

Invited Paper

Optical torque and symmetry

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ABSTRACT

The ability to controllably rotate, align, or freely spin microparticles in optical tweezers greatly enhances the manipulation possible. A variety of different techniques for achieving alignment or rotation have been suggested and demonstrated. Although these methods are diverse, employing specially shaped particles, birefringent particles, multiple trapping beams, complex beam profiles, vortex modes, plane polarised beams, circularly polarised beams, or other methods, the fundamental principle—that optical torque results from the exchange of electromagnetic angular momentum between the trapping beam and the particle—remains the same. The symmetry of both the particle and the beam play a central role in the transfer of angular momentum. We discuss this in detail, with particular attention paid to the special case of optical torque exerted by an incident beam with zero angular momentum.

Keywords: Optical tweezers, laser trapping, light scattering, optical torque

1. INTRODUCTION

Optical forces have been widely used to trap and manipulate microscopic particles for many years.¹ The most common type of optical trap is the single beam gradient trap,² also called *optical tweezers*, in which a high numerical aperture lens, usually a microscope objective, is used to focus a laser beam to a focal spot approximately a wavelength across. Transparent particles with a refractive index higher than that of the surrounding medium are attracted to the bright focal spot of the laser beam. In this way, microscopic objects can be manipulated without physical contact. Optical tweezers are used for a wide variety of applications, including the trapping and manipulation of biological specimens such as living cells and organelles, the study of single molecules such as DNA, and the measurement of piconewton forces and nanometer displacements.

The trapping beam can exert force on a particle within the trap because the electromagnetic fields of the beam carry momentum. The transport of momentum by electromagnetic radiation can be understood classically, or by using a quantum mechanical picture. Momentum is transferred from the beam to the particle by absorption or scattering; the rate of transfer of momentum determines the optical force.

Recently, there has been strong interest in the rotation of microscopic objects, three-dimensionally trapped or otherwise. Since light can carry angular momentum as well as (linear) momentum, transfer of angular momentum can be used to produce optical torque. This introduces the possibility of true three-dimensional manipulation within laser traps—the ability to controllably rotate or orient optically trapped microscopic particles is a major advance in the manipulation possible within a laser trap. This is of interest not only for simple manipulation, but also for the use of rotation as a tool to probe microscopic properties of fluids or biological specimens,^{3–5} and the possibility of developing optically powered and controlled micromachines.^{6,7}

The shape of the particle plays a central role in the interaction with the trapping beam which gives rise to the optical torque. Perhaps the single most important aspect of the particle geometry is its symmetry, and its interaction with the symmetry of the trapping beam, which allows some quite general aspects of the generation of optical torque to be understood. This is our chief goal here.

We can consider two quite distinct cases. Firstly, if we wish to control the orientation of, or rotate, naturally occurring microscopic particles, we have only a very limited choice of particle geometry (a range of different particle shapes might be naturally present), and we are essentially restricted to varying the shape or polarisation

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of the trapping beam to achieve our ends. Therefore, we will examine the interaction of particles of relatively simple shape, such as can be expected to occur naturally, with the beam. Of interest are such questions as how optical torque is generated when the incident beam carries zero angular momentum, with what efficiency angular momentum can be generated, and the relative merits of making use of spin and orbital angular momentum.

The second case that we can consider is that of a microfabricated particle,⁷⁻⁹ for example, such as might be used in an optically-driven micromotor. Here, a great deal of control over the geometry of the particle is possible, and we can explore such questions as what type of particle and beam provide the greatest efficiencies. Noting that while spin angular momentum gives a maximum torque efficiency of $2\hbar$ per photon, whereas orbital angular momentum can be hundreds of \hbar per photon, whether or not optically-driven micromotors of great efficiency exploiting orbital angular momentum can be produced is of considerable interest. More generally, a clear understanding of the role of particle shape in determining the optical torque is essential for the design and optimisation of such devices.

2. THE ANGULAR MOMENTUM OF LIGHT: OPTICAL ROTATION

A laser beam carries angular momentum in two distinct forms: spin angular momentum, associated with the polarisation of the beam, and orbital angular momentum, associated with the spatial structure of the beam.^{10,11} Either, or both, can be zero. The spin angular momentum S varies from $-\hbar$ to \hbar per photon, with $S = -\hbar$ for right-handed circular polarisation, $S = \hbar$ for left circular, $S = 0$ for plane polarisation, and intermediate values for elliptical polarisation. The spin angular momentum is *intrinsic*; it is invariant with respect to translation of the axis about which it is measured. The distinction between spin and orbital angular momentum is important when considering the scattering or absorption processes by which the angular momentum is transferred. The mechanical effects of both on the particle are the same,¹² and either can be (and have been) usefully used for rotation.

Torque results from the scattering (including absorption) of light if either the orbital angular momentum or the spin angular momentum is altered. This is most simply done if the particle absorbs part of a beam with non-zero orbital angular momentum or spin angular momentum.¹³ However, absorption also transfers linear momentum, and the resulting radiation pressure force makes three-dimensional trapping more difficult to achieve. Most critically, the absorption of energy from the trapping beam results in heating, rendering this method unusable in practice if the trapped particle is susceptible to damage due to heating.

Since birefringent particles will act as waveplates, changing the polarisation state of the light as it passes through the particle, such particles will experience torque due to change in the spin angular momentum of the light. This has been used to rotate both naturally birefringent crystals¹⁴ and synthetic polymers¹⁵ without absorption or heating.

Any other particle that acts to change the polarisation of the light will also experience such a torque, even if it is not intrinsically birefringent. For example, small ellipsoidal particles can be characterised by an anisotropic permittivity tensor (uniaxial if the particle is rotationally symmetric about an axis).¹⁶ This effective birefringence is called *shape birefringence*,¹⁷ and has the same effect on the polarisation of the light as an intrinsically birefringent material. Since one can produce a macroscopically birefringent medium by seeding it with aligned elongated objects (perhaps simply its constituent molecules), the equivalence of the two types of birefringence is apparent. This shape birefringence has been used to rotate both biological specimens,^{18,19} which are frequently nonspherical, and artificial particles.²⁰

In both of these cases, the symmetry of the particle—microscopic for intrinsic birefringence, and macroscopic for elongated particles—determines the transfer of angular momentum during scattering.

Specially fabricated particles can be used as optical “windmills” for torque generation.⁷⁻⁹ The rotation results from the incident light being preferentially scattered either clockwise or anticlockwise relative to the particle axis, producing a scattered beam with non-zero orbital angular momentum. It is interesting to note that most transparent particles of this type are essentially microscopic versions of the spiral phase holograms that can be used to produce vortex beams.²¹ Again, key features of the interaction between the beam and the particle are consequences of symmetry.

Finally, specially shaped beams have been used for optical rotation. A beam with a non-axially symmetric focal spot produces a non-symmetric trapping potential, and non-spherical objects will align with the trapping potential. While this method is restricted to non-spherical particles, it does allow a large range of particles, either artificial or naturally occurring, to be rotated. This type of rotation has been achieved through the use of multiple beams, either independently steerable or producing a non-symmetric interference pattern,²² or by spreading the focal spot in one direction, which can be done either by using higher order modes,²³ a cylindrical lens²⁴ or by passing the beam through a slit in which case it will spread through diffraction.²⁵ The multiple beam methods significantly complicate the required experimental setup, and the focal spot spreading methods (and many of the multiple beam methods) result in greatly reduced axial trapping forces due to the non-optimum focussing, making three-dimensional trapping problematic. Since this method depends on the spatial structure of the focal spot of the trapping beam, the torque experienced by the particle is due to the generation of orbital angular momentum.

While both the spin and orbital angular momenta of light are mechanically equivalent, in that they can both result in optical torques, there is one important difference in practical applications—spin angular momentum can be readily measured optically, since it depends only on the polarisation of the light.^{3–5,20} Orbital angular momentum, on the other hand, can in principle be measured optically, in practice this turns out to be possible, but quite difficult.^{26–30}

3. MATHEMATICAL DESCRIPTION OF SCATTERING AND ANGULAR MOMENTUM

A convenient description of monochromatic scattering is provided by the T -matrix formalism.^{31–35} In the T -matrix formalism, the incident and scattered waves are each expanded in terms of a sufficiently complete basis set of functions ($\psi_n^{(\text{inc})}$ for the incident wave, and $\psi_n^{(\text{scat})}$ for the scattered wave, where n is a mode index labelling the discrete modes), each of which is a solution of the Helmholtz equation:

$$U_{\text{inc}} = \sum_n^{\infty} a_n \psi_n^{(\text{inc})}, \quad (1)$$

$$U_{\text{scat}} = \sum_k^{\infty} p_k \psi_k^{(\text{scat})}, \quad (2)$$

where a_n are the expansion coefficients for the incident wave, and p_k are the expansion coefficients for the scattered wave. If the response of the scatterer is linear, the relationship between the incident and scattered waves can be written as a linear system

$$p_k = \sum_n^{\infty} T_{kn} a_n \quad (3)$$

or, in more concise notation, as the matrix equation

$$\mathbf{P} = \mathbf{TA} \quad (4)$$

where T_{kn} are the elements of the T -matrix.

This formalism can be used for a wide range of waves, including scalar and vector waves, with the only restrictions being that the scatterer has a linear response, and that the scattering is elastic and monochromatic, and that a suitable basis set exists. The most convenient starting point for finding a basis is the scalar Helmholtz equation.

A useful method of obtaining solutions to the scalar Helmholtz equation

$$\nabla^2 \psi + k^2 \psi = 0 \quad (5)$$

is the method of separation of variables. This can be done in a number of coordinate systems, but for our purpose here, will only consider such solutions in spherical coordinates—spherical coordinates are well-suited to

the description of scattering by a particle contained entirely within a radius R_0 . A general solution of the scalar Helmholtz equation can be written as a sum over a basis set—the allowed modes of the field—as

$$\psi = \sum_{n=0}^{\infty} \sum_{m=-n}^n a_{nm} h_n^{(2)}(kr) Y_{nm} + p_{nm} h_n^{(1)}(kr) Y_{nm} \quad (6)$$

where a_{nm} and p_{nm} are the mode amplitudes of the incoming and outgoing portions of the field, $h_n^{(1)}$ and $h_n^{(2)}$ are spherical Hankel functions of the first and second kinds, and Y_{nm} are normalised spherical harmonics.

Neither the incoming or outgoing waves alone can represent the incident wave; generally an incident wave, in the absence of the scatterer is both incoming and outgoing. Therefore, it can be convenient to use the regular wavefunctions, $j_n(kr) Y_{nm}$, where j_n are spherical Bessel functions. Unlike the incoming and outgoing wavefunctions, which are singular at the origin, the regular wavefunctions are finite everywhere. Since $j_n = (h_n^{(1)} + h_n^{(2)})/2$, both incoming and outgoing parts are clearly present.

Usually, one expands the incident wave in terms of regular wavefunctions, and the scattered wave in terms of the outgoing wavefunctions. Since we are more interested in the transport properties of the fields (since the optical torque is simply the difference between the incoming and outgoing angular momenta), it is more convenient to use the incoming and outgoing wavefunctions. In this case, the T -matrix differs from the usual, and notably, is equal to the identity matrix when no scatterer is present, while it would be zero in the more common formulation.³¹ In the incoming/outgoing formulation, the “incident” wave expansion is *not* equal to the actual incident wave, although it does specify it uniquely.

Solutions to the vector Helmholtz equation

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = 0, \quad (7)$$

are readily obtained from the scalar solutions. If ψ_n are a complete set of solutions to the scalar Helmholtz equation, the wavefunctions

$$\mathbf{L}_n = \nabla \psi_n \quad (8)$$

$$\mathbf{M}_n = -\hat{\mathbf{a}} \times \mathbf{L}_n \quad (9)$$

$$\mathbf{N}_n = \frac{1}{k} \nabla \times \mathbf{M}_n \quad (10)$$

are a complete set of solutions to the vector Helmholtz equation, where $\hat{\mathbf{a}}$ is a unit vector or a constant vector.³⁶ The \mathbf{L}_n wavefunctions are curl-free, while the \mathbf{M}_n and \mathbf{N}_n wavefunctions are divergence-free. Therefore, for electromagnetic scattering, we only require \mathbf{M}_n and \mathbf{N}_n . We can also note that $\mathbf{M}_n = (1/k) \nabla \times \mathbf{N}_n$.

In spherical coordinates, we then obtain the vector multipole fields, or vector spherical wavefunctions (VSWFs):

$$\begin{aligned} \mathbf{M}_{nm}^{(1,2)}(k\mathbf{r}) &= N_n h_n^{(1,2)}(kr) \mathbf{C}_{nm}(\theta, \phi) \\ \mathbf{N}_{nm}^{(1,2)}(k\mathbf{r}) &= \frac{h_n^{(1,2)}(kr)}{kr N_n} \mathbf{P}_{nm}(\theta, \phi) + N_n \left(h_{n-1}^{(1,2)}(kr) - \frac{nh_n^{(1,2)}(kr)}{kr} \right) \mathbf{B}_{nm}(\theta, \phi) \end{aligned} \quad (11)$$

where $N_n = [n(n+1)]^{-1/2}$ are normalization constants, and $\mathbf{B}_{nm}(\theta, \phi) = \mathbf{r} \nabla Y_n^m(\theta, \phi)$, $\mathbf{C}_{nm}(\theta, \phi) = \nabla \times (\mathbf{r} Y_n^m(\theta, \phi))$, and $\mathbf{P}_{nm}(\theta, \phi) = \hat{\mathbf{r}} Y_n^m(\theta, \phi)$ are the vector spherical harmonics.^{32–35}

One point that we will make use of later is that $-n \leq m \leq n$ for all modes, or, equivalently, we always have $n \geq |m|$.

3.1. Angular momentum

That the vector spherical wavefunctions are an ideal description of the electromagnetic field for our purposes becomes apparent when we realise that they are eigenfunctions of the angular momentum operators J^2 , with eigenvalues $[n(n+1)]^{1/2}$, and J_z , with eigenvalues m . Therefore, the angular momentum properties of these

modes are similar to those of the Laguerre–Gaussian (LG) paraxial modes, having angular momentum about the z -axis of $m\hbar$ per photon. An important difference is that this angular momentum combines both spin and orbital momenta.

The angular momentum flux into or out of the system can be found by integrating the moment of the momentum flux density

$$\mathbf{j} = \frac{1}{2c} \text{Re}(\mathbf{r} \times (\mathbf{E} \times \mathbf{H}^*)) \quad (12)$$

over a sphere surrounding the scatterer. With the fields expanded in terms of VSWFs, much of this integral can be performed analytically, giving a relatively simple result for the angular momentum about the z -axis:

$$\tau_z = \sum_{n=1}^{\infty} \sum_{m=-n}^n m(|a_{nm}|^2 + |b_{nm}|^2 - |p_{nm}|^2 - |q_{nm}|^2)/P \quad (13)$$

in units of \hbar per photon, where

$$P = \sum_{n=1}^{\infty} \sum_{m=-n}^n |a_{nm}|^2 + |b_{nm}|^2 \quad (14)$$

is proportional to the incident power. This is the optical torque exerted on the scatterer. The spin and orbital contributions to the torque can be calculated if desired^{20, 37}; the spin torque is

$$\begin{aligned} \sigma_z = & \frac{1}{P} \sum_{n=1}^{\infty} \sum_{m=-n}^n \frac{m}{n(n+1)} (|a_{nm}|^2 + |b_{nm}|^2 - |p_{nm}|^2 - |q_{nm}|^2) \\ & - \frac{2}{n+1} \left[\frac{n(n+2)(n-m+1)(n+m+1)}{(2n+1)(2n+3)} \right]^{\frac{1}{2}} \\ & \times \text{Im}(a_{nm}b_{n+1,m}^* + b_{nm}a_{n+1,m}^* - p_{nm}q_{n+1,m}^* - q_{nm}p_{n+1,m}^*). \end{aligned} \quad (15)$$

The remainder of the torque is the orbital contribution.

3.2. Strongly focussed laser beams

It is theoretically straightforward to find the VSWF expansion for any monochromatic radiation field using the orthogonal eigenfunction transform,^{38, 39} also known as the generalised Fourier transform. For example, the usual formula for multipole expansion of a plane wave can be derived in this way. Strongly focussed laser beams, however, present some difficulties. These problems are not a result of any deficiency in the basic procedure, but are due to the fact that standard representations of laser beams are not radiation fields—that is, their standard mathematical forms are not solutions of the vector Helmholtz equation, but are solutions of the paraxial scalar wave equation. Since it is sufficient to perform the transform over a surface, rather than over a volume, appropriate choice of the surface avoids most of the problems.⁴⁰

If we consider a paraxial laser beam incident on a lens (the trap objective), this beam can be described by its LG modal decomposition, and its polarisation. Since both the LG modal expansion and the VSWF expansion have similar angular momentum properties, we obtain some simple rules giving us the general features of the VSWF expansion. For a single incident LG_{pl} mode, with $l\hbar$ orbital angular momentum per photon,

- only $m = l \pm 1$ VSWF modes can be nonzero,
- for circular polarisation, either the $m = l - 1$ modes or the $m = l + 1$ modes will be nonzero, but not both,
- and both will be nonzero for plane or elliptical polarisation.

Generally, the expansion can be truncated at $n \approx kr_0$, where r_0 is the radius in the focal plane enclosing all significant portions of the beam. Notably, beams with identical total angular momentum flux, such as a left-circular LG₀₃ beam and a right-circular LG₀₅ beam have similar VSWF mode amplitudes. The VSWF expansion of a beam that is not a single LG mode is simply the sum of the VSWF expansions of its component LG modes.

We should also note that $|m| \leq n$ (a consequence of the necessity of the magnitude of one vector component of the angular momentum being less than or equal to the magnitude of the total angular momentum vector), so

high angular momentum LG modes imply higher values of n as well as m . This gives us a simple non-paraxial equivalent to the paraxial LG modes, incorporating the polarisation as well as the orbital angular momentum and radial structure. Interestingly, the spherical Hankel/Bessel functions are strongly peaked when $kr \approx n$, so the width of the doughnut pattern is proportional to the azimuthal index l of the original paraxial beam, while for a paraxial beam, the width is proportional to \sqrt{l} ; this difference in scaling has been experimentally observed.⁴¹

In general, a plane polarised beam is mirror symmetric, and not rotationally symmetric, even if the equivalent paraxial mode is rotationally symmetric. Once the beam is focussed, the polarisation can no longer be separated from the spatial structure of the beam, and the symmetry of a focussed plane polarised beam is fundamentally different from that of a focussed circularly polarised beam. This deviation from rotational symmetry is even manifested in the intensity distribution in the focal plane.^{40, 42, 43}

3.3. Symmetry and the T -matrix

The T -matrix elements are strongly dependent on the symmetry of the scatterer.³³ We can deduce the principal features from Floquet's theorem, relating solutions to differential equations to the periodicity of their boundary conditions.

If we have a scatterer with n th-order rotational symmetry about the z -axis, an incident mode of azimuthal index m couples to scattered modes with azimuthal indices $m, m \pm n, m \pm 2n, m \pm 3n$ and so on. For scatterers that are mirror-symmetric, upward and downward coupling must be equal, in the sense that, for example, a mirror-symmetric scatterer of 2nd order rotational symmetry (such as a long rod), T -matrix elements coupling from $m = 1$ modes to $m = -1$ modes will have the same magnitudes as the elements coupling from $m = -1$ to $m = 1$ modes. For chiral scatterers, these T -matrix elements will, in general, be different.

For the case of a rotationally symmetric scatterer, this means that there is no coupling between modes with differing angular momenta about the z -axis.^{33, 34, 44} Therefore, it is not possible to exert optical torque on such scatterers except by absorption (or gain)—since the incoming and outgoing angular momenta per photon are the same, the only optical torque can result from a change in the number of photons. This leads to the interesting result that a focussed circularly polarised beam must carry orbital angular momentum. Only in the case of a plane wave can the spin angular momentum actually equal $\pm\hbar$ per photon, and the more strongly focussed the beam is, the smaller the possible spin angular momentum. For the extreme case of a dipole field, the angular momentum is half spin and half orbital.^{10, 37} This conclusion can also be arrived at using a simple semi-classical picture of the transport of spin angular momentum, with exact quantitative agreement with rigorous electromagnetic theory. Therefore, orbital angular momentum can be generated by purely rotationally symmetric elements. This leads to the question of just how the focussed beam carries the orbital angular momentum, and it is interesting to note that the *axial* component of a strongly focussed circularly polarised Gaussian beam has the $\exp(i\phi)$ dependence seen in optically vortices—a longitudinal optical vortex!

We can also note some rotation properties of T -matrices. Since the individual VSWF modes have an azimuthal dependence of $\exp(im\phi)$, a rotation of the coordinate system of Φ about the z -axis must induce a phase change of $m\Phi$ in the VSWF modal amplitudes. The combination of rotation of the incident and scattered fields is equivalent to a phase change of $(m_1 - m_2)\Phi$ for the T -matrix element coupling modes of azimuthal index m_1 and m_2 . If the scatterer is rotating, then $\Phi = \Omega t$, and the scattered light experiences a rotational Doppler shift of $(m_1 - m_2)\Omega$. This rotational Doppler shift accounts for the work done by the field on the spinning particle.

4. ALIGNMENT AND ROTATION OF SIMPLE PARTICLES

If we consider a simple particle such as a mirror-symmetric elongated particle, we immediately note that such a particle possesses 2nd order rotational symmetry, and hence couples modes with $m_1 = m_2 \pm 2n$. We can expect $m_1 = m_2 \pm 2$ to dominate, especially for small particles.

Firstly, we consider the case of such a particle trapped by a plane polarised Gaussian beam ($m = \pm 1$) mirror symmetric plane polarised beam. If the beam is plane polarised along the x -axis, the $m = 1$ and $m = -1$ amplitudes are equal. For simplicity, and without loss of generality, we assume that this is the case. If the long axis of the particle, and the symmetry axis of the beam coincide, the coupling from $m = +1$ to $m = -1$ must be identical, and the scattered modes are of opposing angular momentum are equal. Thus, no torque can be

exerted on the trapped particle. The same considerations apply if the particle is rotated by 90° —in this case, the effect of the rotation is a multiplication by -1 of all relevant T -matrix elements, and the scattered modes of opposing angular momentum are still equal. If we consider a single VSWF mode of amplitude $a_{n,1}$ incident on the particle, in the absence of absorption, energy must be conserved, and hence $p_{n,1} = a_{n,1}(1 - ix)$ and $p_{n,-1} = ix$ where x is a real number.

A rotation of 45° , however, results in multiplication by i and $-i$ of the upward and downward coupling elements, with no change to the coupling from $m = 1$ to $m = 1$, or $m = -1$ to $m = -1$. As a result, if the initial mode amplitudes are real, the real part of, say, the $m = 1$ mode is increased, and the real part of the $m = -1$ mode is decreased, while their imaginary parts remain the same. Thus, the scattered beam now has non-zero angular momentum, and an optical torque is exerted on the particle.

A left circularly polarised incident beam will simply have the $m = 1$ amplitudes reduced, and the $m = -1$ amplitudes will become non-zero, again giving rise to an optical torque. In this case, the torque is independent of the orientation of the particle about the z -axis, since the optical angular momentum is independent of the phase of the mode amplitudes (which is all that will be changed by rotation of the particle). The same conclusion can also be simply deduced from the rotational symmetry of the system.

In the case of weak scattering, as will be typical of most small elongated objects, the coupling will be weak (ie x above will be small). For the circularly polarised beam, this results in loss of power in $m = 1$ modes proportional to x^2 , and an equal gain in the $m = -1$ modes. Therefore, the optical torque will be quite small. For a plane polarised beam at 45° to the particle, the changes in power in each mode are proportional to x , and will generally be much larger. This conclusion is supported both by rigorous electromagnetic calculations and by experimental measurements of the torque in the two case.²⁰

Similar considerations apply to the rotation of birefringent particles—these possess the same symmetry as elongated particles. The major difference is that strongly birefringent particles will result in stronger coupling (ie larger x above), and the difference noted above between the torque exerted by circularly and plane polarised beams need not apply. Energy conservation considerations give the usual maximum angular momentum transfer of $2\hbar$ per photon for a circularly polarised beam, and \hbar per photon for a plane polarised beam. We can note that coupling between the $m = 1$ and $m = -1$ modes of equal n is not sufficient to reverse the polarisation of a beam since $1 - ix$ cannot be zero for real x —a waveplate must couple orders of differing n in order to be effective.

If, instead of trapping the elongated particle in a Gaussian beam, we now use a beam with an elliptical focal spot, the trapping beam has VSWF modes of $m = \pm 1, \pm 3, \pm 5$, etc, and also modes of higher n , due to the larger extent of the focal spot. Again, the overall behaviour will be similar to that of an elongated particle trapped in a plane polarised beam. For weak coupling, we can expect higher torques, since the outward coupling to orders of greater $|m|$ now contribute to the torque rather than resulting in equal angular momenta in the positive and negative m orders. The exact details, however, are likely to be strongly influenced by the exact particle geometry. Modes with $m = \pm 1, \pm 3, \pm 5$, etc will generally exist, due to the overall shape of the beam, regardless of the incident polarisation, and the interference terms that give rise to higher torques in the case of weak coupling will still exist—a large increase in the torque should result for circularly polarised beams.

5. COMPLEX PARTICLES

We can distinguish between two types of complex particles: particles of high-order rotational symmetry that are also mirror symmetric, and particles of high-order or low-order rotational symmetry that are not mirror symmetric—chiral particles. Particles with high-order rotational symmetry couple modes of very different angular momentum, but such modes are not available when the beam waist is small. If the coupling is between modes with $m_1 - m_2 = \Delta m$, the minimum n to support two such modes of equal n is $\Delta m/2$. This places restrictions on the minimum size of such particles if they are to be effective scatterers. This does not seem to be overly limiting, since, for example, a particle with 10th order rotational symmetry (so $\Delta m = 10$, and $n_{\min} = 5$ needs a minimum size parameter (kr) of about 5. As this is only a radius of about one wavelength, the particle can still be quite small.

We can also note that if we can freely choose our operating wavelength, there is little gain in using modes of very high angular momentum. For a beam of high azimuthal index m , the minimum focal spot width is

approximately km in radius. An increase in m does result in the transport of more angular momentum per photon, at the cost of a larger minimum focal spot size, but a decrease in the frequency results in an equivalent gain in the photon flux (each with less energy, but the same angular momentum as before), with the same increase in minimum focal spot size due to increased wavelength. This fundamentally results from the momentum transport being proportional only to the power (recall that the momentum flux is equal to the Poynting vector divided by c). To increase the associated angular momentum requires an increase in the moment arm, and hence an increase in the focal spot size.

For a non-chiral particle of high-order rotational symmetry trapped by a Gaussian beam, the $m = 1$ and $m = -1$ modes can no longer directly interact. Thus, only small torques comparable to the torques exerted on elongated particles by circularly polarised light are expected, regardless of the polarisation of the trapping beam. Such particles thus appear to be of limited utility, at least if trapped by simple beams. A beam sharing the rotational symmetry of the particle will experience such direct coupling, but such a beam requires a complex polarisation structure as well as a complex spatial structure.

Chiral particles, therefore, seem to be possible ideal candidates for highly efficient micromotors and the like.⁷⁻⁹ For such particles, the upward and downward coupling between angular momentum orders are no longer equal. At least some chiral particles closely resemble on-axis spiral holograms of the type used for generating optical vortices,⁹ and it should be possible to achieve moderately high efficiencies. Discounting losses in the production of such incident beams, the optimal strategy appear to be to use an incident beam of angular momentum order m coupled to order $-m$. It is worth noting that such an incident beam does not actually need to be circularly polarised; this is likely to be the simplest option, but the appropriate combination of left-circular beam of orbital angular momentum $(m-1)\hbar$ per photon and a right-circular beam with $(m+1)\hbar$ orbital angular momentum per photon meets the requirements. For the simpler option of a circularly polarised beam, the incident and scattered orders will therefore have differing magnitudes of orbital angular momentum (eg $|m+1|$ versus $|-m+1|$).

Generally, the use of high angular momentum modes ($|m| \approx n$) will result in one-directional coupling within a single set of radial modes (ie for a particular n). This should allow high torque efficiencies to be obtained. Illumination by a focussed LG mode is ideal for driving such a device; since the incident beam and the scattered order into which conversion is desired have similar intensity patterns (the beams themselves having opposite helicity), strong coupling between the two modes should be achievable.

6. CONCLUSIONS

From general principles of symmetry, we have been able to deduce a number of features of the generation of optical torque and its dependence on the geometries of the particle and the beam. For an elongated or birefringent particle, whether rotated by plane polarised light, circularly polarised light, or by a beam with an elliptical focal spot, the shared symmetry results in similar behaviour in all of these cases.

For more complex particles, such a microfabricated optically-driven rotors, it appears that a chiral particle is required for efficiency; nonchiral particles of high-order rotational symmetry do not seem to offer any advantage over simple elongated particles. Chiral particles, on the other hand, offer the possibility of strong one-directional coupling between angular momentum orders, with the result of high torque efficiency. Such a particle, with n th order rotational symmetry, is best driven by a focussed LG mode of helicity of $l \approx n/2$.

Finally, it has been suggested in the past that the essentially unlimited angular momentum of optical vortices, with thousands of \hbar per photon being achievable, can be used for highly efficient optical rotation. This is not likely to be useful for the optical rotation of small objects, since such beams necessarily have focal spots hundreds of wavelengths across (if, however, one wishes to use an optical rotor that is hundreds of wavelengths across ...). In general, the best efficiencies result from the use of focal spots almost the size of the particle, with as much power as possible in the higher angular momentum modes.

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