal to the average amplitude of the grating in the nonuniform array. On the other hand, in thick arrays, the effect of gradual variations in the grating amplitude on the scattered field distribution was demonstrated to be rather noticeable. At the same time, the main feature of EAS—the strong resonant increase in the scattered wave amplitude—is typical for all considered non-uniform arrays. This will allow use of non-uniform periodic arrays with gradually increasing grating amplitude for suppression of edge effects that are expected to be unusually strong during EAS, and for improving the side-lobe structure of the scattered signals.

The tolerance of EAS to small gradual variations (imperfections) in the grating amplitude inside the arrays was determined. It was shown that EAS in narrow arrays is sensitive only to variations of the mean value of the grating amplitude, while the particular shape of the dependence of the grating amplitude on distance from the array boundaries does not matter. In contrast, in wide arrays, the scattered wave amplitude has been demonstrated to depend mainly on local values of the grating amplitude.

It was also shown that DEAS which was previously analysed in arrays with constant magnitude of the grating amplitude [10] also occurs in non-uniform arrays with varying grating amplitude, though it appears to be more sensitive to the particular profile of the grating amplitude than EAS. Two strong simultaneous resonances in DEAS result in a much greater increase in the scattered wave amplitude for the same amplitude of the grating than during EAS. This is the reason why edge effects must be much more significant for DEAS than for EAS, and the use of arrays with gradually varying amplitude is even more important in this case. The sensitivity of DEAS to small grating imperfections was shown to be noticeably stronger than that of EAS, which is related to the much stronger resonance taking place during DEAS.

The approach for the analysis used in this paper, is based on allowance for the diffractive divergence of the scattered wave. Unlike the dynamic theory of scattering [1–5, 8] that is applicable only for the analysis of EAS of bulk electromagnetic waves, the approach used is directly applicable for the analysis of EAS and DEAS of all types of waves, including bulk, guided and surface optical and acoustic waves in uniform and non-uniform

periodic Bragg arrays. Therefore, the obtained results are also applicable to EAS and DEAS of Rayleigh surface acoustic waves in periodic groove arrays. In this case we only need to use in equations (11) and (13) the appropriate coupling coefficients Γ_0 and Γ_1 from the dynamic theory of scattering of Rayleigh waves in periodic groove arrays [15].

The obtained results will be important for development of new EAS- and DEAS-based structures and devices in optical and acoustic signal-processing, communication, instrumentation and sensor design.

Acknowledgment

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APPENDIX

The numerical solution of equations (10) and (12) was carried out using the following boundary conditions at the array boundaries:

$$\begin{aligned} E_0|_{x=0} &= E_{00}, \\ (dE_1/dx)_{x=0} &= 0, \\ (dE_1/dx)_{x=L} &= 0, \end{aligned} \quad (\text{A.1})$$

where E_{00} is the amplitude of the incident wave at $x < 0$. The scattered wave outside the array is represented (similarly to uniform arrays [6–9]) by two plane waves propagating parallel to the array boundaries one on each side of the array (these waves are the solutions to equations (15)). Their amplitudes, A_1 for $x < 0$, and A_2 for $x > L$, are determined from two other conditions

$$A_1 = E_1|_{x=0+0}, \quad A_2 = E_1|_{x=L-0} \quad (\text{A.2})$$

that are actually independent of equations (A.1).

The condition $|E_0|_{x=L-0} = |E_{00}|$, representing the energy conservation in the steady-state EAS, will be satisfied automatically and we do not need to take it into account (see also [6–9]). Since we neglect edge effects at the array boundaries, the conditions implying the continuity of the derivative dE_0/dx across these boundaries are not taken into account (this is the usual approximation in modern theories of Bragg scattering [6–11, 15–19]). The grating amplitude inside the array is assumed to vary either slowly, or very quickly (stepwise variations). These variations are taken into account in the function $\epsilon_1(x)$ in the coupling coefficients $K_{0,1}$. Therefore, we do not need to take into account boundary conditions at interfaces with step-wise variations in the grating amplitude inside the array (figure 1).

For convenience, the coupled wave equations (10) and (12) are rewritten as a set of three linear first-order dif-

ferential equations:

$$\begin{aligned} dE_0/dx - iK_1(x)E_1(x) &= 0, \\ dE_1/dx - E_2(x) &= 0, \\ dE_2/dx + K_0(x)E_0(x) &= 0. \end{aligned} \quad (\text{A.3})$$

This set of differential equations can be approximated by a set of $3 \times (N - 1)$ finite difference equations (FDEs) on a set of N points $\{x_1, x_2, \dots, x_N\}$:

$$\begin{aligned} E_0(x_n) - E_0(x_{n-1}) + \frac{x_n - x_{n-1}}{4} [E_0(x_n) + E_0(x_{n-1})] \\ \times [iK_1(x_n)E_1(x_n) + iK_1(x_{n-1})E_1(x_{n-1})] &= 0 \\ E_1(x_n) - E_1(x_{n-1}) + \frac{x_n - x_{n-1}}{4} [E_1(x_n) + E_1(x_{n-1})] \\ \times [E_2(x_n) + E_2(x_{n-1})] &= 0, \\ E_0(x_n) - E_0(x_{n-1}) + \frac{x_n - x_{n-1}}{4} [E_0(x_n) + E_0(x_{n-1})] \\ \times [-K_0(x_n)E_2(x_n) - K_0(x_{n-1})E_2(x_{n-1})] &= 0 \quad (\text{A.4}) \end{aligned}$$

where $n = 2, 3, \dots, N$. Since the amplitudes E_0 and E_1 are constant outside the array, the points x_1 and x_N are chosen to correspond to the front and rear boundaries of the array, i.e. $x_1 = 0$ and $x_N = L$. Then boundary conditions (A.1) can be written as:

$$E_0(x_1) = E_{00}, \quad E_2(x_1) = 0, \quad E_2(x_N) = 0. \quad (\text{A.5})$$

Equations (A.4) and (A.5) form a set of $3N$ linear algebraic equations with $3N$ variables $E_0(x_1), E_0(x_2), \dots, E_0(x_N), E_1(x_1), E_1(x_2), \dots, E_1(x_N), E_2(x_1), E_2(x_2), \dots, E_2(x_N)$, which was solved numerically for different profiles of varying grating amplitude inside the array (see sections 3–5).

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