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NUMERICAL SIMULATION OF THE TERM STRUCTURE OF INTEREST RATES USING A RANDOM FIELD

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ABSTRACT. In this paper we simulate the term structure of interest rates, where the yield curve is based on forward rates which are modelled as a random field. Term structure models based on random fields offer an improvement on yield curve models based on stochastic differential equations, because they do not require recalibration. In the literature, results concerning random field models of interest rates have been entirely theoretical, and have not discussed the implications for yield curve modelling. The simulation results presented in this paper, to the best of our knowledge, are the first numerical results for random field based interest rate and yield curve models.

1: INTRODUCTION

Over the last two decades a large section of research in finance has been dedicated to the development of valuation models for interest rate derivative securities such as options, caps, collars and swaptions. In this literature, the modelling of interest rates typically begins with specifying a stochastic process for the price dynamics of the bond. The spot price is then related back to the term structure of interest rates through the yield curve. The price dynamics that are represented are usually those of zero-coupon bonds, since the cash flows of any coupon-bearing bond can be seen as linear combinations of the cash flows of a collection of zero-coupon bonds, each with a termination date corresponding to the payment date of a coupon. If we denote by $P(t,T)$ the spot price of a zero-coupon bond at time t that pays one dollar at maturity date T, then the T-maturity interest rate at time T, $Y(T;t)$ can be set equal to

$$Y(T;t) = \frac{-\ln P(t,T)}{T-t}. \tag{1.1}$$

This function $Y(\cdot;t)$ is known as the *yield curve at time* t, and represents the relative yields at time t of bonds with different maturities.

It should be noted that not all models representing the price dynamics of the zero-coupon bond are suited to modelling the yield curve. The simplest model of interest rate dynamics that can be used to fit the yield curve, is that by Ho & Lee [25]:

$$dr(t) = \eta(t)\,dt + \beta^{1/2} dW(t), \tag{1.2}$$

where $\beta > 0$ and $\eta(t)$ is a function of time, $W(t)$ is a Wiener process of finite dimension, and $r(t)$ is the spot interest rate. The Ho & Lee model was the first *no-arbitrage model* of the term structure of interest rates, i.e. careful choice of the

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drift term $\eta(t)$ will result in theoretical bond prices which are the same as market prices. However the main drawback of this type of model, is that the volatility structure of the spot rate is assumed to be constant, hence this model implicitly assumes that the yield curve at time t will be parallel to yield curves for any time in the future.

This is a *one-factor model of interest rates*, i.e. the term structure of interest rates depends solely on their spot rate. A more general one-factor model has been developed by Heath, Jarrow & Morton ([24]; HJM) who have shown that the yield curve can be derived from the dynamics of the forward rates of interest. If we let

$$F(t,T) = -\frac{\partial \ln P(t,T)}{\partial T} \tag{1.3}$$

denote the instantaneous forward rate of interest at time t for a bond of fixed maturity T, so that

$$P(t,T) = \exp\left\{-\int_t^T F(t,u)\,du\right\}, \tag{1.4}$$

then the yield curve can be derived as follows:

$$Y(t,T) = \frac{1}{T-t}\int_t^T F(t,u)\,du. \tag{1.5}$$

Notice that as $t \to T$, $P(t,T) \to 1$, so that bond prices converge to their face value upon maturity. The dynamics of the forward rate are defined as follows:

$$dF(t,T) = \alpha(t,T)\,dt + \sigma(t,T)\,dW(t), \tag{1.6}$$

where $F(t,T)$ is the forward rate, $W(t)$ is Wiener process of finite dimension and $\alpha(t,T)$ and $\sigma(t,T)$ are the model parameters for the drift and volatility. We note that $r(t) = F(t,t)$, hence all single factor models based on the spot rate – the Vasicek [48] and Cox-Ingersoll-Ross (Cox et al. [14]) that use constant coefficients, as well as the model by Ho & Lee [25] that was introduced above, the Hull & White model [26], and the lognormal models of Black, Derman & Toy [4] and Black & Karasinski [5], which employ more sophisticated trend and volatility specifications – can be subsumed within the HJM model.

Because there is no reason to believe that the yield curve will depend only on a single factor, this paper concentrates on an alternative approach for modelling interest rate term structure based on multi-factor models. Empirical research (e.g. Litterman & Scheinkman [32], Stambaugh [46], Pearson & Sun [41], Chen & Scott [11, 12]) seems to favour the use of multi-factor models for modelling the term structure of interest rates. There are two emerging schools. One group has attempted to model the short rate as given by the function $r(t) = R(X(t))$, where $X(t)$ is some finite dimensional state space vector comprising various economic indices which may affect interest rates. The dynamics of this state variable are given as

$$dX(t) = \mu(X(t))\,dt + \sigma(X(t))\,dW(t), \tag{1.7}$$

where $W(t)$ is an n-dimension Wiener process, $\mu : D^n \to \mathbb{R}^n$ and $\sigma : D^n \to \mathbb{R}^{n\times n}$, $D^n \subset \mathbb{R}^n$, satisfy sufficient regularity conditions for the existence and uniqueness of solutions. Thus a zero-coupon bond maturing at time T will have at $t \leq T$ a market price given by

$$P(X,t) = E\left[\exp\left\{-\int_t^T R(X(s))\,ds\right\}\middle| X(t)\right]. \tag{1.8}$$

In this type of model the state variable can either be related directly to the yield curve, by modeling the term structure as an affine function of $X(t)$ subject to regularity conditions (e.g. Duffie & Kan [15, 16]) or indirectly via a non-affine model (e.g. Brennan & Schwartz [6], Chan [10], El Karoui et al. [17], Constantinides [13], and Jamshidian [21]).

The other school is based on the work of Kennedy [27, 28], Goldstein [19] and Musiela [40, 7], in which the forward rates are expressed as

$$F(t,T) = \mu(t,T) + X(t,T), \tag{1.9}$$

where $X(t,T)$ is a Gaussian random field with zero mean and bounded covariance, given by

$$Cov(X(t_1,T_1), X(t_2,T_2)) = c(t_1 \wedge t_2, T_1, T_2), \tag{1.10}$$

for some appropriate function c, and $\mu(t,T)$ is the deterministic trend, which is defined so that $\mu(0,T) = F(0,T)$. This function c will be symmetric in t_1 and t_2 and positive semi-definite in (t_1,T_1) and (t_2,T_2). Adler [1, 2] provides conditions on c which ensure the continuity of the Gaussian random field.

Kennedy [27] has shown that given the structure of the covariance function (1.10), and subject to a necessary and sufficient conditions on the drift surface that the following statements are equivalent:

1. For each $t \geq 0$, the discounted bond price process $\{Z(t,T); \mathcal{F}_t, 0 \leq t \leq T\}$ is a martingale, where

$$Z(t,T) = P(t,T)\exp\left(-\int_0^t r(u)\,du\right), \tag{1.11}$$

   given $r(t) = F(t,t)$.

2.

$$\mu(t,T) = \mu(0,T) + \int_0^T [c(s \wedge v, v, T) - c(0,v,t)]\,dv, \quad \forall\, 0 \leq s \leq T. \tag{1.12}$$

3.

$$P(t,T) = E\left[\exp\left(-\int_t^T r(u)\,du\right)\middle| \mathcal{F}_t\right], \quad \forall 0 \leq t \leq T. \tag{1.13}$$

Since the random field driving the dynamics is Gaussian, once both the initial term structure $\{\mu(0,T), T \geq 0\}$ has been determined and the covariance structure between forward rates (i.e. between bonds of different maturities) specified, then the full distribution of the forward surface rate and the bond price surface will be completely determined.

Specifying the covariance function of the random field $X(t,T)$ (1.10) as a function of $t_1 \wedge t_2$ ensures that the random field has *independent increments in the direction of* t, i.e. for any $0 \leq t \leq t' \leq T$ the random variable $X(t',T) - X(t,T)$ is independent of the σ-field $\mathcal{F}_t = \sigma\{X(u,v); u \leq s,\ u \leq v\}$. Kennedy [28, p.109–110] has shown that the martingale property on $Z(t,T)$ is related to the independent increments property of the Gaussian random field, i.e. if the martingale property holds, then the random field $X(t,T)$ possesses a covariance function given by (1.10).

In this paper we will simulate the term structure of interest rates for this type of model and present simulation results of two theoretical term structure models, one based on the papers by Kennedy [27, 28], the other based on a model by Goldstein [19, 20]. The simulations use a numerical technique developed in McDonald [35] and McDonald and Beard [36], for modeling the evolution of a process on a random field. This numerical technique uses the method of lines to convert the random field into a random vector process. We then draw from the standard menu of finite difference procedures to simulate the resulting vector SDE. As a consequence our method is considerably simpler than many of the existing techniques in use for simulating SPDEs and also benefits from drawing from a well defined collection of finite difference procedures for simulating SDEs (see Kloeden and Platen [30] provides a comprehensive discussion of these techniques).

The remainder of the paper is split into two sections. The first of these provides two models for simulation. Simulation results for each of these models, in the form of surface plots of respective yield curves, will be provided in the appendix of this paper. The last section motivates the technique used for the numerical simulation of the yield curve for Kennedy-Goldstein term structure models. The results of these simulations suggest that as σ the coefficient controlling diffusion decreases, the term structure will generate a surface which resembles the fluctuations of the business cycle. These results also suggest that as the level of volatility increases, this noise serves to dampen the incidence of cyclical effects recorded in the term structure. The co-movement of the term structure of interest rates with fluctuations in the business cycle suggests a possible application of this type of model to macroeconomic dynamics.

We should also state that the range of possible applications for this simulation procedure are very large. The technique employed in this paper has been used to simulate the stochastic Fisher equation from population biology and the stochastic Hodgkins-Huxley equation for modelling neuro-electrical transmissions [35, 36]. Other applications of SPDEs for which this technique could be applied, include the modelling ground water transportation in soil physics and fleet dynamics for fisheries modelling. We are currently exploring the application of our technique for simulated likelihood to address stock-flow problems in panel data. The obvious application of these simulated likelihood procedures for continuous spatio-temporal panel data would be for modelling the evolution of the term structure of interest rates.

2. Fitting the yield curve from a random field

To explain why modelling innovations with random fields will remove any requirement for the recalibration of the yield curve, we begin by recalling that for either single or finite factor models of interest rates, the selection of an appropriate drift function $\eta^*(t)$, the *risk-neutral drift*, will determine that the theoretical and actual market prices are equivalent. The example of this, which was presented in the introduction, was the Ho & Lee model (1.2). Using this model as an example, Wilmott [50] has shown that the slope of the yield curve is equal to one-half of this risk-neutral drift, while the curvature of the yield curve at the short-end is proportional to the derivative of this drift term. In other words, both the slope and the curvature of the yield curve are dependent on the form of η^*.

For the model presented in the introduction (1.1), the yield curve is only a function of the term structure, with the current time t acting as a parameter. Hence the only fluctuations recorded by the yield curve will be a consequence of the term structure. Hence, if the drift term η^* was to remain constant throughout time as well, its form would imply a dramatic flattening of the yield curve, which is not shown to occur empirically. As a consequence of this, the yield curve must be recalibrated to account for the changing structure of the drift equation. The risk-neutral drift captures this evolution of the term structure over time.

This is circumvented under the random field model of the instantaneous forward rate which was presented in the introduction. Recalling the random field model,

$$F(t,T) = \mu(t,T) + X(t,T), \quad 0 \le t \le T, T \ge 0,$$

where the drift $\mu(t,T)$ and random field $X(t,T)$ as previously defined, Kennedy has shown that the interest rate term structure $Y(t,T)$ can be expressed as a Gaussian random field with covariance function

$$\begin{aligned} &Cov\left(Y(t_1,T_1), Y(t_2,T_2)\right) \\ &= \frac{1}{(T_1-t_1)(T_2-t_2)} \int_{t_1}^{T_1}\int_{t_2}^{T_2} c(t_1 \wedge t_2, u, v)\, du dv. \end{aligned} \tag{2.1}$$

If we exploit the connection between the instantaneous yield curve at time t and the forward rate $F(t,T)$, this leads to a model of evolution of interest rate term in terms of the forward rate:

$$\begin{aligned} \frac{\partial Y(t,T)}{\partial t} &= \frac{1}{T-t}\left[F(t,T) + Y(t,T)\right] \\ &\quad - \frac{\partial F(t,T)}{\partial t} + (T-t)\frac{\partial Y(t,T)}{\partial t \partial T}. \end{aligned} \tag{2.2}$$

Furthermore, under Kennedy's model, all available information at time t is assumed to be contained in the σ-field

$$\mathcal{F}_t = \sigma\left\{F(u,v); 0 \le u \le t, u \le v\right\};$$

i.e. at any point in time, the entire yield curve is observable. This contrasts with finite-factor models, where it is assumed that there are a small number of bonds of particular maturities for which the dynamics are specified and bond prices may be calculated: e.g. for $0 < s_1 < s_2 < \ldots < s_k$ the evolution of $P(t,t+s_1), P(t,t+s_2), \ldots, P(t,t+s_k)$ and the spot rate $r(t) = F(t,t)$ for $t \ge 0$. Suppose that $\mathcal{R}_t$ is a sub-σ-field such that $\mathcal{R}_t \subset \mathcal{F}_t$ and the conditional distributions of $\{F(t,v); v \ge t\}$ given $\mathcal{R}_t$ are Gaussian. Then with this restricted information the bond prices will be

$$\hat{P}(t,T) = E\left[P(t,T) | \mathcal{R}_t\right] = e^{-\int_t^T \hat{F}(t,u)du}, \tag{2.3}$$

where

$$\int_t^T \hat{F}(t,u)\, du = \int_t^T E\left[F(t,u) | \mathcal{R}_t\right] du - \frac{1}{2}\mathrm{Var}\left(\int_t^T F(t,u)\, du | \mathcal{R}_t\right). \tag{2.4}$$

If we denote by $\hat{c}(t,u,v) = \mathrm{Cov}(F(t,u), F(t,v) | \mathcal{R}_t)$ the covariance function conditional on the filtration $\mathcal{R}_t$, then it follows that

$$\int_t^T \hat{F}(t,u)\, du = \int_t^T E\left[F(t,u) | \mathcal{R}_t\right] du - \int_{u=t}^T \int_{v=t}^u \hat{c}(t,u,v)\, du dv$$

which implies that

$$\hat{F}(t,T) = E[F(t,u)|\mathcal{R}_t]\,du - \int_t^T \hat{c}(t,u,T)\,du. \tag{2.5}$$

Now consider the special case of a one-factor model where $\mathcal{R}_t = \sigma\{r(t)\}$, and suppose that the *Markov property* holds for the random field, i.e. it has the *independent increments property.* Then

$$E[F(t,T)|\mathcal{R}_t] = \mu(t,T) + \frac{c(t,t,T)}{c(t,t,t)}(r(t) - \mu(t,t)) \tag{2.6}$$

and

$$\hat{c}(t,u,v) = c(t,u,v) - \frac{c(t,t,u)\,c(t,t,v)}{c(t,t,t)}. \tag{2.7}$$

Recalling that

$$\mu(t,T) = \mu(0,T) + \int_0^T [c(t\wedge v,v,T) - c(0,v,T)]\,dv,$$

this then leads to

$$\int_t^T \mu(t,u)\,du = \int_t^T \mu(0,u) + \int_t^T\int_0^u [c(t\wedge v,v,u) - c(0,v,u)]\,dudv. \tag{2.8}$$

It then follows that $\hat{P}(s,t)$ may be represented as

$$\hat{P}(s,t) = a(t,T)\,e^{-b(t,T)r(t)}, \tag{2.9}$$

where

$$b(t,T) = \int_t^T \frac{c(t,t,u)}{c(t,t,t)}du \tag{2.10}$$

and

$$\begin{aligned}\ln a(t,T) &= \ln\left(\frac{P(0,T)}{P(0,t)}\right) + b(t,T)\,F(0,t)\\ &\quad -\frac{c(t,t,t)}{2}b^2(t,T) + d(t,T),\end{aligned} \tag{2.11}$$

with

$$d(t,T) = \int_{u=v}^T\int_{v=0}^t \left[\frac{c(v,v,t)\,c(t,t,u)}{c(t,t,t)} - c(v,v,u)\right]dudv. \tag{2.12}$$

Kennedy states that a random field $F(t,T)$ is said to satisfy the *first Markov property* if it satisfies the usual definition of a Markov process upon holding T constant. The second Markov property is satisfied by $F(t,T)$ if $F(t_1,T_1)$ and $F(t_2,T_2)$ are conditionally independent, given $F(t_2,T_1)$, with $0 \leq t_1 < t_2$ and $t_2 \leq T_1 \wedge T_2$. The process $F(t,T)$ is said to be Markov in direction T if $F(t,T_1)$ and $F(t,T_3)$ are conditionally independent, given $F(t,T_2)$, with $0 \leq t \leq T_1 \leq T_2 \leq T_3$. Under the situation where all three of these properties hold together, then the random field is said to be strictly Markov. Kennedy [28, p. 117-118] has shown that the covariance function attain the form

$$c(t,T_1,T_2) = \sigma^2\exp[\lambda t + (2\mu - \lambda)(T_1\wedge T_2) - \mu(T_1 + T_2)]. \tag{2.13}$$

In addition, providing that the σ-field is *strictly Markov*, then $d(t,T) \equiv 0$. When

$$E[P(t,T)|\mathcal{R}_t] = E[P(t,T)|\mathcal{Q}_t],$$

where $\mathcal{Q}_t = \sigma\{r(u); 0 \leq u \leq t\}$, the Ho & Lee model is attained if

$$c(s,u,v) = \sigma^2 s, \quad \forall u,v, \tag{2.14}$$

and the Hull & White model can be derived if

$$c(s,u,v) = \frac{\sigma^2 \sin(\alpha s) e^{-\alpha(u \vee v)}}{\alpha}, \quad \forall u,v. \tag{2.15}$$

The connection between these finite factor models and the Kennedy-Goldstein model can be understood through the Radon-Nikodym theorem. Given that

$$\mathcal{F}_t = \sigma\{F(u,v); 0 \leq u \leq t, u \leq v\}$$

defines the σ-field for the Kennedy-Goldstein model, and $\mathcal{R}_t$ is a sub-σ-field of $\mathcal{F}_t$, such that it defines the filtration for the evolution of $P(t,t+s_1)$, ..., $P(t,t+s_k)$, $0 < s_1 < s_2 < ... < s_k$, and the spot rate $r(t) = F(t,t)$ for $t \geq 0$. Then by the Radon-Nikodym theorem it is possible to construct an equivalent σ-finite measure P_t and Q_t on $(\;, \mathcal{F}_t)$ and $(\;, \mathcal{R}_t)$ respectively, such that there exists a measurable function $f(\cdot)$ where

$$Q_t = \int_A f(F(u,v))\, dP_t$$

and $A \in \mathcal{F}_t$. The function $f(\cdot)$ is the Radon-Nikodym derivative and is denoted

$$f = \frac{dQ_t}{dP_t}$$

Via the Girsanov theorem, Kennedy has been able to parametrize the drift and volatility of the finite factor model in terms of the equivalent measure of the random field. This same technique underlies algorithm employed for estimating Bayes models (Phillips [42] and Phillips and Ploberger [43]) and Markov switching [22, 23]. Hence, there is a direct connection between the relationship of the single and finite-factor models to the infinite dimensional approach of Kennedy and Goldstein, and the success of approaches like the Bayes modelling approach of Phillips and Markov switching for estimating the parameters of the yield curve.

3. More general formulations of term structure as a random field

Kennedy has stated that one model which adheres to the properties given in the introduction, is where the random component $X(t,T)$ is a Gaussian random field with covariance function

$$c(t,T_1,T_2) = \sigma(t)\, \tau(T_1 \vee T_2), \tag{3.1}$$

where $\sigma, \tau : [0,\infty) \to [0,\infty)$ are continuous and monotone increasing and monotone decreasing respectively such that $\sigma(0) = 0$. The assumption that τ is decreasing corresponds to the assumption that $F(t,T)$, $0 \leq t \leq T$, decreases as the maturity in time t increases. For fixed t

$$\mathrm{Corr}(X(t,T_1), X(t,T_2)) = \sqrt{\tau(T_1 \vee T_2)/\tau(T_1 \wedge T_2)}. \tag{3.2}$$

Thus for fixed t and T_1 it can be shown that as T_2 increases from $T_2 = T_1$, the correlation between $F(t,T_1)$ and $F(t,T_2)$ decays to 1.

Following Kennedy, we assume that the spot rate has a constant volatility $a > 0$,then assuming σ and τ are differentiable,

$$\sigma'\tau - \sigma\tau' = a^2. \tag{3.3}$$

By setting $\tau(t) = e^{-\lambda t}$, $\lambda > 0$, and setting the boundary condition $\sigma(0) = 0$, integrating the differential equation gives

$$\sigma(t) = a^2 \sinh(\lambda t) \lambda. \tag{3.4}$$

In this case

$$\begin{aligned} \operatorname{Cov}(r(s), r(t)) &= \operatorname{Cov}(F(s,s), F(t,t)) \\ &= \sigma(s \wedge t)\tau(s \vee t) \\ &= \frac{a^2}{\lambda} e^{-\lambda(s\vee t)} \sinh(\lambda(s \wedge t)) \\ &= \frac{a^2}{2\lambda}\left[e^{-\lambda|s-t|} - e^{-\lambda(s+t)}\right], \end{aligned} \tag{3.5}$$

which shows that the stochastic component of $r(t)$, $X(t,t)$, follows an Ornstein-Uhlenbeck (O-U) process

$$dX(t,t) = -\lambda X(t,t)\,dt + a dW(t), \tag{3.6}$$

where $W(t)$ is standard Brownian motion.

It then follows that

$$dr(t) = d\mu(t,t) - \lambda(r(t) - \mu(t,t))\,dt + a dW(t), \tag{3.7}$$

where

$$\mu(t,t) = \mu(0,t) + \frac{a^2}{\lambda} e^{-\lambda t} \int_0^t \sinh(\lambda v)\,dv \tag{3.8}$$

$$= \mu(0,t) + \frac{a^2}{\lambda} e^{-\lambda t}(\cosh(\lambda t) - 1). \tag{3.9}$$

Upon substituting in $\mu(t,t)$ we get

$$dr(t) = \lambda\left[\frac{e^{-\lambda t}}{\lambda}\frac{d\left(e^{\lambda t}\mu(0,t)\right)}{dt} + \frac{a^2}{\lambda^2}e^{-\lambda t}\sinh(\lambda t) - r(t)\right]dt + a dW(t). \tag{3.10}$$

Providing that $\mu(0,T) \to \mu(0)$ as $T \to \infty$, then it follows that $r(t)$ has mean reverting to the value $\mu(0) + a^2/2\lambda^2$.

Following Kennedy, we define $X(t,T)$ as follows:

$$X(t,T) = W(\sigma(t), \tau(T)). \tag{3.11}$$

where W is the standard Brownian sheet, and the σ and τ functions are translation parameters. (See Walsh [49] for a list of scaling, inversion and translation transformations for the Brownian sheet.) In the case where $\sigma(t) = \sigma^2 t$, $\sigma^2 > 0$, Kennedy has shown that $F(t,T)$ performs a Brownian motion with variance parameter σ^2 and drift

$$\begin{aligned} \mu(t,T) &= \mu(0,T) + \sigma^2 \int_0^T (t \wedge v)\tau(v \vee T)\,dv \\ &= \mu(0,T) + \frac{\sigma^2\tau(t)\left[T^2 - (T-t)^2\right]}{2}. \end{aligned} \tag{3.12}$$

If we will let $\tau(T) = e^{-\lambda T}$, $\lambda > 0$, so that

$$\text{Corr}(F(t,T_1), F(t,T_2)) = e^{-\lambda|T_1-T_2|/2}, \tag{3.13}$$

then we can see that the correlation structure between the forwards will decay to zero as $|T_1 - T_2| \to \infty$. In addition $t^2\tau(t)$ converges as $t \to \infty$, which implies that this model possesses the mean reverting property.

Goldstein has suggested an alternative model for the forward rate

$$dF(t,T) = \left(\mu(t,T) + \lambda \frac{\partial^2}{\partial T^2} F(t,T)\right) dt + \sigma(t,T)\, dW(t,T), \quad \lambda > 0, \tag{3.14}$$

where $W(t,T)$ is the standard Brownian sheet. To generate such a process, the yield curve must be sufficiently smooth to ensure that $\frac{\partial^2}{\partial T^2} F(t,T)$ exists. Goldstein begins with the O-U sheet $dZ(t,T)$, where

$$dZ(t,t) \sim N(0, dt)$$

with the remainder of the field generated by

$$dZ(t,T) = e^{-\rho(T-t)} dZ(t,t) + \sqrt{2\rho} \int_t^T e^{-\rho(T-s)} dz(s,t), \tag{3.15}$$

where $dz(s,t)$ satisfies

$$E[dz(s,t)] = 0, \quad \text{Cov}[dz(s_1,t), dz(s_2,t)] = \begin{cases} ds\,du_1 & \text{if } u_1 = u_2 \\ 0 & \text{otherwise.} \end{cases} \tag{3.16}$$

He then integrates the O-U sheet:

$$\begin{aligned} dV(t,T) &= \sqrt{2\rho^2} \int_{-\infty}^T e^{-\rho(T-u)} dX(t,u)\, du \\ &= e^{-\rho(T-u)} dV(t,t) + \sqrt{2\rho^2} \int_s^T dX(s,u)\, e^{-\rho(T-u)} du. \end{aligned} \tag{3.17}$$

This process gives the following correlation structure:

$$\text{Corr}[dV(t,T_1), dV(t,T_2)] = e^{-\rho\tau}(1 + \rho\tau), \tag{3.18}$$

where $\tau = |T_1 - T_2|$; its Taylor expansion is given by

$$\begin{aligned} \text{Corr}[dV(t,T_1), dV(t,T_2)] &= \left(1 - \rho\tau + \frac{1}{2}(\rho\tau)^2 + \ldots\right)(1 + \rho\tau) \\ &= 1 - \frac{1}{2}(\rho\tau)^2 + \ldots, \end{aligned} \tag{3.19}$$

which is differentiable. However to accommodate $\frac{\partial^2}{\partial T^2} F(t,T)$, its correlation function must be at least second order differentiable (see Adler [1]). Hence Goldstein integrates over $dV(t,T)$,

$$dZ(t,T) = \sqrt{\frac{4\rho^2}{3}} \int_{-\infty}^T du\, dV(t,u)\, e^{-\rho(T-u)}, \tag{3.20}$$

to generate the correlation structure

$$\text{Corr}(dZ(s,T_1), dZ(s,T_2)) = e^{-\rho\tau}\left(1 + \rho\tau + \frac{1}{3}(\rho\tau)^2\right), \tag{3.21}$$

which satisfies this property. The covariance function for $dZ(t,T)$ is defined as follows

$$\begin{aligned} &\mathrm{Cov}\left(dZ\left(t,T_1\right), dZ\left(t,T_2\right)\right) \\ = \; & e^{-\rho|T_1-T_2|}\left(1+\rho\left|T_1-T_2\right|+\frac{1}{3}\rho^2\left|T_1-T_2\right|^2\right) ds. \end{aligned}$$

upon substituting $|T_1 - T_2|$ for τ. Goldstein has shown that this process leads to the following equations for the drift and volatility terms in (3.14):

$$\mu(t,T) = \frac{\sigma^2}{\rho-\kappa}\left(e^{-2\rho(T-t)} - e^{-(\rho+\kappa)(T-t)}\right), \tag{3.22}$$

and

$$\sigma(t,T) = \sigma e^{-\kappa(T-t)} \tag{3.23}$$

respectively. This random field is therefore non-Gaussian. The results of both these models are shown in the appendices. In both models it can be seen that a reduction in the size of σ will lead to an increase in cyclical volatility.

4. A NUMERICAL PROCEDURE FOR THE SIMULATION OF THE TERM STRUCTURE AS A RANDOM FIELD

The approach that has been adopted in this paper, is to model the dynamics of the term structure as a stochastic partial differential equation (SPDE). The procedure we will be employ is based on the method of lines (MOL). Method of Lines is a technique for solving PDEs by reducing them to system of ordinary differential equations (ODEs), usually by application of finite difference or finite element techniques. MOL is attractive as a technique, because it does not change the characteristics of the problem being solved. For example, if the original problem is an initial value problem, then the resulting system of ODEs will also form an initial value problem. Likewise, if the original problem is a boundary value problem, then the resulting system of ODEs will also be a boundary value problem. The reader is recommended to consult Ames [3] for a review on the procedure and its application to solving boundary condition problems in deterministic PDEs.

We will be using MOL in the same way, i.e. it will be used to transform the boundary condition problem for the SPDEs, into boundary condition problems for a system of SDEs. A review of other numeric procedures for solving SPDEs can be found in these papers. We begin by defining an equidistant mesh on $[0, T_{\max}]$:

$$s_n = nh_s, \quad \text{with } n = 0, 1, ..., N+1 \text{ and } h_s = \frac{T_{\max} - T_{\min}}{N+1}.$$

The SPDEs defined in the last section, now becomes systems of SDEs. For the first model:

$$dF(t,s_n) = \mu(t,s_n)\,dt + \sum_{n=1}^{N} \sigma(t,s_n)\,dW(t,s_n), \quad n = 1,2,...,N, \tag{4.1}$$

where

$$\mu(t,s_n) = \mu(0,s_n) + \frac{\sigma^2\tau(t)\left[s_n^2 - (s_n-t)^2\right]}{2} \tag{4.2}$$

with $\mu(0, s_n) = F(0, s_n)$ and

$$\sigma(t, s_n) = \sigma^2 t, \quad \sigma^2 > 0. \tag{4.3}$$

The second model is given as follows:

$$\begin{aligned} dF(t, s_n) &= \mu(t, s_n)\,dt + \lambda\left(\frac{F(t, s_{n+1}) - 2F(t, s_n) + F(t, s_{n-1})}{h_s^2}\right) dt \\ &\quad + \sum_{n=1}^{N} \sigma(t, s_n)\,dW(t, s_n) \end{aligned} \tag{4.4}$$

for $n = 1, 2, ..., N$, where

$$\mu(t, s_n) = \frac{\sigma^2}{\rho - \kappa}\left(e^{-2\rho(s_n - t)} - e^{-(\rho+\kappa)(s_n - t)}\right), \tag{4.5}$$

and

$$\sigma(t, T) = \sigma e^{-\kappa(T-t)}, \tag{4.6}$$

with

$$\sigma, \kappa > 0, \ 0 < \rho < 1, \text{ and } \lambda > \frac{\kappa}{\kappa^2 + \rho^2}. \tag{4.7}$$

The boundary values for these systems are given as follows:

$$\begin{aligned} r(T, T) &= 0, P(T, T) = 1, \\ P(t, T) &> 1, \quad \forall\, 0 \le t \le T, \\ r(t, T) &> 0, \quad \forall\, 0 \le t \le T. \end{aligned}$$

We note that each equation in the system is in the general form of a SDE driven by a d-dimensional Wiener process:

$$dy = f(t, y)\,dt + \sum_{i=1}^{d} g_i(t, y)\,dW_i(t), \quad y(t_0) = y_0,$$

where $W_i(t)$ is a standard Wiener process whose increment $\Delta W_j(t) = W_j(t + \Delta t) - W_j(t)$ is $\Delta W_j(t) \overset{\text{iid}}{\sim} N(0, \Delta t)$. We define an equidistant grid on $[0, s_n]$ for each of the N equations which compose the system:

$$t_m = m h_t, \quad \text{with } m = 0, 1, ..., M+1 \text{ and } h_t = \frac{t_{\max_n} - t_{\min_n}}{M+1}.$$

We note $t_{\min_n} = 0$ for all $n = 1, ..., N+1$, while $t_{\max_n} = s_n$. We now solve each of these systems of equations by using an Euler method based on the backward-difference approximation – this is the backward Euler-Maruyama method, which is an implicit Euler procedure. This contrasts with work by Tian et al. [47] which also uses MOL to an SPDE model in stochastic hydrology. The approach employed by Tian et al. uses a semi-implicit Euler procedure based on the method of Burrage and Tian [8, 9]. We suggest that our procedure has an intuitive appeal in that the discretization of the space converts the random field model to a vector process, containing information about the forward rates of bonds of different maturities. While the use of the backward Euler method is appropriate for solving the problem of the contingent claim on the bond.

Applying the backward Euler-Maruyama to (4.1) and (4.4) gives:

$$F(t_{m-1}, s_n) = F(t_m, s_n) - \left(\mu(t_m, s_n) \Delta t + \sum_{n=1}^{N} \sigma(t_m, s_n) \Delta W(t_m, s_n) \right), \tag{4.8}$$

where

$$\mu(t_m, s_n) = \mu(0, s_n) + \frac{\sigma^2 \tau(t_m) \left[s_n^2 - (s_n - t_m)^2 \right]}{2} \tag{4.9}$$

with $\mu(0, s_n) = F(0, s_n)$ and $\tau(s_m) = e^{-\lambda s_n}$ and

$$\sigma(t_m, s_n) = \sigma^2 t_m, \quad \sigma^2 > 0. \tag{4.10}$$

for the first model. The second model is given as follows:

$$\begin{aligned} F(t_{m-1}, s_n) &= F(t_m, s_n) - \mu(t_m, s_n) \Delta t \\ &\quad -\lambda \left(\frac{F(t_m, s_{n+1}) - 2F(t_m, s_n) + F(t_m, s_{n-1})}{h_s^2} \right) \Delta t \\ &\quad - \sum_{n=1}^{N} \sigma(t_m, s_n) \Delta W(t_m, s_n), \end{aligned} \tag{4.11}$$

where

$$\mu(t_m, s_n) = \frac{\sigma^2}{\rho - \kappa} \left(e^{-2\rho(s_n - t_m)} - e^{-(\rho+\kappa)(s_n - t_m)} \right), \tag{4.12}$$

and

$$\sigma(t_m, s_n) = \sigma e^{-\kappa(s_n - t_m)}, \tag{4.13}$$

with σ, κ, ρ, and λ as defined earlier, for $n = 1, 2, ..., N$ and $m = 1, 2, ..., M$ respectively. The boundary conditions are as previously stated.

We will model the noise term by a white noise process $\eta_{t_m, s_n} \sim N(0, 1)$, hence by the standard normal transformation

$$\Delta W(t_m, s_n) = \frac{1}{\sqrt{\Delta t_m}} \eta_{t_m, s_n}. \tag{4.14}$$

This is arrived at by using the spectral decomposition of the spatial covariance function (1.10) defined for the Brownian sheet $W(t, T)$ is given by

$$Cov(W(t_1, T_1), W(t_2, T_2)) = c(t_1 \wedge t_2, T_1, T_2). \tag{4.15}$$

As noted earlier this function c will be symmetric in t_1 and t_2 and positive semi-definite in (t_1, T_1) and (t_2, T_2). In addition to ensure the continuity of the random field the covariance function must also be bounded, hence it will have a spectral decomposition

$$c(t_1 \wedge t_2, T_1, T_2) = \sum_{m=1}^{\infty} \lambda_m f_m(T_1) f_m(T_2), \tag{4.16}$$

where λ_m and $f_m(T)$ are respectively the ith eigenvalue and eigenvector of the spatial covariance function. Following Ghanem and Spanos [45] this will imply that the Brownian sheet can be approximated by the following truncated Wiener-Itô chaos expansion

$$W(t,T) = \sum_{m=1}^{N} \sqrt{\lambda_m}\eta_{t_m s_n}(t) f_m(T) \tag{4.17}$$

where η_{t_m,s_n} is defined at the beginning of the paragraph.

The results of the simulation for both of these models are contained in the Appendix. The simulations for these models were completed using MATLAB 5.2.

For both models, it was found that as the coefficient σ increases (i.e. as the volatility increases) with respect to time, there is a reduced incidence in cyclical behaviour which would reflect a co-movement in the business cycle. In turn, lower volatility provides a cyclical pattern to the term structure along the temporal dimension. The coefficient σ controls the diffusion of noise across time. These results suggest that as the level of volatility increases, this noise serves to dampen the incidence of cyclical effects recorded in the term structure.

It is also well known that noise can be used as a techniques for stabilizing partial differential equations; Kwiecinska [31] has presented an application of this to stabilizing the dynamics of the heat equation. The implications of this to business cycle modelling, with respect to the arguments of Working [51], Slutsky [44] and Magill [37] is that when the stochastic environment is noisy, it acts to dampen the dynamics of the capital accumulation process, i.e. the dynamics movements of asset prices. As the business cycle is the dual process of the dynamics of capital accumulation, this may explain a reduced incidence in business cycle activity which is associated with fluctuations in the term structure.

Another implications for business cycle modelling is the effect of varying the λ coefficient in the Goldstein model. Apart from this one coefficient λ, which governs the degree of curvature in the yield curve, the Kennedy and Goldstein models are quite similar in structure. This is also confirmed if we examine the simulations of the Goldstein model in which we keep λ constant and vary σ: as the coefficient σ is reduced the degree of induced cyclical behaviour increases. However, when holding σ constant and varying λ we see that as the λ coefficient becomes larger the slope of yield curve switches sign. This yield curve inversion related to correlation between bonds of different maturities and has important implications for both interest rate policy, and when taken in conjunction with the Brace-Gatarek-Musiela [7, 40] model of long bonds, the pricing of options like caps and caplets.

As we stated in the introduction a natural extension of our simulation procedure is to simulated likelihood and the econometric estimation continuous spatio-temporal panel data models. Particularly since the mathematics behind random fields was initially developed in applications using ANOVA (see Walsh [49] for an interesting account of this connection). As stated earlier, the most immediate application of this methodology would for the estimation of parameters for term structure and bond pricing models of interest rates that use SPDEs to model their underlying behaviour. Calibration procedures based on our yield curve simulations reduce the need for recalibration of yield curve models by allowing for spatio-temporal correlation. However beyond this, these simulated likelihood techniques would eventually provide the basis for a mean-square optimal control algorithm for SPDEs. This is essentially our long-term research program.

5. Conclusion and Discussion

In this paper we have presented simulation results of two theoretical term structure models. As the yield curve is based on forward rates which are modelled as a random field, we have introduced a new numerical technique for modeling the evolution of a process on a random field.

As was stated earlier, this numerical technique uses the method of lines to convert the random field into a random vector process. Our method is considerably simpler than many of the existing techniques in use for simulating SPDEs and also benefits from drawing from a well defined collection of finite difference procedures for simulating SDEs.

The results of these simulations suggest that as σ decreases, the term structure will generate a surface which resembles the fluctuations of the business cycle. These results also suggest that as the level of volatility increases, this noise serves to dampen the incidence of cyclical effects recorded in the term structure. The co-movement of the term structure of interest rates with fluctuations in the business cycle suggests a possible application of this type of model to macroeconomic dynamics.

When we examined the simulations of the Goldstein model in which there is coefficient λ controlling the curvature of the yield curve, we see that when we hold σ constant and increase λ, the slope of yield curve switches sign. This yield curve inversion related to correlation between bonds of different maturities and has important implications for both interest rate policy, as well as for the pricing of options on long bonds, like caps and caplets.

We have also state that random field term structure models offer an improvement on the yield curve models based on SDEs, because they circumvent the problem of recalibration which is associated with these models. It was stated that while Markov switching or Bayesian modelling methodologies are effective for modelling interest rate term structure, their effectiveness is due to incomplete information.

Taken together with the simulation results, this would suggest that an econometric modelling approach that incorporates a spatial-temporal aspect, would be useful for modelling the evolution of the term structure of interest rates. The development of an estimation method for this type of model, is likely to require the simulation of SPDEs as part of the estimation process. As we stated in the last section, we anticipate that this will lead us to mean-square control procedure for SPDEs.

6. Appendix: Simulation results for term structure model

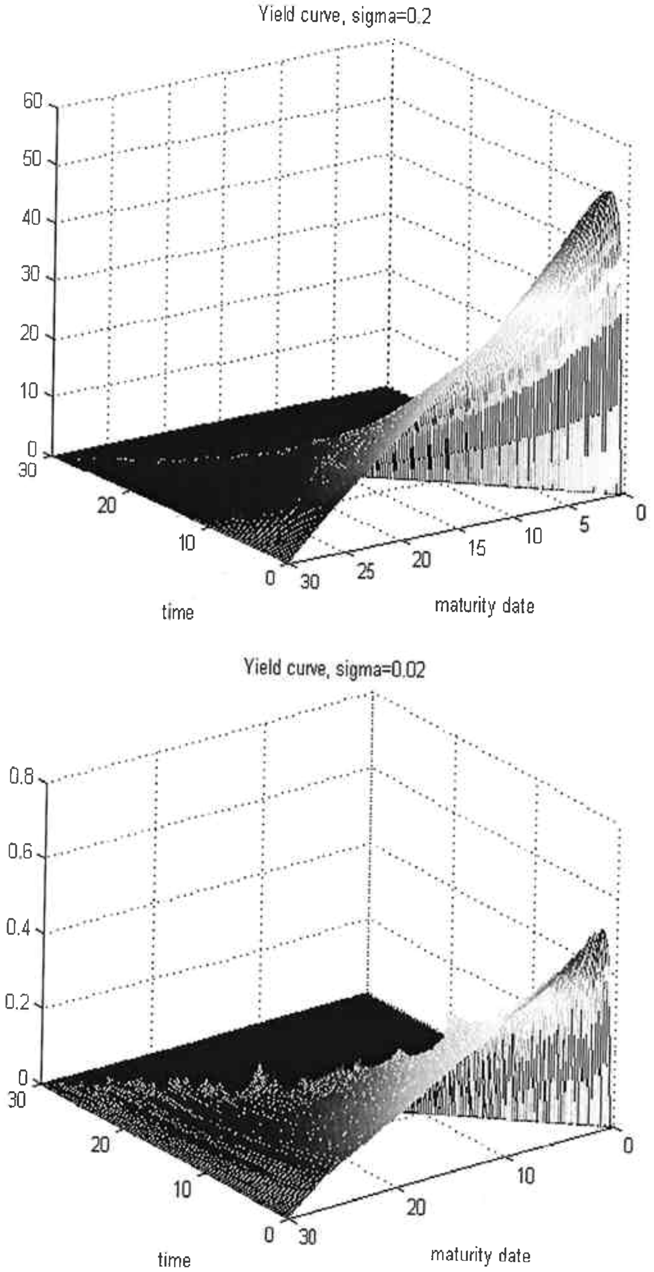

Figure 1. Term structure for the Kennedy model varying sigma

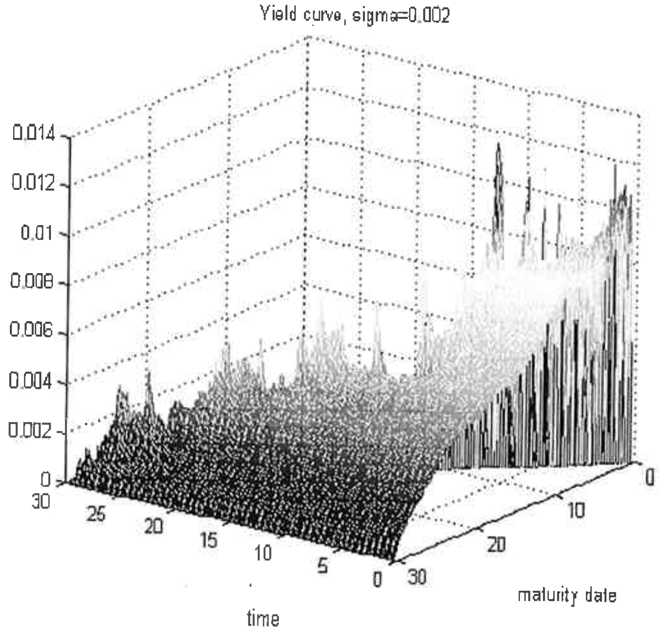

FIGURE 2. Term structure for the Kennedy model varying sigma

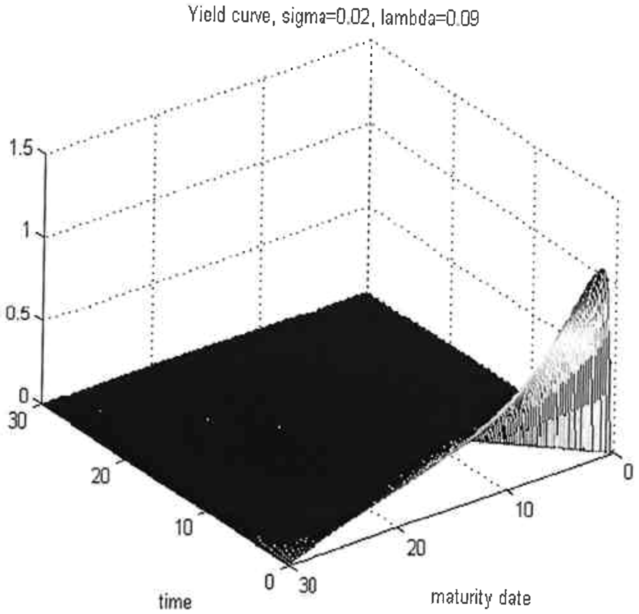

FIGURE 3. Term structure for the Goldstein model varying lambda

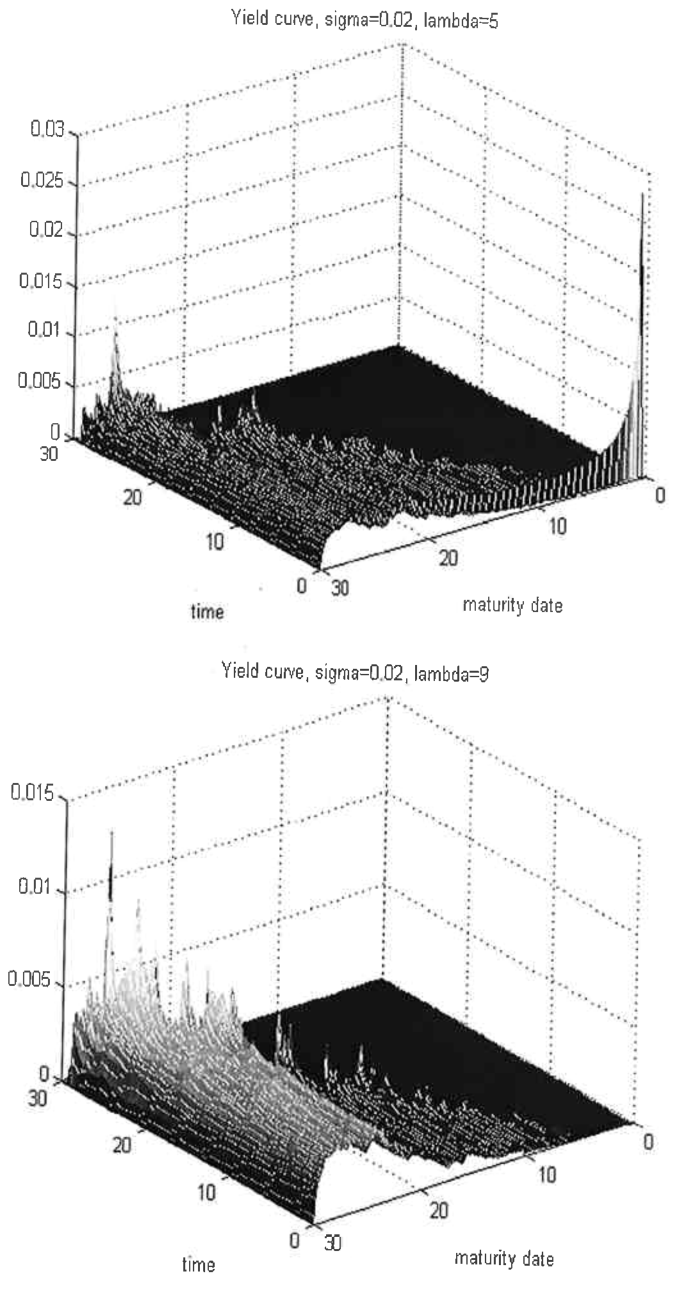

FIGURE 4. Term structure for the Goldstein model varying lambda

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