This is an e-print of:
Timo A. Nieminen, Norman R. Heckenberg and Halina Rubinsztein-Dunlop
"Measurement of rotation speed of birefringent material and optical torque from polarisation of transmitted light"
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Abstract

The rotation speed of, and optical torque acting on, an optically trapped birefringent particle can be determined from the polarisation of the transmitted light. This can be used to determine, for example, viscous drag torque.

Author's notes:

A more complete paper covering the method published in this paper was published:
T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein Dunlop
"Optical measurement of microscopic torques"
Journal of Modern Optics **48**, 405–413 (2001).
It is probably unwise to talk about "circular and plane-polarised components", since the circular and plane basis vectors for polarisation are not mutually orthogonal. That is, a circular basis vector is not orthogonal to either plane basis vector, and vice versa.

This method has been implemented:
Alexis I. Bishop, Timo A. Nieminen, Norman R. Heckenberg, and Halina Rubinsztein Dunlop
"Optical application and measurement of torque on microparticles of isotropic nonabsorbing material"
Physical Review A **68**, 033802 (2003)
Alexis I. Bishop, Timo A. Nieminen, Norman R. Heckenberg, and Halina Rubinsztein Dunlop
"Optical microrheology using rotating laser-trapped particles"
Physical Review Letters **92**, 198104 (2004)
The first of these papers includes formulae for calculation of torque from measurements of circular/plane polarisation (without any problems with non-orthogonal mixed bases).

It is claimed in this paper that negative uniaxial materials are stably trapped in 3D with the optic axis normal to the beam axis. This is incorrect, and it is in fact *positive* uniaxial materials (eg, as used in the microrheology paper cited above) that are trapped stably in the proper orientation (see Arthur La Porta and Michelle D.Wang, "Optical torque wrench: Angular trapping, rotation, and torque detection of quartz microparticles", *Physical Review Letters* **92**, 190801 (2004), which also describes an implementation of the method published in this paper.

Measurement of rotation speed of birefringent material and optical torque from polarisation of transmitted light

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Introduction

A microscopic birefringent particle can be simultaneously trapped and rotated by a circularly polarised laser beam [1]. A laser-trapped birefringent particle will act as a wave-plate (a calcite particle approximately 3 μm thick is a $\lambda/2$ plate for 1064 nm light), and will alter the polarisation of incident light. If the particle is composed of a negative uniaxial birefringent material (ie with $n_o > n_e$), such as calcite, the particle can also be stably trapped in the correct orientation by the same beam that is used to rotate it. As an elliptically polarised incident beam passes through the particle, the polarisation, and therefore the angular momentum carried by the beam is altered. The difference between the incident and outgoing angular momenta determines the torque acting on the particle to cause rotation. This torque can induce high rates of rotation, in excess of 350 Hz [1], with very little heating. The maximum rotation rate results when the incident light is completely circularly polarised.

For simple cases, such as flat disc-shaped particles, the torque can be simply calculated. For more complex shapes, there are considerable difficulties in performing such calculations. The torque can be calculated from measurements of the rotation rates in cases where the size and shape of the particle are known, and the viscosity of the surrounding medium is known. The rotation rate can be measured by observing the variation in the intensity of the reflected light at a point which, especially in the case of irregular particles, depends on the orientation of the particle. For smooth regular particles, this would be much more difficult. However, if the size of the particle, or the viscosity of the surrounding medium is not known, it is not possible to determine the optical torque in this way. One difficulty is accounting for the effects on viscosity of heating due to the beam, or changes in the viscosity due to the presence of nearby surfaces. Another method of determining the optical torque would therefore be highly desirable.

Measurement of rotation speed and optical torque

When a laser beam is used to rotate a trapped birefringent particle at the maximum rotation rate, the incident beam is circularly polarised, and the outgoing beam is, in general, elliptically polarised. The rate of input of angular momentum associated with a laser beam is

$$dL/dt = \sigma_z P/\omega \tag{1}$$

where P is the beam power, ω is the angular frequency, and σ_z is the degree of circular polarisation. A purely circularly polarised beam has $\sigma_z = \pm 1$, and a plane polarised beam has $\sigma_z = 0$. The torque acting on the trapped particle is the difference between the incident and outgoing angular momenta, and is given in terms of the polarisation of the outgoing $\sigma_{\rm zout}$ by

$$\tau = (1 - \sigma_{\rm zout})P/\omega \tag{2}$$

where the incident polarisation $\sigma_{\rm zin} = 1$.

We can also note that the plane of polarisation of the plane-polarised component of the outgoing beam exiting a rotating waveplate will be rotating at twice the frequency of the rotating waveplate. If the outgoing beam is a (rotating) purely plane-polarised beam, rotating at 2Ω, and power P, and is passed through a polariser, the measured power will be $P_{\rm m} = (1 + \cos 4\Omega t)P/2$. By measuring this power, the rotation rate Ω of the trapped particle can be simply determined. This will still be the case for an elliptically polarised beam, as the same variation at a frequency of 4Ω will be observed. The angular momentum associated with this rotation of the plane of polarisation will be negligible as $\Omega \ll \omega$.

In the general case, there will be an elliptically polarised outgoing beam, consisting of both plane- and circularly polarised components. The power of the two components will be $P_{\rm c} = |\sigma_{\rm zout}|P$ for the circularly-polarised component, and $P_{\rm p} = (1 - |\sigma_{\rm zout}|)P$ for the plane-polarised component. The power measured after passing this beam through a polariser will be

$$P_{\rm m} = 1 + (1 - |\sigma_{\rm zout}|) \cos 4\Omega t P/2. \tag{3}$$

Observation of the variation of the transmitted power therefore allows the determination of the rotation period of the trapped particle, and the degree of (but not the direction of) circular polarisation.

If, instead of a plane-polariser being placed in the beam path before the power is measured, a reversed circular polariser is used (eg a quarter-wave plate followed by a plane-polariser appropriately oriented), the direction of circular polarisation can be determined. In the case where the trapping beam is right-circularly-polarised ($\sigma_{\rm zin} = +1$), the light emergent from the particle can be described in terms of left- and right-polarised components $P_mathrmL$ and $P_mathrmR$, where $P_mathrmL = (1 - \sigma_{\rm zout})P/2$ and $P_mathrmR = (1 + \sigma_{\rm zout})P/2$. If the output beam is predominantly right-polarised, $\sigma_{\rm zout} > 0$, and $PR > P_{\rm L}$. A left-polarised beam has $\sigma_{\rm zout} < 0$, and $P_mathrmL > P_{\rm R}$. In this way, the direction of circular polarisation can be determined, and $\sigma_{\rm zout}$ as opposed to merely $|\sigma_{\rm zout}|$ can be found. Once $\sigma_{\rm zout}$ is known, the optical torque acting on the particle can be found using equation (2).

Applications

A simple method of measuring the rotation speed and the optical torque applied to a laser trapped birefringent particle has been outlined above. This method can be used even if the viscosity of the medium in which trapping is performed is unknown, and provides a means to measure this viscosity. Thus, this method is suitable for employment in a micro-rheometer, which could be simply constructed by trapping a birefringent probe particle of known size and shape, for example, a sphere of known radius, in the fluid of interest. As the optical torque can be controlled by varying the power, the probe particle rotation speed can be varied, allowing, for example, the investigation of non-linear properties of the fluid.

In a known trapping medium, the viscosity will, in general, still be unknown, as it is strongly dependent on the temperature, which, due to heating by the trapping beam, will

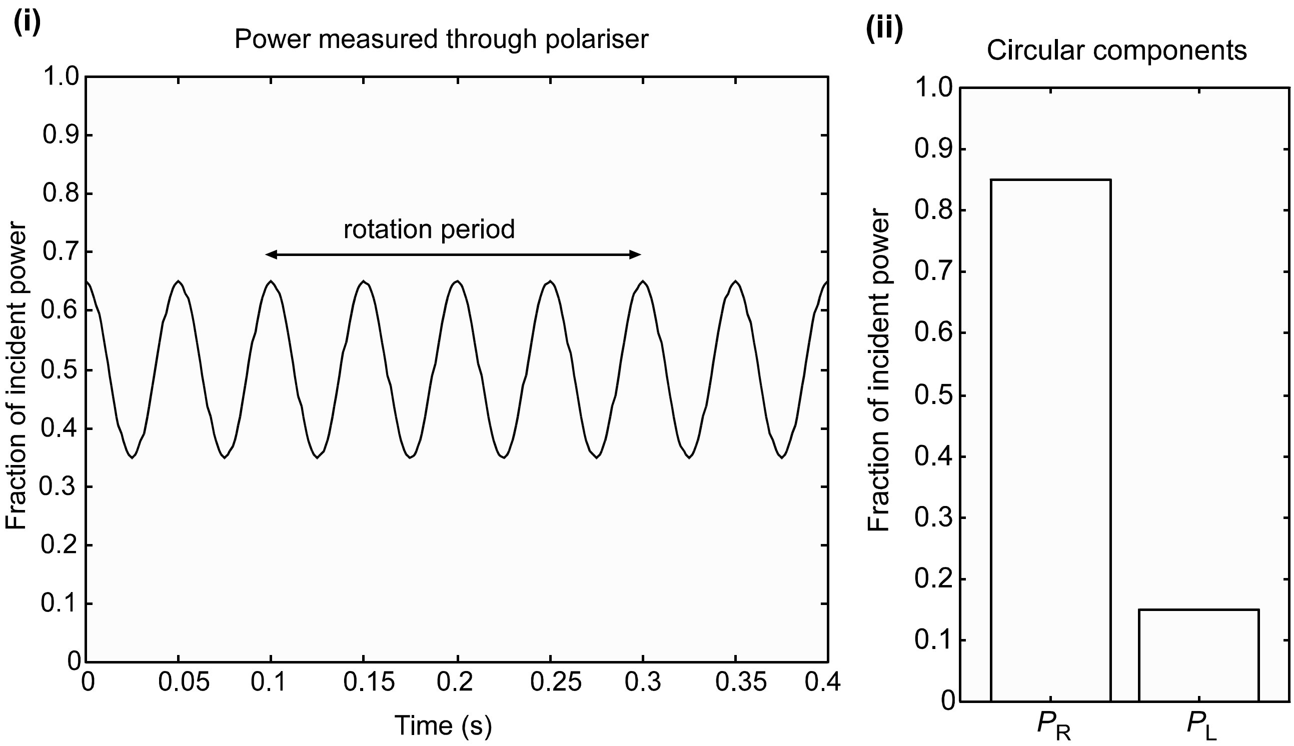

Figure 1: Measured power and circular polarisation. The power measured through a polariser and the powers measured through a reversed circular polariser are shown for a beam with $\sigma_{\text{zin}} = +1$, $\sigma_{\text{zout}} = +0.7$ and particle rotation frequency $\Omega = 5$ Hz. The mean power measured through the polariser is half the power incident on the particle.

be unknown. Measurement of the viscosity of the medium is a means of determining the temperature; measurement of the heating is of great importance if biological specimens are to be trapped. With a transparent particle, and a transparent medium, the heating will be small, but measurement of the variation of viscosity with respect to the beam power allows the temperature to be determined as a function of the incident power.

Conclusions

It is shown that both the rotation speed of and the optical torque acting on an optically trapped birefringent particle can be measured. Some applications of this are presented in detail.

References

[1] M.E.J. Friese, T.A. Nieminen, N.R. Heckenberg and H. Rubinsztein-Dunlop, *Nature* **394** 348–350 (1998)