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Laser trapping of non-spherical particles

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Abstract

Optical trapping, where microscopic particles are trapped and manipulated by light [1] is a powerful technique. The single-beam gradient trap (also known as optical tweezers) is widely used for a large number of biological and other applications [2, 3].

The forces and torques acting on a trapped particle result from the transfer of momentum and angular momentum from the trapping beam to the particle. Despite the apparent simplicity of a laser trap, with a single particle in a single beam, exact calculation of the optical forces and torques acting on particles is difficult, and a number of approximations are normally made. Approximate calculations are performed either by using geometric optics, which is appropriate for large particles, or using small particle approximations. Neither approach is adequate for particles of a size comparable to the wavelength. This is a serious deficiency, since wavelength sized particles are of great practical interest because they can be readily and strongly trapped and can be used to probe interesting microscopic and macroscopic phenomena.

The lack of suitable theory is even more acute when the trapping of non-spherical particles is considered. Accurate quantitative calculation of forces and torques acting on non-spherical particles is of particular interest due to the suitability of such particles as microscopic probes. These calculations are also important because of the frequent occurrence of non-spherical biological and other structures, and the possibility of rotating or controlling the orientation of such objects.

The application of electromagnetic scattering theory to the laser-trapping of wavelength sized non-spherical particles is presented.

1 Introduction

Optical trapping, the trapping and manipulation of microscopic particles by a focussed laser beam, is a widely used and powerful tool. The most common optical trap, the single-beam gradient trap (optical tweezers) consists of a laser beam focussed by a lens, typically a high-numerical aperture

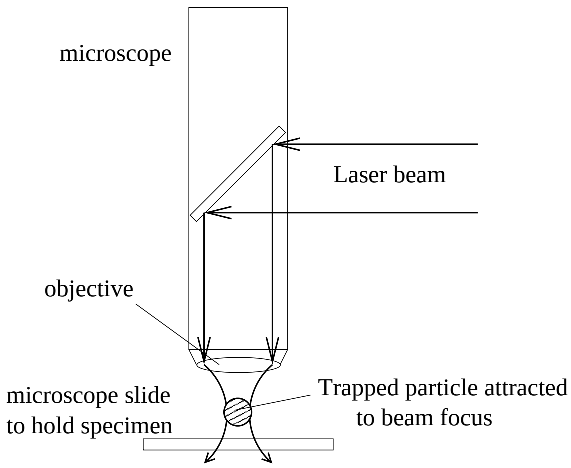

Figure 1: Typical laser tweezers.

microscope objective, with the same microscope being used to view the trapped particles (see Fig. 1). The trapped particle is usually in a liquid medium, on a microscope slide.

Although simple trapping and manipulation are sufficient for many applications, the use of optical trapping for quantitative research into physical, chemical, and biological processes, typically using a laser-trapped particle as a probe, requires an accurate quantitative theory of optical trapping. The minimisation of damage to trapped biological specimens also indicates the desirability of using theoretical results to design traps in order to minimise the power absorbed by the trapped particle.

The concept of optical trapping is based on a gradient force causing small particles to be attracted to regions of high intensity in a tightly focussed laser beam. Other optical forces, due to absorption, reflection, and scattering are termed scattering forces. Both the gradient and scattering forces result from the transfer of momentum from the trapping beam to the particle. Optical torques can also be produced by the transfer of angular momentum from the beam. This can result from birefringence or scattering with a non-zero angular component relative to the particle centre [4]. However, despite this apparent simplicity, exact calculation of the optical forces is difficult, and a number of approximations are usually made. Approximate calculations use geometric optics for large particles (radius $r > 5\lambda$), or assume that a small particle ($r < \lambda/2$) acts as a Rayleigh scatterer or a point-like polarisable particle. This leaves a large range of particle size for which no adequate theory exists. This is unfortunate, since particles of a size comparable to the wavelength are of great practical interest because they can be readily and strongly trapped, and used to probe interesting microscopic and mesoscopic phenomena.

An accurate quantitative theory of optical trapping of wavelength-scale particles is highly desirable. Recent theoretical efforts have individually eliminated some of the deficiencies due to the various approximations usually used [5, 6, 7], but there still exists no general correct theory.

The lack of suitable theory is even more acute when the trapping of non-spherical particles is considered. Non-spherical particles are of particular interest due to their suitability for use as microscopic probes, and the frequent natural occurrence of non-spherical biological and other structures. The possibility of rotating or controlling the orientation of non-spherical particles greatly extends the range of manipulation possible in an optical trap.

2 Calculation of forces

An optical trap consists of a trapping beam incident on a particle. The incident beam is typically a strongly focussed Gaussian beam, but other types of beams, such as Laguerre-Gaussian (doughnut) beams can be used, or multiple trapping beams can be used. The incident field carries momentum and angular momentum, and if scattering by the particle changes the momentum or angular momentum, there will be a corresponding force or torque acting on the particle. This optical force is used to trap the particle.

If the trapping field and the scattered field produced by the particle are known, the momentum of each field can be calculated since

$$\mathbf{P} = \epsilon_0 \int \mathbf{E} \times \mathbf{B} d^3x \tag{1}$$

Similarly, the angular momentum can also be found:

$$\mathbf{L} = \epsilon_0 \int \mathbf{x} \times (\mathbf{E} \times \mathbf{B})\, d^3x \tag{2}$$

The optical force and torque can be calculated from the changes in momentum and angular momentum. Since, usually, the trapping field is known beforehand, it is only necessary to calculate the scattered field.

A general theory of laser trapping will be applicable to

- all particle compositions - transparent, absorbing, reflective, birefringent, conductive, non-uniform
- all particle shapes - spherical, non-sperical, complex regular shapes, irregular
- all trapping beams - Gaussian beams, Laguerre-Gaussian (doughnut) beams, multiple beams,

and will be correct for large, small, and wavelength-scale particles.

The scattering in a system involving wavelength-scale particles must be dealt with in terms of electromagnetic theory, i.e. the Maxwell equations must be solved. The field can be assumed to be monochromatic and the amplitudes constant with respect to time with no loss of generality. The transparency, absorptivity, reflectivity, and birefringence of the particle are all readily dealt with by considering the electromagnetic properties of the particle.

Where simple techniques exist for calculating the scattering of a plane wave by the type of particle being considered, the decomposition of the trapping beam into a plane wave spectrum [8] suggests the following algorithm:

a. Decompose trapping beam into a plane wave spectrum

b. Calculate the scattering of a plane wave for all (needed) orientations of the particle

c. Combine the scattered fields of the plane wave components to find the total scattered field

d. Find the optical force and torque from the momentum and angular momentum of the trapping and scattered fields

For geometrically simple particles, eg, rotationally symmetric particles such as spheroids and cylinders, the theory is well known, and efficient numerical techniques are available. For the limiting case of non-birefringent spherical particles, only one orientation needs to be considered. The computation of scattering for varying orientations needs to be performed only once for any given particle, so this algorithm is ideal for repeated calculations of force acting on the same particle, for example, at varying positions within the beam. Therefore, this is an ideal method for the calculation of force and torque as a function of position with a trap.

For particles of arbitrary shape and composition, the scattering, and thus the optical force and torque can be found using a finite difference or finite element [9] method to solve the Maxwell equations. Since the illumination is monochromatic, and the particle can be assumed to be in equilibrium at all times, FDFD methods are ideal, as long as the calculation is not excessively large. A plane wave representation of the trapping beam can still be used in this case, or, particularly if the calculation will only be performed at a few positions within the beam, the EM field of the total beam can be used [10].

3 Conclusion

Electromagnetic scattering theory can be used to calculate the forces and torques acting on a laser-trapped particle. The usual approximations used for optical trapping calculations (geometric optics or small particle approximations, paraxial beams, etc) that serious limit accuracy for wavelength-scale particles can be eliminated.

For particles for which efficient and fast computational techniques exist for the calculation of scattering, force and torque calculations are straightforward, and can be readily and efficiently used to calculate the force and torque as a function of the position and orientation of the particle within the trap.

Wavelength-scale non-spherical particles are commonly encountered in practical applications of laser trapping, so accurate and efficient techniques to calculate forces and torques for such particles are of particular interest.

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